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Application of machine learning in the process of commander decision support in the military fuel distribution system

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Abstract

Providing energy to troops requires maintaining optimal fuel levels across all management stages, especially within Garrison Support Units and Regional Logistic Bases. The article examines the fuel distribution system supported by a program that predicts commanders' actions using input data from subordinate units. To aid decision-making, Garrison Support Units implemented neural network variants to model logistical activities, training, peacetime operations, or combat, and segment fuel supply accordingly. The Neural Network Toolbox from MATLAB (MathWorks) was used for computations. The study presents the Garrison Support Units operational assumptions, the role of commanders as agents, and factors affecting fuel distribution. It also outlines the development of the Logistic Decision Support System dashboard, which enables entering decision variables, neural network coefficients, and weights to forecast fuel consumption and plan future operations based on environmental and operational data. The article includes MATLAB simulation results, analysing neural network algorithms and neuron counts per layer to determine the most effective configuration for decision-making optimisation. Results show that the Bayesian regularisation algorithm achieved the lowest mean square error across all data sets and the highest prediction accuracy measured by the root mean squared error. The regression coefficient confirmed a strong correlation between predicted and actual outcomes, demonstrating the Bayesian regularisation algorithm's effectiveness in supporting logistical fuel management decisions.

Keywords: Machine learning, Root mean squared error; Bayesian regularisation; Decision-making process support; Energy in military logistics.

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1. Introduction

The increasing complexity of fuel supply chains and the pressure for resilience, efficiency and environmental sustainability mean that artificial intelligence and machine learning (ML) methods are rapidly being adopted in logistics (including military). In the literature of recent years, there has been a clear shift from classic optimisation problems to predictive analytics, integration of heterogeneous data, and models supporting short-term operational decisions. This is particularly important in the context of the military fuel distribution system, where the commander's decisions are burdened by demand uncertainty, infrastructure constraints, and spatio-temporal dynamics. The concept of a modular Logistic Decision Support System (LDSS) adopted in this paper, based on multilayer neural networks and the selection of learning algorithms such as Bayesian regularisation, Levenberg-Marquardt (LM), Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton, or resilient backpropaga-

Nomenclature

BR – Bayesian regularisation
GSU – Garrison Support Unit
LDSS – Logistic Decision Support System
MATLAB– Matrix Laboratory
ML – machine learning
MSE – mean square error
R – regression coefficient
RLB – Regional Logistic Base
RMSE – root mean squared error

tion (RP), remains consistent with the directions observed in research from practical implementations of short-term prediction to critical reviews of data quality, standardisation and model reliability.

The article aims to research the application of an artificial intelligence algorithm to support decision-making processes in the management of fuel resources, based on the adopted model of elements of the military logistics system. When starting the research, previous publications [1−4] characterised the logistics space within a selected scope. The need to develop solutions supporting the decision-making process of the commander (decision-maker) based on machine learning algorithms was indicated, including Broyden–Fletcher–Goldfarb–Shanno quasi-Newton, Levenberg-Marquardt, Bayesian regularisation and the resilient backpropagation algorithm.

The innovative nature of the article is manifested, among other things, by the fact that variables that have not been collected in the IT logistics system so far are indicated, as well as the method of developing the architecture of the LDSS program for implementation. The author indicated a number of dependent variables that he detected while working with the data that were analysed, namely the indicated fuel level in the Garrison Support Unit (GSU), the intensity of operations, field data, meteorological data or variables illustrating the capabilities of GSU.

The proposed model of support for the decision-making process in the fuel distribution system is based on a neural network, for which the basic input parameters and initial states of the system have been indicated, limited only to fuel distribution. The model of the modular multi-layered one-way neural network (MNN) seems to be the solution with a small and acceptable error to provide courses of action for the management staff in the indicated positions of the military logistics system. Nevertheless, in order for the indicated information technology (IT) solution to be effective, the artificial intelligence (AI) system must be taught through the experience of decision-makers and historical data, in order to present several optimal variants of fuel resource management. Finally, the thesis in which the reinforcement learning (RL) model improves the quality and accelerates the process of developing an appropriate variant of the decision-maker's action was analysed.

Bibliometric studies of logistics in Saudi Arabia [5] synthetically show the global shift towards sustainable logistics, machine learning, and the integration of technology (AI, blockchain) into traditional supply chain paradigms. The authors, analysing over 7.6 thousand publications, identify the growing importance of chain resilience and international cooperation as an accelerator of innovation. These conclusions are relevant for military logistics: the development of predictive and decision-making capabilities in the fuel system requires combining classic procedures with machine learning (ML) models and connecting multi-domain data (operational, environmental, infrastructural), which directly corresponds to the concept adopted in this paper of extending the input vectors of the network to include meteorological, terrain and operational factors.

A strong practical trend is represented by the work of Eichenseer et al. [6], in which a data-driven ML model for a five-day forecast of the number of "delivery positions" for workforce planning in logistics was developed and validated. The model, tested in the company, exceeded both practical expert forecasts and auto ML systems, especially in the short term. The value of models tailored to the specifics of the process (custom ML) in relation to universal tools is a key here from the perspective of the military GSU. This approach reinforces the decision to use a specialised multilayer perceptron (MLP) architecture in LDSS and to select training algorithms for generalisation stability with limited samples and strong operational variability. A five working days forecast horizon (week) seems to be a natural "decision window" for fuel distribution planning and personnel dispatch.

For this article, several literature items were analysed, where, among others, Nguyen et al. [7] present a critical review of ML models for estimating propulsion power and fuel consumption of marine vessels, an area inherently related to fuel and energy (power) prediction (FEP) and exposed to environmental disturbances. The authors emphasise the need for "data-centric AI": standardisation of metrics (beyond mere accuracy), benchmarks, and policies for maintaining trustworthiness in models. For military applications, this means the need to:

a) unambiguous definition of quality measures (root mean squared error/mean square error (RMSE/MSE), regression coefficient (R), but also resistance to regime changes and out-of-distribution),
b) monitoring of procedures and re-training,
c) documenting data and model decisions (auditability), which in this article is reflected in the adopted set of metrics and comparisons of BR/LM/BFGS/RP algorithms and the conclusion about the advantage of research and development (R&D) in terms of generalisation.

The cost and decision perspective in commercial logistics was developed by Yaiprasert and Hidayanto [8], who used ensembles of ML methods to explore cost strategies on simulated data. Despite the synthetic nature of the collection, the paper shows two lessons useful for LDSS:

a) ensembles increase predictive resilience to data variance and heterogeneity in operational conditions,
b) It is possible to use simulations to "seal" sparse regions of the state space, which is important when real data (e.g., intensity of activities, procedural constraints) are not yet fully collected in departmental systems.

This approach can complement a trained MLP network with a simulation component or an MLP + ensemble hybrid for critical inventory thresholds.

Zhou et al. [9], on the other hand, propose a three-stage geoinformation model, geospatial information system (GIS), multiple-criteria decision making / multiple-criteria decision analysis (MCDM), or social network analysis (SNA) network, for the selection of locations for urban green logistics centres. Despite the civilian context, the construction of the method (combining environmental, economic, technological, and social criteria with network analysis) is analogous to the needs of military planning for the deployment of depots and GSU in operational space.

Finally, a broad overview of sustainable aviation fuel (SAF) supply chains by Liang et al. [10] highlights that the energy transition in transportation requires a combination of mathematical programming, ML and multi-criteria optimisation. The identified barriers: cost, complexity of multi-stage production, instability of raw material supply and regulatory discrepancies, translate into military implications:

a) the need to take alternative fuels scenarios into account in planning (compatibility, availability, chain risk),
b) expansion of the decision-making module with multi-criteria functions (cost-risk-environmental footprint-operational readiness),
c) tracking policies and standards, which support the approach to building an LDSS decision-making dashboard with weights and ratios reflecting operational priorities.

