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Structural damage identification using frequency and modal changes

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Abstract. The problem of identification of a structural damage is considered. The identification of location and/or dimensions of a damaged area or local defects and inclusions is performed using the measurements of vibration frequency and eigenvalues of real structure and the corresponding finite element model. The proper distance norms between the measured and calculated structural response are introduced and minimized during the identification procedure.

Key words: damage identification, modal analysis.

1. Introduction

The prediction of a location and a degree of damage in existing engineering structures is of great importance from the point of view of their serviceability and safety. A visual inspection and an extensive testing can be employed to locate and measure the degradation of structure by non-destructive techniques such as an acoustic emission, ultrasonic methods, thermography, or the modal testing. In the present paper, we study the damage or fault detection by analyzing the dynamic response of structures. In particular, the structural modal parameters, such as natural frequencies with their related modes are considered. In fact, eigenfrequencies and mode shapes vary with structural stiffness and their variation can be used in order to identify the stiffness reduction and damage localization. Two typical identification problems are: i) a specification of a single crack (an localized damage) with its size, location and orientation as unknown parameters, and ii) the specification of a distributed damage within a structure regarded as a distributed stiffness reduction. A related problem is concerned with the effect of damage on structural performance and safety.

There are numerous papers devoted to these classes of problems. An extension review of damage identification techniques in structural and mechanical systems, based on changes of vibration response, was provided by [1]. In most papers it is assumed that the eigenfrequencies and eigenmodes of an undamaged structure are known and their variations induced by damage are used in the identification procedure. However, when the accurate measurement of mode shapes is not feasible, then the damage should be identified by using only measured eigenfrequencies and their variation with respect to the reference (undamaged) model. The eigenmodes of undamaged structure are assumed as known and their variation for damaged structure is assessed analytically or neglected, cf. Dems and Mróz [2]. However, when the mode shape changes are measured, then the identification procedure is augmented by using modal assurance criteria, mode curvature or an energy distribution in consecutive structural elements, and variation of position of mode lines. The papers by Pandey et al. [3], Hearn and Testa [4] provide good examples of these approaches. A general sensitivity study of natural frequencies and modes with respect to cracks and holes was presented, for instance, by Gudmundson [5], Garstecki and Thermann [6] as well as Dems and Mróz [7] for geometrically non-linear vibration and stability problems. The damage identification using some additional control parameters in order to increase the sensitivity of structural response with increasing damage and to maximize the assumed distance norm between the measured and structural model response was also presented in [2].

Apart from the above mentioned methods, other approaches can be also used in order to determine the degree of structural damage. Some interesting approach basing on image analysis of cracked specimen was presented by Glinicki et al. [8].

2. Damage identification using frequency changes

When the damage identification is based on the free frequencies measurement, their variations are used in the identification procedure.

Consider the undamaged structure shown in Fig. 1a and the damaged one shown in Fig. 1b. These structures can be treated either as the real structure or as its finite element model, obtaining using the finite element approach, cf. Hinton and Owen [9]. The stiffness and mass matrices of real structure can be obtained, if necessary, from measurements, while the respective matrices for finite element model follow from calculations.

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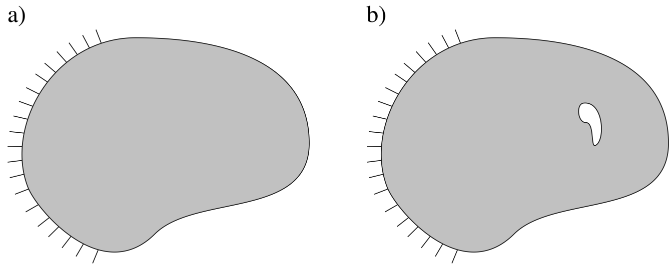

Fig. 1. Undamaged (a) and damaged structure (b)