In conclusion, the literature confirms the three pillars adopted in this work. First, short-term prediction on operational data (Eichenseer et al. [6]) is a key component of resource planning and should be the core of the GSU commander's support module; the choice of training algorithms with good generalisation with limited samples (Bayesian regularisation) is methodologically justified and, in our experiments brings the best compromise of accuracy/robustness, which has been demonstrated empirically. Secondly, the integration of spatial and multi-criteria data (Zhou et al. [9]) indicates that the extension of LDSS to include the GIS/MCDM/SNA component will support decisions on inventory manoeuvre and supply priorities under conditions of infrastructural and environmental constraints. Third, the trend towards sustainability and resilience (Alasmari and Alzahrani [5], Liang et al. [10]) requires that decision-making tools take into account both performance metrics (RMSE, MSE, R) and chain risk and environmental impact criteria, while maintaining the principles of "data-centric AI" and model transparency (Nguyen et al. [7]). In this context, the LDSS module, which is being developed and is based on MLP and enriched with regularisation, validation and monitoring mechanisms, is a way to authenticate operational recommendations for the GSU decision-maker, in accordance with the literature.

2. Description of the research problem

The research focused on mapping the logistics space with which agents can interact with the indication of states and transitions between individual layers of the network dedicated to the selected GSU. The agent is the commander of the selected GSU (in our solution, we assume only one decision-making level for simplicity). He observes the changing situation of fuel security in a given area of responsibility, introducing GSU into the states through actions (decisions). The operational situation in a given area of responsibility complements the environment of operations. Actions that can be taken by the decision-maker are: receiving, dispensing, or withholding the dispensing of fuel.

The decision-maker performs actions to maintain the fuel at the required level in the unit's tanks. The agent has limited transportation resources, is constrained by the load capacity of the supplied units, and must also deliver fuel within the timeframe specified in the demand of a given military unit. The time in the proposed model is discrete, and a training set based on data from five years of GSU operation was used for prediction. Then, 70% of the data was used for training the network; 15% of all data was used to validate the network for generalisation and to stop the training process before overfitting occurred, and a further 15% was used as a test set to independently assess the network's ability to generalise.

The time is discrete, and one week of fuel management in the GSU has been taken as the time step. The prediction of fuel distribution and decision-maker actions was set ten months ahead. The purpose of the program in the MATLAB environment is to train the neural network in such a way that it is able to reduce the mean square error of the deviation of real values from those predicted in the network training process as much as possible. For the purpose of studying the degree of mapping of the prediction results of the GSU input variables, the parameters of the neural network were introduced, such as the number of neurons in the network layers and the function according to which the training was conducted. As a parameter that was predicted, it was a series of fuel level data in the GSU. In subsequent tests for the LDSS commander's desktop, the number of predicted parameters (e.g. intensity of operations, number of available personnel, availability of equipment, etc.) should be increased simultaneously on the basis of the input data. The neural network learning process involves appropriately adjusting the weights between neuronal connections in the network layers to minimise errors and improve the quality of predictions. Reducing prediction errors can be achieved by using a feedback loop (Fig. 1) backpropagation, i.e. comparison of the obtained results with the desired values. The error calculated in this way leads to a weight correction to minimise this error in subsequent steps (iterations). These processes take place in the so-called epochs, and each epoch is a single passage through the entire set of learning data. The most important steps in the network training process are: initialisation of weights, feed forward, error calculation, backpropagation, updating of weights, and repeating the process in steps from 2 to 5 over many epochs. Thanks to these measures, the network becomes more and more accurate ("learns"), and its predictions are closer to the actual results. Finally, there is a process of completing learning after reaching a certain criterion, e.g. after reaching a satisfactory level of accuracy, in our case of mapping the prediction results, the parameters of the neural network were changed, such as: internal delays, feedback delay, number of epochs, the function according to which the training was carried out, and the size of hidden lay-

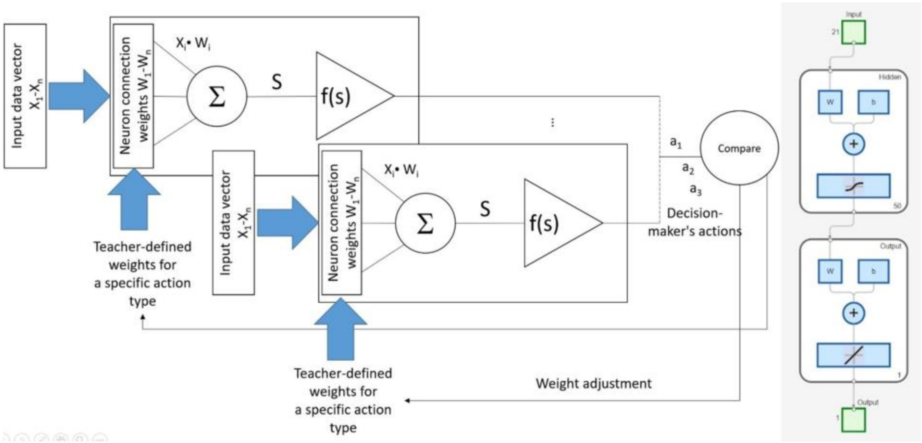

Fig. 1. Neural network model to predict fuel consumption for the selected GSU. The function according to which the network was trained was: Bayesian regularisation, Levenberg-Marquardt, BFGS quasi-Newton and resilient backpropagation.

ers. Below is a brief description of four algorithms used in the study.

2.1. Bayesian regularisation

A modification of a classical algorithm with feedback propagation that optimises both model fit and complexity by regularising weights. In this algorithm, the error function has two components – a matching error (MSE) and a penalty for heavy weights. The weights and regularisation parameter are estimated using Bayesian methods. It is known for high resistance to overfitting and excellent generalisation, especially with a large number of neurons, without needing separate cross-validation. However, it requires longer training times than non-regularised methods. This approach is chosen as a benchmark for the quality of generalisation. It is particularly effective when using many neurons and dealing with the risk of overtraining, which is important in studies with a network of 50 neurons in the hidden layer. It offers automatic selection of the regularisation parameter and reduces manual hyperparameter tuning [11].

2.2. Levenberg–Marquardt

An optimisation algorithm that is a hybrid of the Gauss–Newton method and gradient fall adjusts the learning step by switching seamlessly between the fast Gauss–Newtonian confluence (near the minimum) and the stability of the gradient method (far from the minimum). Its advantages are very fast convergence for small and medium-sized networks and high accuracy with a small number of neurons. High memory requirements. For large networks, it can become unstable or very slow. Widely recognised as the fastest algorithm for training small and medium-sized MLP networks. It handles regression problems and achieves low error in a small number of epochs very well. Ideal for comparison with BR in terms of trade-off of learning time vs. accuracy [12].

2.3. BFGS quasi–Newton

An advanced optimisation algorithm that approximates the inverse Hessian matrix (second-order derivatives) without fully calculating it. Updates the Hessian approximation in each iteration, using gradient and weight changes. Its advantages include faster convergence than a pure straight gradient and good results for medium-sized networks and well-conditioned problems. However, it may lose efficiency with very large networks. High memory requirements. It was chosen as a classic, robust optimisation algorithm, allowing us to assess how it performs against newer and more adaptive methods [13].

2.4. Resilient backpropagation

A variation of backpropagation that ignores the size of a gradient based only on its character. A very stable algorithm in conditions where the gradient is scaled badly. Good for problems with large differences in input values. It does not use the gradient size information, so it can reach very low error values more slowly. A simple but effective gradient sign method. It is in contrast to methods that require calculations of the Hessian matrix or regularisation. In studies, it acts as a benchmark of stability at different numbers of neurons [14].