Let assume that the stiffness and mass matrices of a discretized linear elastic structure are symmetric and the free vibration state is described by the equation specifying the eigenvalue problem. This equation describing the behavior of undamaged discretized structure has the form:

$$(\mathbf{K}_0 - \lambda_{0i}\mathbf{M}_0)\Phi_{0i} = 0, \quad i = 1, 2, \ldots, n, \tag{1}$$

where the eigenvalues $\lambda_{0i} = \omega_{0i}^2$ are the squares of free frequencies, $\mathbf{K}_0$ and $\mathbf{M}_0$ denote the stiffness and mass matrices and Φ_{0i} are the corresponding eigenmodes. The eigenvalues follow from (1) and can be expressed by the Rayleigh quotients as follows:

$$\lambda_{0i} = \frac{\Phi_{0i}^T\mathbf{K}_0\Phi_{0i}}{\Phi_{0i}^T\mathbf{M}_0\Phi_{0i}}, \quad i = 1, 2, \ldots, n \tag{2}$$

or, using the $\mathbf{K}_0$ – orthogonality and $\mathbf{M}_0$ – normalization of all eigenvectors, in the equivalent form:

$$\delta_{ij}\lambda_{0j} = \Phi_{0i}^T\mathbf{K}_0\Phi_{0i}, \quad \Phi_{0i}^T\mathbf{M}_0\Phi_{0j} = \delta_{ij}, \quad i, j = 1, 2, \ldots, n, \tag{3}$$

where δ_{ij} denotes the Kronecker's symbol.

Assume next that the stiffness matrix $\mathbf{K}$ of a damaged structure can be expressed as follows:

$$\mathbf{K} = \mathrm{K}_o + \delta\mathbf{K}, \tag{4}$$

where the stiffness matrix variation $\delta\mathbf{K}$ has the form:

$$\delta\mathbf{K} = -\sum_{l=1}^{n_e}\mathbf{K}_{0l}^e\delta k_l, \qquad 0 \le \delta k_l \le 1 \tag{5}$$

and δk_l are the non-dimensional parameters specifying the fraction of stiffness reduction in damaged elements and $\mathbf{K}_{0l}^e$ is an element stiffness matrix of undamaged structure. For damaged structure we can write:

$$(\mathbf{K} - \lambda_i\mathbf{M})\Phi_i = 0. \tag{6}$$

Now the mass matrix of damaged structure can be written as $\mathbf{M} = \mathbf{M}_o + \delta\mathbf{M}$ and its eigenvalues are $\lambda_i = \lambda_{0i} + \delta\lambda_i$ and the variation of eigenvectors of damaged structure ($\Phi_i \cong \Phi_{0i}$) is neglected. It is usually assumed that $\delta\mathbf{M} = 0$, $\mathbf{M} = \mathbf{M}_o$ and the damage affects only the stiffness matrix. Thus, the Eq. (6) can be rewritten in the form:

$$[\mathbf{K}_0 + \delta\mathbf{K} - (\lambda_{0i} + \delta\lambda_i)\mathbf{M}_0]\Phi_{0i} = 0. \tag{7}$$

The variation of eigenvalues of damaged structure, following from (7), can be expressed by the Rayleigh quotients and now it equals:

$$\delta\lambda_i = \Phi_{0i}^T\delta\mathbf{K}\Phi_{0i}, \quad i = 1, 2, \ldots, n, \tag{8}$$

where once again the mode normalization and orthogonality specified by (3) is used. Let us note that one only needs to calculate or measure the eigenvalues and associated eigenmodes in order to estimate the eigenvalues of damaged structure and their variations.

Let us introduce the relative change of i-th eigenvalue as the measure of damage of structure, namely:

$$DI_i = \frac{\lambda_{0i} - \lambda_i}{\lambda_{0i}} = 1 - \frac{\lambda_i}{\lambda_{0i}} = -\frac{\delta\lambda_i}{\lambda_{0i}}, \tag{9}$$

where DI_i can be called the 'damage index' associated with i-th natural frequency [2]. The variation of λ_i specified by (8), in view of (5), can be now expressed as follows:

$$\delta\lambda_i = -\sum_{l=1}^{n_e}\Phi_{0i}^T\mathbf{K}_{0l}^e\Phi_{0i}\delta k_l = -\mathbf{D}_{il}\delta k_l, \tag{10}$$

where $\mathbf{D}_{il}$ is the $(n\mathrm{x}n_e)$ eigenvalue sensitivity matrix. Equation (10) can be used in specification of δk_l by generating inverse or generalized inverse solution. This solution can be constructed by minimizing the properly defined norm of $\delta\mathbf{K}$ and $\delta\lambda$ and is discussed in details in [10]. Now, the damage index DI_i expressed by (9), in view of (10), takes the form:

$$DI_i = -\frac{\delta\lambda_i}{\lambda_{0i}} = \frac{-\mathbf{D}_{il}\delta k_l}{\lambda_{0i}}, \quad i = 1, 2, \ldots, n. \tag{11}$$

Besides the damage index DI_i associated with i-th natural frequency, additionally the 'global damage index' can be also introduced and it is expressed as the sum of damage indices for first n natural frequencies, given in the form

$$I_g = \sum_{i=1}^{n} DI_i. \tag{12}$$

The identification problem can thus be stated as follows:

$$\min_{\mathbf{d}}.\, G(\mathbf{d}) \quad \text{subject to} \quad (\mathbf{K} - \lambda_k\mathbf{M})\Phi_k = \mathbf{0}, \quad k = 1, 2, \ldots, n, \tag{13}$$

where $G(\mathbf{d})$ denotes the proper defined distance norm between the calculated (for the model) and measured (for the real structure) structural responses and $\mathbf{d}$ is a set of parameters defining the location, size and orientation of local defect or damage area. In particular, the problem (13) can be formulated as:

$$\min_{\mathbf{d}}.\, G_1(\mathbf{d}) = \frac{1}{2}\left[I_g(\mathbf{d}) - I_g^m(\mathbf{d})\right]^2 \quad \text{subject to} \quad (\mathbf{K} - \lambda_k\mathbf{M})\Phi_k = \mathbf{0}, \tag{14}$$

where $I_g(\mathbf{d})$ and $I_g^m(\mathbf{d})$ denote the calculated and measured global indices (12). Another form of identification problem (13) can be based on the calculated and measured eigenvalues, and it is expressed in the form:

$$\min_{\mathbf{d}}.\; G_2(\mathbf{d}) = \frac{1}{2}\sum_{j=1}^{n}(\lambda_i - \lambda_i^m)^2 \quad \text{subject to} \quad (\mathbf{K} - \lambda_k\mathbf{M})\Phi_k = \mathbf{0}. \tag{15}$$

The square norms (14) and (15) were successfully used in identification of location and magnitude of beam or plate damage. The illustrative examples of using both norms can be found in Dems & Mróz [2, 7, 11].

3. Damage identification using modal changes

The idea of identification presented in the previous Section is based on the concept of measurement of free frequencies of structure. Another approach to damage identification can be based on measurements not only the eigenvalues associated with free vibration problem but also the corresponding eigenmodes. Consider once again the free vibration problem for the undamaged discretized elastic structure, described by Eq. (1). The eigenvalues following from (1) are expressed by Eqs. (3). The respective eigenvalues for the damaged structure problem are expressed by (6) and then:

$$\delta_{ij}\lambda_j = \Phi_i^T \mathbf{K}\Phi_i, \qquad \Phi_i^T \mathbf{M}\Phi_j = \delta_{ij}, \qquad i,j = 1,2,\ldots,n, \tag{16}$$

where once again the mode normalization and orthogonality conditions are used.