An important gap highlighted by the review authors (Nguyen et al. [7]) is the insufficient standardisation of data and benchmarks, and the lack of widely accepted measures of model confidence. In military conditions, this gap is reinforced by the limited availability of structured data on the intensity of operations, procedural constraints, or environmental parameters, which has also been identified in this study, and directly motivates the expansion of the range of collected input variables (21+ features) and the procedure for dividing the sets into training/validation/test. In addition, the literature indicates the potential for reinforcement learning (RL) for problems in which decisions and rewards are spaced over time; combined with an MLP network trained on historical data, RL can support fuel dispensing/halting/receiving tactics, balancing exploration and exploitation in a dynamic environment, a direction that has been outlined as the next stage in LDSS development.

In light of the above, the GSU commander's decision support model proposed in the paper using the MLP architecture, BR/LM/BFGS/RP algorithms, an extended set of input features, and a dashboard module with weights and coefficients is in line with the best practices identified in the literature, and responds to the key challenges:

a) short-term prediction of demand and inventory levels,
b) integration of multi-domain data (operational, environmental, infrastructural),
c) transparency and standardisation of model evaluation,
d) possibility of further hybridisation with RL and ensemble methods for resistance to data variability.

The states of the GSU logistics system were determined on the basis of historical observations from 2019−2023. It was assumed that the logistics system worked optimally and the agents acted in accordance with the adopted strategy. The number of times GSU was in a situation where the fuel level for GSU was adequate (Fig. 2) and was counted week by week.

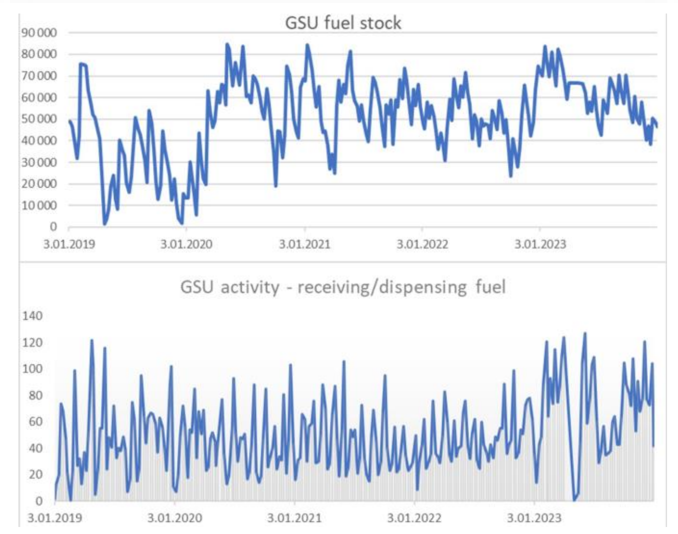

Fig. 2. Historical data on the state of diesel fuel levels and activity (fuel receiving/dispensing) in GSU in 2019−2023, every week (260 weeks – as of the first day of the week).

Using the collected historical data, it is possible to analyse changes in fuel levels for a selected fuel composition, GSU depot or a single storage facility. Changes in fuel levels reflect many factors, such as the number and size of units supplied in a given area, their operational and training activity, or the ability to maintain and restore stocks. Therefore, the detailed information contained in the database of the Integrated Multi-Level IT System of the Ministry of National Defence (ZWSI RON – ”Zintegrowany Wieloszczeblowy System Informatyczny Resortu Obrony Narodowej”) on the change in the level of fuel stocks in the actual area and time for specific military units (MU) may constitute sensitive military data. While maintaining the overt nature of the study for analytical research, an "artificial" model was adopted, built from existing elements of the system, but in fact coming from various unrelated fuel supply regions. This model consists of a superior fuel depot in the Regional Logistic Base (RLB), GSU and three supplied MUs of different sizes: brigade, regiment and battalion (Fig. 3).

The task of GSU is to dispense fuel to MU in the amount and time in accordance with the demand. Of course, there are many factors affecting the quality and certainty of this task [1]. However, the basic element determining success or failure from the perspective of GSU is maintaining an appropriate level of fuel in the warehouses. Taking into account the specificity of peacetime military logistics, going below a certain minimum level should be considered a failure, even if this resource could still secure the reported needs of MU. This is related to the operational need to maintain the necessary reserves. Due to the need to manoeuvre the fuel resource between the internal GSU storage facilities, it is advisable not to exceed the specified maximum level of fuel storage capacity. Exceeding this level should be considered a failure, even if the storage capacity has not been fully used. We consider all activities leading to maintaining the state between the minimum and the maximum as a success.

To sum up, the neural network is designed to learn from historical data, taking into account the input data entered by the decision-maker in LDSS for the assumed number of weeks, how the GSU processes are carried out, and to present a prediction of the unit behaviour in terms of fuel distribution based on the decisions that the decision-maker will make for a given period of time.

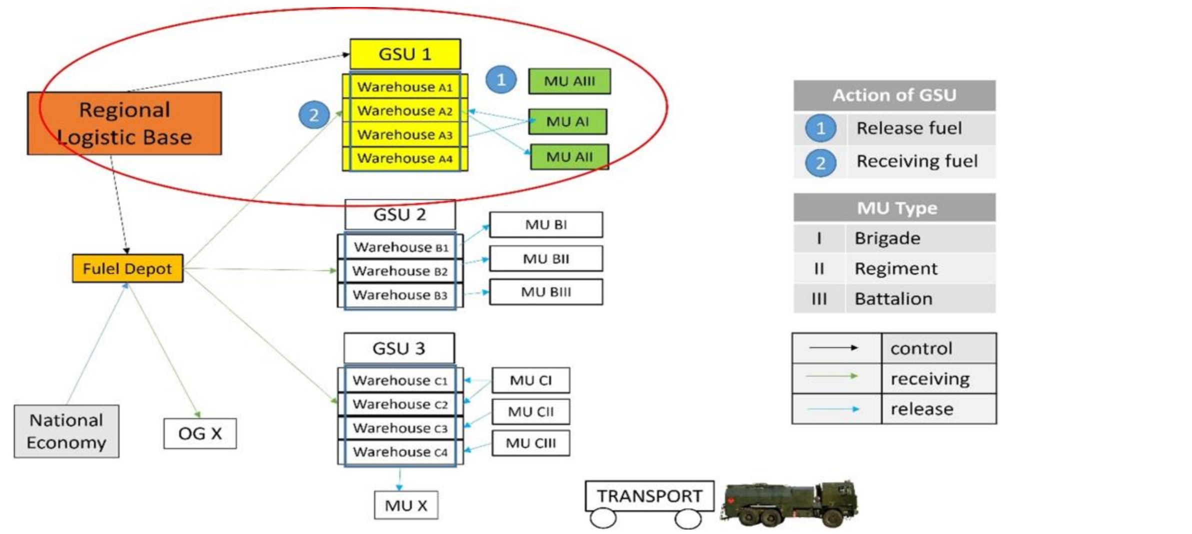

Fig. 3. The process of dispensing and receiving fuel by GSU.

The analysis covered data from selected MUs and GSU warehouses, which have been artificially linked by logistical relationships, but are a reliable model of the fuel supply system for research purposes. The probability of the initial state obtained thanks to the aggregation of historical data guarantees that, for the last five years, day by day, the tasks of supplying fuel have been performed in accordance with the adopted logistics policy of the troops. The operating environment affected the logistics system and caused successes and failures in procurement. The Logistics Decision Support System (LDSS) is a predictive module that, based on historical data and operational assumptions, e.g. exercises and policy, will allow for predicting the level of GSU fuel through the prism of the actions assumed at a given time and the operational situation. The system, based on the assumed coefficients, is to show how the decision-maker will lead GSU in the future.