Assume, as previously, that $\mathbf{K} = \mathbf{K}_o + \delta\mathbf{K}$, $\mathbf{M} = \mathbf{M}_o$ and $\lambda_i = \lambda_{0i} + \delta\lambda_i$ are the respective stiffness matrix and eigenvalues of the damaged structure. Moreover, the eigenvectors of that structure are now assumed as $\Phi_i = \Phi_{0i} + \delta\,\Phi_i$. Since the damage affects only the stiffness matrix and not the mass matrix, it follows from (3) and (16) that the difference of eigenvalues, neglecting the higher order terms, can be expressed as follows:

$$\delta\lambda_i = \lambda_i - \lambda_{0i} = \Phi_i^T \delta\mathbf{K}\Phi_i + \delta\Phi_i^T \mathbf{K}_0\Phi_i + \Phi_i^T \mathbf{K}_0\delta\Phi_i. \tag{17}$$

It follows from (17), that now the variation of i−th eigenvalue depends not only on the variation of structural stiffness matrix but also on the variation of i-th eigenmode, that should be calculated. Let us note that in order to use now the norms (14) or (15) for damage identification, there is a need to solve the free vibration problem at each iteration step of constrained minimization algorithm, what is a time consuming task. Moreover, the application of distance norms based not on eigenvalues but eigenmodes of damaged structures suffer the same inconvenience. To overcome this difficulty, one can express the eigenvectors for actual model of damaged structure as the sum of eigenvectors for undamaged model and their increments due to damage growth. Next, these increments can be calculated using different simplified approaches, where there is no need to resolve the free vibration problem at each iteration step.

Now, let us discuss briefly two approaches to approximate calculation of eigenmode variation associated with damage growth within structure domain.

The first approach is based on Eq. (6) describing the behavior of damaged structure that can be rewritten in the form:

$$[(\mathbf{K}_0 + \delta\mathbf{K}) - (\lambda_{0i} + \delta\lambda_i)\mathbf{M}_0](\Phi_{0i} + \delta\Phi_i) = 0. \tag{18}$$

Making use of (1) and neglecting the second order term $\delta\lambda_i\delta\Phi_i \cong 0$, it follows from (18), that:

$$\delta\Phi_i \cong (\mathbf{K}_0 + \delta\mathbf{K} - \lambda_{0i}\mathbf{M}_0)^{-1}(\delta\lambda_i\mathbf{M}_0 - \delta\mathbf{K})\Phi_{0i}, \tag{19}$$

where $\delta\lambda_i$ can be calculated from (8) or can be obtained from measurement. Thus, using this approach, we overcome the need to solve the free vibration problem at each iteration step, but instead of that we need to invert the non-singular matrix $\mathbf{K}_0 + \delta\mathbf{K} - \lambda_{0i}\mathbf{M}_0$, what is also the time consuming problem.

The other approach uses the static correction method to calculate the increments of eigenmodes due to damage growth [12]. Following the analysis presented in [12], we can write:

$$\begin{aligned} \mathbf{\Phi}_i &= \mathbf{\Phi}_{0i} + \delta\mathbf{\Phi}_i, \\ \delta\mathbf{\Phi}_i &= -\mathbf{B}\mathbf{\Phi}_{0i} + \mathbf{B}^2\mathbf{\Phi}_{0i} - \mathbf{B}^3\mathbf{\Phi}_{0i} + \ldots, \end{aligned} \tag{20}$$

where

$$\mathbf{B} = \mathbf{K}_0^{-1}\delta\mathbf{K}. \tag{21}$$

Let us note, that in order to calculate the eigenmode Φ_i and its variation $\delta\Phi_i$ we need only once to calculate the eigenvectors and associated eigenmodes for undamaged structure and moreover to invert only once the stiffness matrix $\mathbf{K}_0$ for undamaged structure. This will save considerably the time of calculations necessary at each iteration step of identification procedure.

Taking into account only the linear terms in the second equation of (20), the eigenmode of damaged structure and its variation can be expressed as:

$$\begin{aligned} \Phi_i &\cong (\mathbf{I} - \mathbf{K}_0^{-1}\delta\mathbf{K})\Phi_{0i}, \\ \delta\Phi_i &\cong \mathbf{K}_0^{-1}\delta\mathbf{K}\Phi_{0i}. \end{aligned} \tag{22}$$

The eigenvalues of damaged structure and its variation follow from (17) and, in view of (22), can be rewritten in the form:

$$\begin{aligned} \lambda_i &= \lambda_{0i} + \delta\lambda_i, \\ \delta\lambda_i &= \Phi_{0i}^T\{(\mathbf{I} - \mathbf{K}_0^{-1}\delta\mathbf{K})^T\delta\mathbf{K}(\mathbf{I} - \mathbf{K}_0^{-1}\delta\,\mathbf{K}) + \\ &+ (\mathbf{K}_0^{-1}\delta\mathbf{K})^T\mathbf{K}_0(\mathbf{I} - \mathbf{K}_0^{-1}\delta\mathbf{K}) + \\ &+ (\mathbf{I} - \mathbf{K}_0^{-1}\delta\mathbf{K})^T\mathbf{K}_0\mathbf{K}_0^{-1}\delta\mathbf{K}\}\Phi_{0i}. \end{aligned} \tag{23}$$

The damage identification problem can be still formulated by (14) or (15) using the better approximations (23) of eigenvalues and then the global damage index than those discussed in the previous Section. However, knowing also the approximation of eigenmodes (22) for damaged structures, the other identification functional based on eigenmode measurements can be introduced.

Firstly, one can use the square of distance norm being the ‘angular distance’ between the eigenvectors of real damaged structure and its model, cf. Fig. 2, given in the form:

$$G_3 = \sum_{k=1}^{m}\left[\arccos\left(\frac{\sum\limits_{j=1}^{p}\Phi_{kj}\Phi_{kj}^m}{\sqrt{\sum\limits_{j=1}^{p}(\Phi_{kj})^2\sum\limits_{j=1}^{p}\left(\Phi_{kj}^m\right)^2}}\right)\right]^2 \tag{24}$$

where Φ_{kj} and Φ_{kj}^m denotes the model and measured eigenvectors, respectively, and p denotes the number of components of eigenvectors and m is the number of considered eigenmodes. The functional (24) attains its global minimum equal to zero, when the eigenvectors of real structure and its model coincide. Thus, the optimization problem can be now formulated as:

$$\min_{\mathbf{d}} G_3(\mathbf{d}) = \sum_{k=1}^{m}\left[\arccos\left(\frac{\sum\limits_{j=1}^{p}\Phi_{kj}\Phi_{kj}^{m}}{\sqrt{\sum\limits_{j=1}^{p}(\Phi_{kj})^2\sum\limits_{j=1}^{p}\left(\Phi_{kj}^{m}\right)^2}}\right)\right]^2$$
$$\text{subject to } (\mathbf{K}-\lambda_k\mathbf{M})\Phi_k = \mathbf{0}. \tag{25}$$

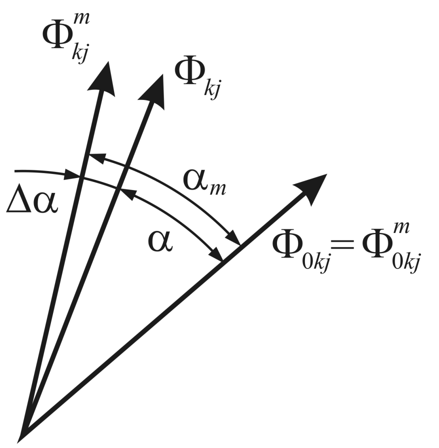

Fig. 2. Angular' distance norm between the eigenvectors $\Delta\alpha$ and differences of eigenvectors α and α_m