The minimum and maximum level of maintaining fuel stocks was adopted, which made it possible to assess fuel inventory levels and determine the probability of the logistics system for a given week. The behaviour of the GSU decision-maker is influenced by logistical space-time – a complex concept that is superimposed by many factors, including the dynamics of actions, manoeuvres of forces, surprise and dispersion. Military logistics must keep up with the changes in this environment, and the commanders of logistics subunits must make decisions in a short time by analysing large amounts of data.

Providing fuel in the theatre of military operations is only one of the elements of the military logistics system. For the purposes of creating a decision-making model, we assume that securing the fuel needs of the operating MU requires several logistics processes based on the resources available in the operational space over time (based on GSU's resources as well as allied and civilian resources). According to this, it is necessary to overcome the logistic space-time [15] to ensure the appropriate level of fuels in time (T) in the indicated locations (X) with input data such as:

a) Environmental:
   - temperature,
   - humidity,
   - the amount of rainfall in a given area,
   - characteristics of the terrain,
   - season,
   - altitude.

b) Structural:
   - accessibility of land, sea, and airspace with landing strips,
   - transmission line capacity (fuel tonnage over time),
   - storage capability: equipment parameters, tonnage, dispensing capacity, and filling and distribution time.

c) Operational (space, time, resources, and information):
   - duration of the operation,
   - the number of main equipment, infrastructure, and soldiers in the area of responsibility,
   - the dynamics of activities,
   - loss factor [1].

Based on the area of the logistics operation, we can distinguish the general function of the space-time of the operation [15].

There are several phases of the decision-making process. At the outset, it is necessary to identify the decision-making situation, which should be characterised by all factors that affect the decision-maker's verdict. Then, we will formulate the decision-making problem that the decision-maker faces. We must include a definition of: a decision-maker; decision options; factors limiting the decision-making space, and the reasons shaping the assessment of decision-making options. The first phases of a decision problem are mainly descriptive and are based on the coefficients and weights of the input variables of the neural network.

With the help of network inputs, you can determine the elements of a set of acceptable decision options, as well as indicate optimal options. Finally, you need to determine different subsets of the set of options: acceptable, satisfactory and optimal, and make a decision on this basis. The determination of different subsets of the set of options is based on the use of, among others, single- and multi-criteria optimisation methods. The Logistics Decision Support System – LDSS is to be a computerised system that supports decision-making in the area of logistics at the appropriate decision-making level of the military logistics system.

3. Neural network selection and the learning process

For the purpose of predicting the actions to be carried out by the decision-maker, a model of energy consumption prediction based on neural networks was used. The MATLAB software was used for this purpose. It includes, among other things, the Neural Network Toolbox library, which enables the construction and use of neural networks for forecasting. For this purpose, you can use ready-made tools, or use the basic command line.

To build a neural network, you need to follow these steps:

- collect data,
- create a network,
- configure the network,
- initialise scales,
- test the network,
- use the network for prediction.

We assume that the artificial neural network will generate the GSU fuel level signal taking into account the above conditions, with 260 lines of historical data set containing 21 input variables such as time, intensity of activities in the GSU operating area, including the number of fuel acceptance/dispensing operations, availability of equipment, number of personnel in the GSU system, variable specifying procedural restrictions (regulations, instructions), type of terrain, meteorological data (wind speed, temperature, humidity, variable that determines who is the recipient (size of the unit, e.g. brigade, regiment, battalion), time of fuel dispensation, execution of the movement of the GSU fuel resource, state of the GSU warehouse.

The prediction assumes the determination of 10 consecutive values of fuel levels in the GSU for the decision-maker or other data (intensity of activities, dispensing or suspension of fuel dispensing, etc.) necessary to decide for the next 10 weeks of GSU action planning.

An array of input and output data containing 21 input variables and n-waveforms of output parameters necessary to predict the operation of GSU, which in the MATLAB environment will be used for prediction using a multilayer one-way network (multilayer perceptron).

The supervised learning method was used in the research, in which the network parameters are selected on the basis of a comparison of the values at the network output with the set values for the recorded, actual level of fuel dispensed in GSU. Training in this case consists of minimising the error function depending on the differences between the set values and the actual network output for training data.

A variant of the network with one layer of hidden neurons for different numbers of neurons was studied. It was assumed that there would be 10, 20 and 50 neurons in the hidden layer.

Four learning algorithms were adopted for the task, including three second-order algorithms:

– BFGS quasi-Newton (BFG),
– Levenberg-Marquardt (LM),
– Bayesian regularisation (BR),
– resilient backpropagation algorithm (RP).

Sigmoid neuronal activation functions in the latent layer were used.

The program randomly divided the data into three sets:

– 70% – a training set,
– 15% – a set used to validate a network in terms of its ability to generalise and stop the learning process before the phenomenon of overtraining occurs,
– 15% – a test set to perform an independent test of the network's ability to generalise.

The results were evaluated using:

– regression coefficient (R^2) measuring the correlation between the exit signal and the given target (the closer the value to 1, the better the result);
– MSE determining the quality of processing for all sets used in the study;
– RMSE mean squared error, indicating the accuracy of the forecast in a given model.

The terms dispensing or adoption cover specific decision-making processes leading to such "final" actions.

We assume that an artificial neural network model based on the algorithm of a neural network of learning with a teacher (Fig. 1) is designed to indicate, based on historical data, whether the decision-maker should perform one of three types of actions: dispense fuel, suspend dispensing fuel, or accept fuel. At the input of the neural network, to predict the decision-maker's actions, we need to collect specific data in IT systems, which are elements of the input vector of the model [1].

To train the neural network to predict fuel level changes at GSU, a group of environmental, operational and infrastructural variables were used. According to the assumptions of the LDSS system, it should be assumed that the decision-maker will introduce indicated groups of variables into the program, which will be used to create a matrix of validation data in order to predict how the fuel level parameter in GSU will behave. An artificial intelligence algorithm will indicate a different distribution of GSU fuel levels based on historical data and the relationship between the layers of the network. The decision-maker, thanks to the illustration as in Fig. 4 or thanks to the indicators of differences between the levels assessed by the decision-maker and the one indicated by the network, will allow the decision-maker to react or take action on refuelling in a given week.