The other approach is based on the correlation of difference between the eigenvectors of damaged and undamaged structures for real structure and its model. In this case, the identification functional is expressed in the form:

$$G_4 = -\frac{\sum\limits_{k=1}^{m}\sum\limits_{j=1}^{p}\Delta\Phi_{kj}\Delta\Phi_{kj}^{m}}{\sqrt{\sum\limits_{k=1}^{m}\sum\limits_{j=1}^{p}(\Delta\Phi_{kj})^2\sum\limits_{k=1}^{m}\sum\limits_{j=1}^{p}\left(\Delta\Phi_{kj}^{m}\right)^2}}, \tag{26}$$

where $\Delta\Phi_{ki}^{m} = \Phi_{ki}^{m} - \Phi_{0ki}^{m}$ and $\Delta\Phi_{ki} = \Phi_{ki} - \Phi_{0ki}$ denote the difference of eigenvectors for real structure and its model. The functional G_4 attains the global minimum equal to -1 for full correlation of real structure and its model and maximum equal to 1 when there is no correlation. The optimization problem can now be stated in the form:

$$\min_{\mathbf{d}} G_4(\mathbf{d}) = -\frac{\sum\limits_{k=1}^{m}\sum\limits_{j=1}^{p}\Delta\Phi_{kj}\Delta\Phi_{kj}^{m}}{\sqrt{\sum\limits_{k=1}^{m}\sum\limits_{j=1}^{p}(\Delta\Phi_{kj})^2\sum\limits_{k=1}^{m}\sum\limits_{j=1}^{p}\left(\Delta\Phi_{kj}^{m}\right)^2}}$$
$$\text{subject to } (\mathbf{K}-\lambda_k\mathbf{M})\Phi_k = \mathbf{0}. \tag{27}$$

4. Identification algorithm

To perform the identification task (14), (15), (24) or (27), different identification systems can be used. They can be based on gradient-oriented or evolutionary algorithms or can constitute a hybrid identification system composed from in series connected evolutionary and gradient-oriented algorithms, shown in Fig. 3. The first module performs the initial identification using the floating point evolutionary algorithm starting from randomly selected initial model solution.

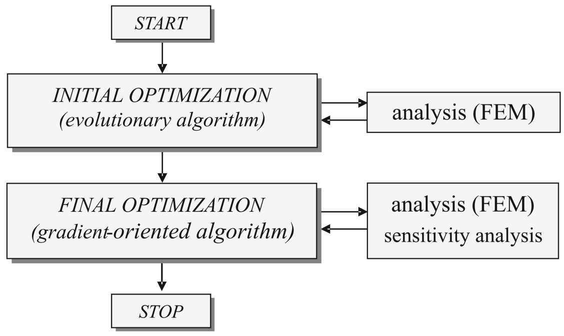

Fig. 3. Flowchart of hybrid identification system

The termination of the algorithm was established by the fitness convergence. The fitness function, being the measure of design quality, was assumed in the form:

$$f_i = e^{\left(-a\frac{(G_i - G_{\min})}{(G_{\max}-G_{\min})}\right)}, \tag{28}$$

where G_i denotes the value of respective objective functional associated with i-th individual in the current population, and $G_{\max}$ and $G_{\min}$ are its maximal and minimal values in this population. The definition of the fitness function guarantees its non-negativity and makes the difference of individual fitness more controllable, which is an important factor for the selection stage. The positive factor a is used to control the probability of the individuals being selected to create a new population. The increasing value of a causes higher probability for selection of the individual with higher value of fitness function. The negative sign in front of a converts a minimum problem to problem of maximization of fitness function. Next, in order to increase the efficiency of identification process, the variable metric algorithm starting from the last, best solution generated by evolutionary algorithm is used in the second module of a system. The finite element method is applied in both modules to model the real damaged structure and to perform the analysis step, where the eigenvalues and/or eigenvectors of actual model of structure or their approximations are calculated. In addition, this method is also used in final identification module for performing the sensitivity analysis of eigenvalues and eigenvectors in order to obtain the gradient information for identification functional.

It is worth to add that also only the evolutionary or gradient-oriented algorithm can be used during the identification procedure.

5. Illustrative examples

To illustrate the presented idea of identification procedure two simple examples of damage identification within a beam will be presented. Consider a beam shown in Fig. 4. The damage in the beam was modeled by decreasing the bending stiffness of one of beam element. On this stage of analysis the response of real beam was also modeled using finite element approach and in order to make the 'measurements' more realistic, some randomly distributed error of magnitude 1% was introduced to calculated values of eigenfrequencies and eigenvectors. The response of actual finite element model of the 'real' beam was calculated exactly. To increase the structural response and its sensitivity to the damage, some additional rigid or elastic support can be introduced within beam domain, as it is shown in Fig. 4.

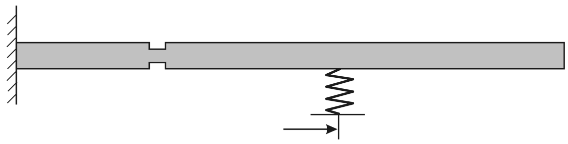

Fig. 4. Beam with additional rigid support

Firstly, we shall present the example of identification based on the functional (15). The beam was modeled using 20 finite elements with 21 nodes equally spaced. To increase the structural response, the additional supports were assumed to be installed consecutively at 3 nodes: 5, 11 and 17. Figure 5 presents graphically the results of identification for a beam with element 10 weakened, so that $(\mathrm{EI})_{10} = 0.6\mathrm{EI}$. In the numerical process, the sensitivity gradient identification was applied and stiffness moduli of different number of finite elements $(\mathrm{EI})_i$ were identified. The prediction 1 was generated for seven varying element stiffness moduli, and the prediction 2 was obtained for 5 varying stiffness moduli.

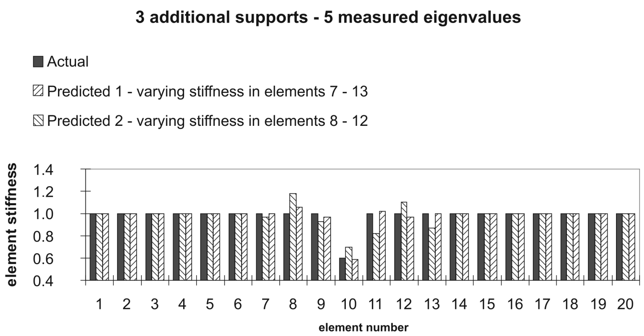

Fig. 5. Identification of a cantilever beam with damage in element 10

Let us note that there was no constraint set on the stiffness variation, and then the model predicts some element stiffness moduli higher than *EI*. The analysis presented here was based on the work of Dems and Mróz [2] which has been concerned with the damage identification method using parameter dependent evolution of natural frequencies.

In the second example we present the application of eigenmodes in identification procedure and use the functional (24) or (26). Thus, we solve the identification problem (25) or (27) using only the evolutionary algorithm. We consider once again the beam shown in Fig. 4, now simply supported on the both ends with no additional supports. The 'measured' node amplitudes of vibrating beam was disturb with random error not increasing the 1% of amplitude magnitude. The influence of the random measurement error on the values of functional (24) and (26) for varying location of damage element in finite element model of real structure is depicted on the Fig. 6a and 6b, respectively. On both figures, the continuous line shows the value of respective functional for the case of "exact" measurements (with no error), while the dotted lines show their values for three cases of the randomly distributed measurement error with its maximal value not increasing 1% of the amplitude magnitude at measuring points. It can be seen that the character of plot of both functional for error free and error weighted measurements are very similar the global minima are located very closely.