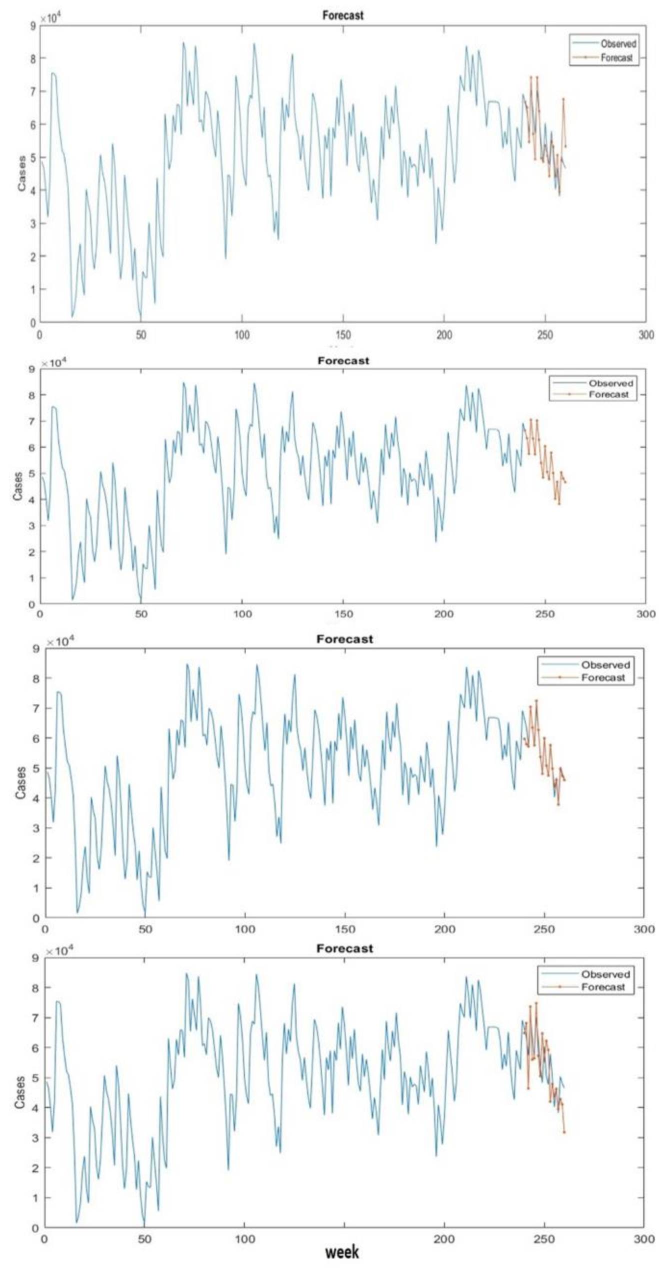

Fig. 4. Comparison of the fit plots of the simulated and measured results and the mean squared error for the study using a multilayer unidirectional network with 50 neurons using the BFG, BR, LM and RP algorithms.

A similar analysis can be carried out for other GSU parameters, such as the number of personnel (200−300 people), the intensity of activities, which consequently translates into changes in the fuel level in GSU. There are dependencies between the variables, thanks to which we can determine dependent variables, such as the time of fuel dispensing from the GSU, which depends on the intensity of activities (the number of fuel trading operations per week was 0−127), the number of available personnel (accepted in the study in the range of 200−300 people) and the amount of fuel dispensed (historical data). In this case, the decision-maker will receive information whether his calculations are correct. In addition, it will be possible to determine from historical data whether an increased intensity of activities should be expected in a given week beyond that assumed by the decision-maker.

Figure 4 should be interpreted in such a way as to look for differences between the indications of the GSU parameter from the prediction and the validation set. The regression error in this case will indicate whether these values differ so significantly that the network is trained to provide the right variants of action for the decision-maker.

Below, the results of research using the BFG, BR, LM and RP algorithms for a multilayer neural network with one hidden layer and 50 neurons are presented in the form of graphs in Figs. 5−8. Similar data analyses were carried out for networks with 10 and 20 neurons, but in this configuration, the differences in results for individual algorithms are most representative.

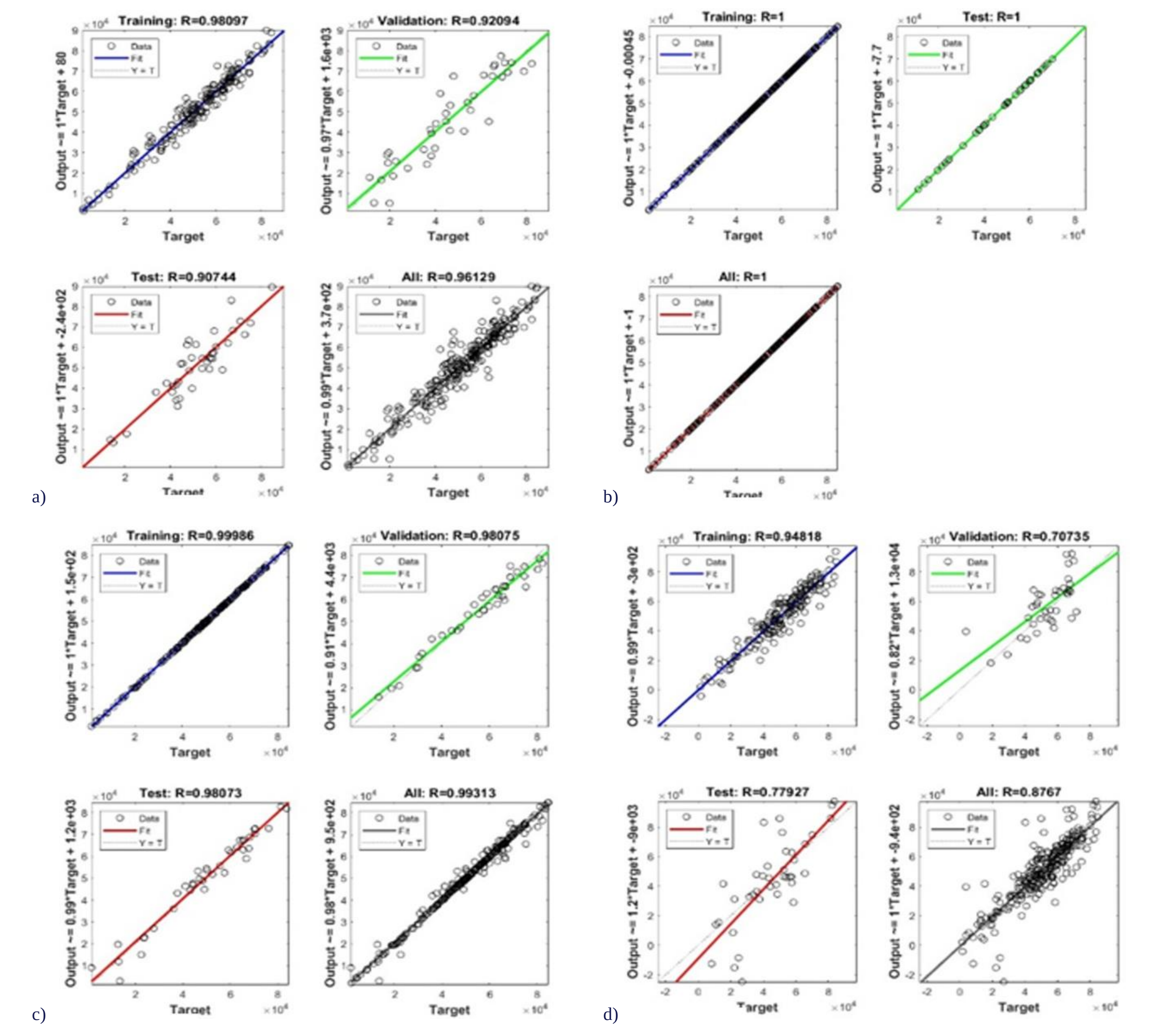

Fig. 5. Regression error plots for individual versions of networks with BFG (a), BR (b), LM (c) and RP (d) learning algorithms with 50 neurons.

In the results obtained, the BR algorithm obtained the best parameters mainly due to the specificity of its operation:

1. Regularisation mechanism minimises overlearning:
   - BR modifies the error function to simultaneously minimise the matching error and the complexity of the network scales.
   - This way, the network does not learn training data "by heart", but generalises better to test and validation data.
   - effect: the highest R coefficient (0.999) and the lowest MSE in validation.
2. Learning stability with a large number of neurons:

- In the configuration (50 neurons, 1 layer), the network has a very large number of weights → a high risk of overtraining with classical methods.
- BR copes with this better than BFG, LM or RP, because it automatically adjusts the degree of regularisation based on the data.

3. Data noise immunity:
   - Because there is a "penalty" component for too high weights in the target function, the BR-trained network does not over-adjust to single outliers. In practice, this translates into the highest number of zero-error hits in the test.
4. Better fit in regression problems:
   - BR is particularly effective in continuous regression because it not only optimises the MSE error, but also improves the input-output correlation.
   - The results show that the differences between LM, BFG and BR are the largest in the validation MSE, suggesting that BR is better at predicting values for data that the network has not seen before.

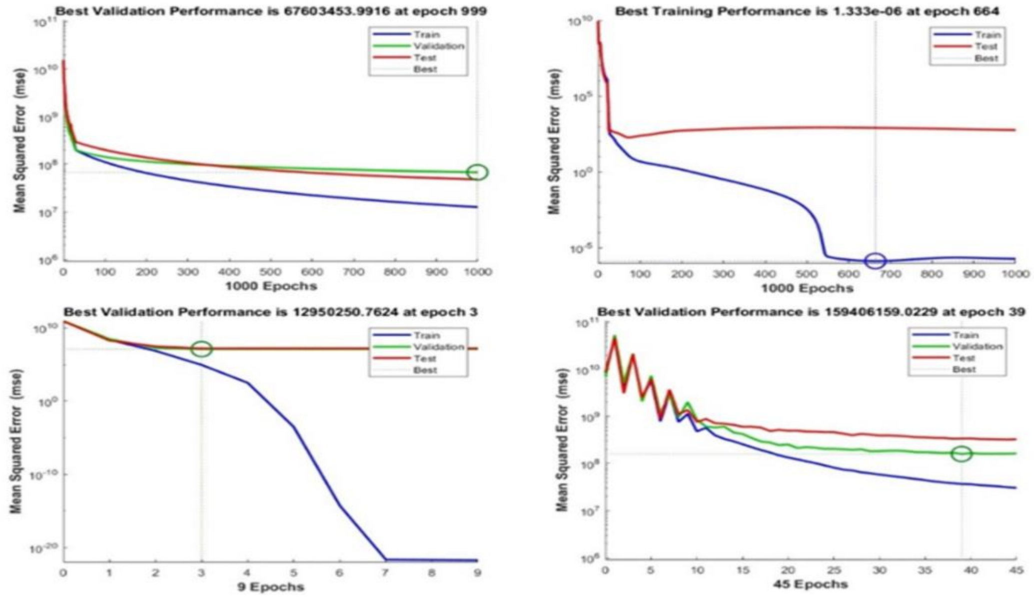

Fig. 6. Evaluation of the quality of validation for a multilayer neural network with one hidden layer and 50 neurons, using the BFG, BR, LM and RP algorithms.

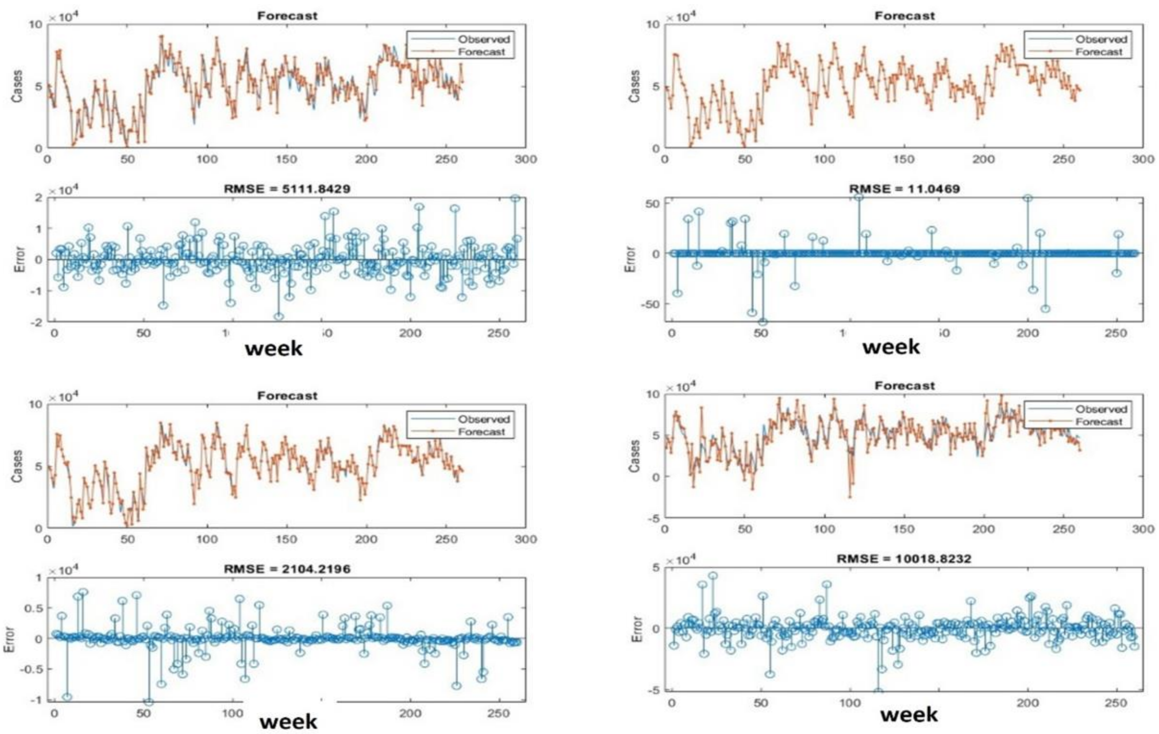

Fig. 7. Comparison of the fit plots of the simulated and measured results and the mean squared error for the study using a multilayer unidirectional network with 50 neurons, using the BFG, BR, LM and RP algorithms.

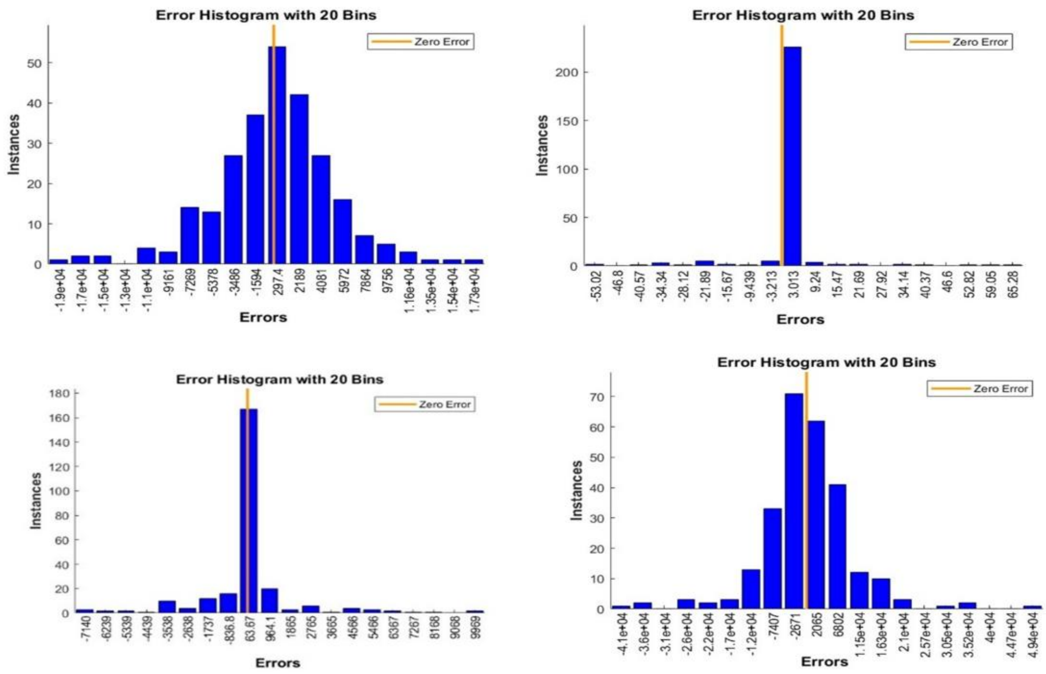

Fig. 8. Regression error histograms for consecutive neural networks with 50 neurons using the BFG, BR, LM and RP algorithms.