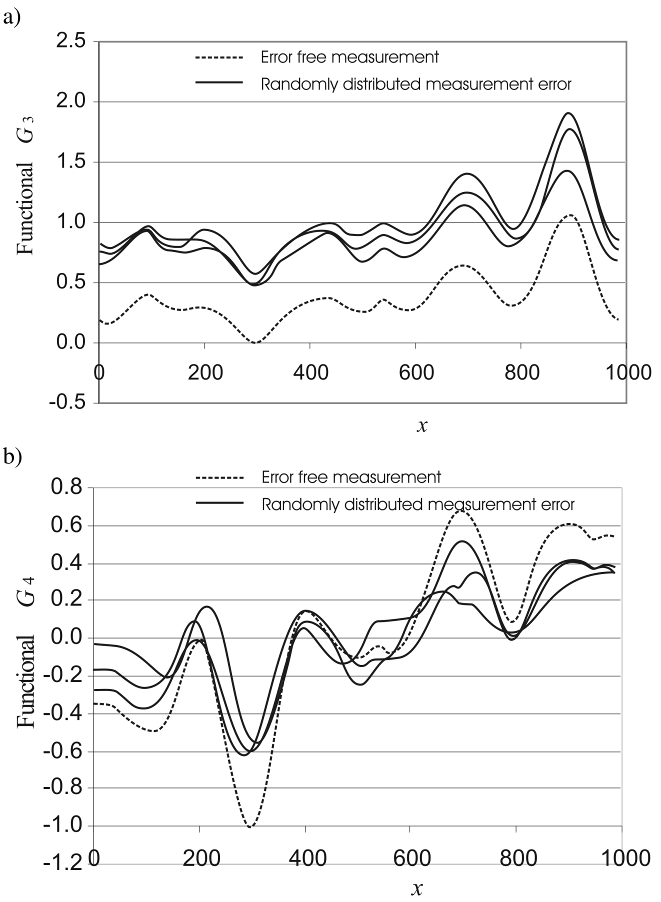

Fig. 6. Plot of functional G_3 (a) and G_4 (b) versus damage location in a model beam

The identification solutions of damage located in real structure at $x = 300$ mm, basing on functional (24) and (26) were performed using the evolutionary algorithm. The calculations were repeated 1000 times with randomly selected starting population. The averaged value of identified location was equal $x = 299.10$ mm for functional (24) and $x = 299.06$ mm for functional (26), while the standard deviation were equal to 9.054 and 9.116, respectively.

6. Concluding remarks

The present paper is concerned with the damage identification method using parameter dependent various structural response. In particular, the evolution of natural frequencies and eigenvectors due to occurrence of damage or its growth constitutes the basis for nondestructive identification of location, orientation and size of internal defects and cracks within structure domain.

Four different identification funtionals were introduced and the proper identification problems were formulated basing on these functionals. Using the concept of damage indices, being the measures of relative changes of eigenvalues due to damages, two first identification functionals were based on the measurements of eigenvalues of real structure and calculated eigenvalues of its discrete model. When the eigenvectors of real structure and its discrete model were also available, another two identification functionals were introduced based on the angular distance norm between the measured and calculated eigenvectors, as well as on the correlation factor between these vectors. In these two last cases, in order to avoid the time consuming solution of eigenvalue problems at each iteration step during the identification procedure, two approaches for approximate calculation of eigenmode variations associated with damage growth were proposed.

To solve the formulated identification problems, the novel hybrid identification system was proposed. This system was composed from evolutionary and gradient-oriented algorithms, with exponential fitness function being the measure of design quality.

To increase the accuracy and convergence of identification procedure in presented examples, some additional control parameters, associated with additional rigid or elastic support, were also introduced in order to increase the sensitivity of the measured and structural model responses. The obtained results indicate that the presented methods can be combined with parameter dependent eigenvalues and eigenmodes measurements and calculations.

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