4. Summary

BR performed best for the network with 50 neurons, as its built-in regularisation mechanism reduced overtraining and increased generalisation. This is especially important with large numbers of neurons and potentially complex inputs.

The results obtained for different neural network structures and learning algorithms are summarised in Table 1. Based on the conducted research, Fig. 9 depicts formulated trend conclusions from data for 10, 20 and 50 neurons in BFGS, BR, LM and RP algorithms:

1. Accuracy of fit (R):
   – Highest R (1.0),
   – BR and LM with 10 neurons (perfect match),
   – LM maintains R = 1.0 also with 20 neurons,
   – BR with 50 neurons still R ≈ 1.0,
   – BFGS – stable around 0.98 with 10 and 20 neurons, a slight drop to 0.96 with 50 neurons,
   – RP – a marked decrease with the number of neurons: 0.97 → 0.92 → 0.87.

   Conclusion: BR and LM are very accurate on small and medium-sized networks; with larger networks, BR maintains an advantage over LM and other algorithms.
2. Mean squared error (RMSE and MSE):
   – BR – extremely low MSE in each case (in the order of 10^{-4} – 10^{-6}), which suggests high stability and lack of overlearning,
   – LM – with 10 and 20 neurons, low RMSE (1.65 and 0.36), but with 50 neurons, a significant increase in RMSE ($2.1\cdot10^{3}$) and MSE ($1.3\cdot10^{8}$), which may suggest a problem with fitting or overlearning,
   – BFGS – increasing RMSE and MSE with the number of neurons,
   – RP – high RMSE and MSE in any scenario, deteriorates with larger networks.

   Conclusion: BR has a definite qualitative advantage, LM is good for smaller networks, and BFGS and RP lose accuracy as neurons grow.
3. Influence of the number of neurons:
   – BR – virtually independent of the number of neurons in terms of fit quality (R ≈ 1.0, low MSE),
   – LM – great up to 20 neurons, but at 50 neurons, there is a degradation in quality,
   – BFGS – moderately stable up to 20 neurons, later larger errors,
   – RP – quality decreases significantly as neurons grow, which may indicate a problem with weight propagation in large networks.

Table 1. Results for three networks and four learning algorithms (the best fit data for the BR algorithm is marked in green).

Algorithm	**BFGS quasi_Newton**			**Bayesian regularisation**			**Levenberg-Marquardt**			**Resilient backpropagation**		
Indicator	RMSE	MSE	R	RMSE	MSE	R	RMSE	MSE	R	RMSE	MSE	R
10 neurons	$2{,}14\cdot10^{3}$	$6{,}14\cdot10^{6}$	0,98	0,21	$1{,}13\cdot10^{-4}$	1	1,65	7,03	1	$2{,}84\cdot10^{3}$	$1{,}9\cdot10^{7}$	0,97
20 neurons	$2{,}4\cdot10^{3}$	$1{,}18\cdot10^{7}$	0,98	27,5	$2{,}6\cdot10^{-6}$	0,99	0,36	0,38	1	$5{,}58\cdot10^{3}$	$8{,}12\cdot10^{7}$	0,92
50 neurons	$5{,}11\cdot10^{3}$	$6{,}76\cdot10^{7}$	0,96	11,04	$1{,}3\cdot10^{-6}$	1	$2{,}1\cdot10^{3}$	$1{,}3\cdot10^{8}$	0,99	$1{,}02\cdot10^{4}$	$1{,}6\cdot10^{8}$	0,87

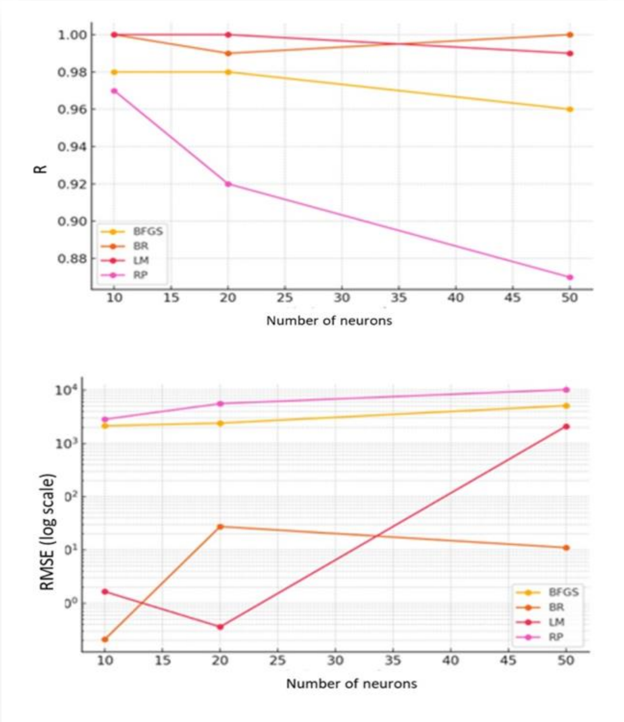

Fig. 9. Change in R and RMSE for the studied algorithms depending on the number of neurons.

The main consumer of fuel in the adopted model of the supply region by GSU are vehicles and devices based on the operation of internal combustion engines. An analysis of the available scientific literature [16−18] in recent years indicates a significant influence of air temperature and humidity on the fuel consumption of internal combustion engines. This relationship was the reason for including these parameters in the database to be studied. The research analyses the collected data in terms of the impact of temperature and humidity on fuel consumption in GSU. In order to determine the relationship between fuel consumption in a logistics unit and weather conditions, a correlation and regression analysis was performed, using variables: ambient temperature, relative humidity and intensity of activities measured as the number of fuel extraction operations (Fig. 10). The direct relationship was assessed using the Pearson correlation coefficient and the linear regression model.

The results clearly indicate that fuel consumption strongly depends on the intensity of operations of military units in the area of responsibility of GSU, which is confirmed by a positive correlation ($y \approx 0.51$). This means that an increase in the number of fuel withdrawal operations is significantly linked to an increase in fuel consumption.

On the other hand, the correlations between fuel consumption and temperature ($r \approx -0.05$) and humidity ($r \approx 0.03$) are close to zero, indicating that there is no direct linear relationship between the variables in question. Adding temperature and humidity to the regression model did not increase its explanatory power ($R^2 \approx 0.26$), and the coefficients assigned to them turned out to be statistically insignificant ($p > 0.5$). This means that the impact of meteorological parameters on fuel consumption is not due to direct impact, but may be a secondary effect depending on planning and seasonality of activities.

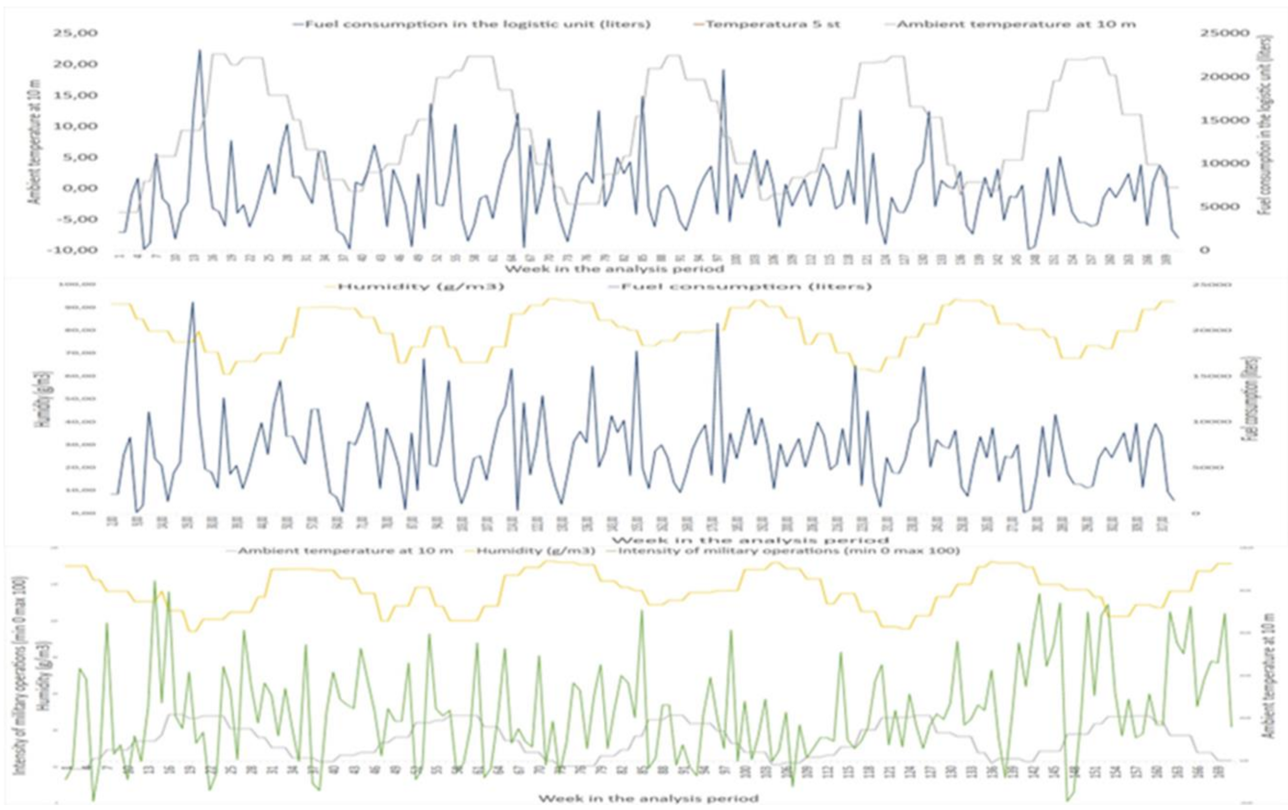

Fig. 10. Influence of air temperature and humidity on the fuel consumption in GSU

Fuel consumption in the tested model unit does not directly depend on air temperature or humidity, but is strongly correlated with the intensity of logistics activities. Weather conditions may indirectly affect consumption by modifying training activities, but they are not an independent factor determining fuel consumption. Even the use of the non-linear random forest model did not allow for a satisfactory quality forecast of fuel consumption based on meteorological data. The most important predictor remains the intensity of fuel withdrawal operations, which indicates that the demand for fuel is determined mainly by training and operational activities. Weather factors, on the other hand, can play an indirect role – they affect the planning of exercises and military operations, but they are not an independent determinant of the level of consumption.

5. Conclusions

In order to create software supporting the decision of the GSU decision-maker in the future, neural networks based on the most commonly used algorithms were trained. The Levenberg-Marquardt and Bayesian regularisation algorithms within the framework of approximation tasks are considered to be the best in obtaining MSE from all the algorithms used [16]. It should be noted that the Levenberg-Marquardt algorithm, which in many studies shows its advantage over other algorithms, turned out to be less useful in this case. LM is considered the fastest, not too complex, and is used almost exclusively for training a medium-sized unidirectional network with a single neuron at the output layer. Unfortunately, the amount of data collected for the purpose of training the network to predict activities in GSU was small, mainly because so far the ZWSI RON system (Integrated Multi-Level IT System of the Ministry of National Defense) has not collected data such as the intensity of activities in the area of the GSU's operation, the type of terrain, the availability of equipment, the variable that determines the limitations related to procedures (regulations), meteorological data (wind speed, temperature, humidity) probability of moving the GSU fuel resource. The scale of the amount of data that must be implemented into the military logistics information system is significant; however, the presented article is intended to indicate the need to take actions that will lead to the expansion of the number of collected parameters of the military logistics system. In this case, probably too small a dataset to train and a decidedly small network size did not allow for fully showing the advantages of this algorithm, including its speed.

The use of the Bayesian regularisation algorithm turned out to be the most effective method in terms of prediction ability, much better than the Levenberg-Marquardt algorithm, which is slightly inferior to this algorithm. Such a favourable result for R&D was influenced by, among others, greater resistance to learning, less cross-validation [17], and better flexibility and quality of generalisation [18].

In the case of BFGS algorithms, the quasi-Newton algorithm is considered to be a more complex algorithm, but its properties are perfect for small networks, due to the complexity of the calculations it performs. The results confirmed this by placing the algorithm in third place in the study.

The resilient RP backpropagation algorithm, despite the advantage of eliminating errors in the training set, did not guarantee good results in the obtained prediction.

It turned out that the quality of learning using this algorithm, given the relatively small size of the network used for the research, gives worse results than other methods.

To sum up, it should be stated that a multilayer unidirectional perceptron with a sigmoid function of neuronal activation in the hidden layer and with the use of the Bayesian regularisation learning algorithm can be successfully used to predict fuel management in GSU.

Proper construction of a database for GSU and entering geolocation and meteorological data into the ZWSI RON system will significantly improve the ability of the neural network to generalise, and thus fit into the decision-making model in GSU by creating tools for the development of a logistic decision support system – LDSS.

Regardless of the above, during the study of the problem of optimisation of prediction using machine learning algorithms, it can be noted that the next step in optimising this type of software, mainly in terms of the quality and speed of prediction, are reinforcement learning (RL) algorithms. Based on research conducted on RL algorithms in, e.g. the game "Tetris" [19,20], the key factor in reinforcement learning is the time delay between the action and the reward. Based on the literature on the use of RL algorithms in computer games, it should be stated that the worst performance is achieved by networks trained to predict actions several dozen steps away from the next reward. It can be said that the longer the interval between the action and the reward, the game requires a much more deliberate strategy, and the shorter the pause, the more the game is reactive to the changing situation of the operating environment. RL-based learning occurs much faster [21] in reflex-based games than in games that require a lot of strategy, which makes sense and is in line with the author's research. The results show that using AI to combine an agent neural network trained on historical GSU data and a strategy-based RL algorithm is a very beneficial approach in terms of improving decision-making efficiency. Reinforcement learning differs from supervised learning in that it does not require the presentation of labelled input/output pairs and does not require direct correction of suboptimal actions. Instead, we focus on finding a balance between exploring unknown solutions and the decision-maker's experience to maximise the long-term reward, whose feedback may be incomplete or delayed [22].

Taking into account the above, the use of a neural network to predict activities in a business branch turned out to be very useful and allows decision-makers to make decisions for the next weeks of GSU's operation with the indication of variables to be predicted. On the other hand, supplementing the analysis with RL algorithms in the future will allow us to further optimise this process and significantly speed it up. The use of the proposed machine learning methods is aimed at developing a logistics decision support system – LDSS in the future, which will be designed to support decision-making processes in the area of logistics at the appropriate decision-making level of the Armed Forces logistics system.

The method of predicting fuel consumption of the logistics system in GSU described in the article can be successfully used to determine other thermodynamic parameters, such as the pressure at the inlet to the turbine of a jet engine, the distribution of air flow on the steam turbine blade depending on the parameters of the external environment, as well as to predict fuel consumption in power plants or other energy generation systems. Thanks to the use of backpropagation, and comparison of the obtained results with the desired values and continuous calculation of the error, the network leads to weight correction to minimise error in subsequent steps (iterations). These processes take place in the so-called epochs, and each epoch is a single passage through the entire set of learning data. The method described in the article can be used by readers for their own research and lead to many interesting solutions to research problems in the field of thermodynamics.

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