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Algorithms of parallel calculations in task of tolerance ellipsoidal estimation of interval model parameters

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Abstract. The methods of the tolerance ellipsoidal estimation for the tasks of synthesis of the tolerances to parameters of radio-electronic circuits and possibility of its parallelization are considered. These methods are the result of the task of estimation the solutions of an interval system of linear algebraic equations (ISLAE) which is built according to given criteria of optimality. The numerical algorithm is proposed for solving the tolerance ellipsoidal estimation tasks with a possibility of parallelization.

Key words: interval model, parameters identification, tolerance ellipsoidal estimation, parallel algorithm.

1. Task statements

The methods of tolerance ellipsoidal estimation are used for the tasks of tolerance synthesis of the parameters of radio-electronic circuits [1], and for the tasks of design the technical devices for surgical operations assisting in particular for decreasing risk of damage the function-of important organs when the surgical area of safe operation is necessary to define. The indicated tasks are the result of the mathematical task estimating the solutions of an interval system of linear algebraic equations (ISLAE). It is described in [2, 3] that quality criteria for methods of receiving tolerance estimations of model parameters are: minimal computational complexity, maximal volume of tolerance estimation the parameters area and minimal computational resources for realization the resulting model. It is described in [3, 4] that the ellipsoidal estimations are the most effective estimations of ISLAE solutions and mostly satisfy the indicated criteria. Let's consider the mathematical definition of the indicated estimation task.

Let the construction of interval models has such data:

$$X = (\vec{x}_i),\ [\vec{Y}] = ([y_i^-; y_i^+]), \quad i = 1, \ldots, N, \tag{1}$$

where y_i^-, y_i^+ are lower and upper bounds of output characteristic values; $\vec{x} \in R^n$ is a vector of input variables.

Let's assume that the "input-output" model structure given as a linear equation in relation to parameters has the following form:

$$\widehat{y}(\vec{x}) = b_0 + b_1\varphi_1(\vec{x}) + \ldots + b_m\varphi_m(\vec{x}), \tag{2}$$

where $\widehat{y}(\vec{x})$ is a predicted value of output characteristic; $\vec{b}$ is a vector of unknown model parameters; $\vec{\varphi}(\vec{x})$ is a vector of the known base functions; $\widehat{y}(\vec{x}_i) \subset [y_i^-, y_i^+]$.

For the model construction it is necessary to estimate the parameters b_i, $i = 0, \ldots, m$ that is to find the solutions of ISLAE

$$y_i^- \le b_0 + b_1\varphi_1(\vec{x}_i) + \ldots + b_m\varphi_m(\vec{x}_i) \le y_i^+, \quad i = 1, \ldots, N. \tag{3}$$

In space of parameters, in the case of compatibility of the system (3), the solution of ISLAE is an area of parameters values in kind of polyhedron

$$\Omega = \{\vec{b} \in R^m \,\big|\, y_i^- \le \vec{b}\vec{\varphi}^T(\vec{x}_i) \le y_i^+,\ i = 1, \ldots, N\},$$

which is showed in Fig. 1.

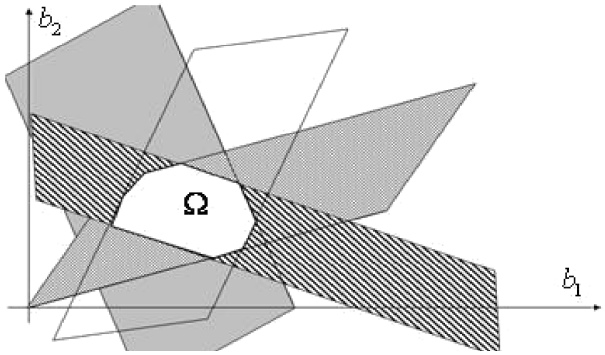

Fig. 1. Illustration of area Ω for $m = 2$, $N = 4$

For estimating the area of model parameters a tolerance ellipsoid is used of a following kind

$$Q_m = \{\vec{b} \in R^m \,\big|\, (\vec{b} - \vec{\overline{b}})^T H (\vec{b} - \vec{\overline{b}}) \le r\}, \tag{4}$$

where $\vec{\overline{b}}$ is a center of weight; H is a matrix which sets the ellipsoid configuration; $r \le 1$.

Thus corridor of interval models $[\widehat{y}(\vec{x}_i)]|_{\vec{b} \in Q_m}$ which is built on the basis of the tolerance ellipsoidal estimation (TEE) of parameters area is to be in kind:

$$[\widehat{y}(\vec{x}_i)]|_{\vec{b} \in Q_m} = [\vec{\varphi}^T(\vec{x}_i) \cdot \vec{\overline{b}} - \Delta_{\widehat{y}(\vec{x}_i)}|_{\vec{b} \in Q_m};\ \vec{\varphi}^T(\vec{x}_i) \cdot \vec{\overline{b}} + \Delta_{\widehat{y}(\vec{x}_i)}|_{\vec{b} \in Q_m}], \tag{5}$$

where

$$\Delta_{\widehat{y}(\vec{x}_i)}|_{\vec{b} \in Q_m} = r^{0,5} \times \sqrt{\vec{\varphi}^T(\vec{x}_i) H^{-1} \vec{\varphi}(\vec{x}_i)}. \tag{6}$$

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Then the task of the tolerance ellipsoidal estimation the parameters of model (2) are to be defined in kind:

$$V(Q_m) \xrightarrow{Q_m} \max, \qquad Q_m \subseteq \Omega, \tag{7}$$

where $V(Q_m)$ is a volume of tolerance ellipsoid (4) which is proportional to the value $\det(rH^{-1})$.

2. Numerical solving method

Evidently, a task of tolerance ellipsoidal estimation is a complex nonlinear task. For its solving the next sequence is offered:

1 – search the TEE configuration;

2 – search the suboptimal TEE with taking into account all limitations.

For many tasks a matrix of TEE configuration may be unknown, however there is information about some m-dimensional parallelepiped, so-called saturated block which forms the configuration of estimations area. The large group of guaranteed estimation methods is developed for estimating the area of interval model parameters. Although they do not allow to get the tolerance estimations, however it is possible to use their some scientific results and, taking into account the features of tolerance area, to offer the methods of tolerance area search.

After research on the multiple estimations of parameters area it is shown that the solution of m equations from ISLAE (3) is m-dimensional parallelepiped which enables a search of the tolerance ellipsoid with maximal volume.

In paper [5] it is proposed the guaranteed estimation of parameters area to search a m-dimensional parallelepiped with minimal volume which is built on the basis of the choice of m equations as saturated block from a system (3), and after that taking into account the rest equations from system (3) for the guaranteed estimation clarifying.

As a result of modification the method for search the guaranteed estimation of parameters area offered in [5] and taking into account the features of tolerance estimation, gets the method of a search the tolerance estimation of parameters area as m-dimensional parallelepiped $\widetilde{\Omega}_m$ from which it is simple on the basis of lemma in [6] to go to the ellipsoidal estimation.

Formally the task of search the tolerance area as m-dimensional parallelepiped $\widetilde{\Omega}_m$ on the basis of criterion (7) it is possible to present it in the following manner:

$$V_{\widetilde{\Omega}_m} \xrightarrow{\widetilde{\Omega}_m} \max, \tag{8}$$

$$\widetilde{\Omega}_m \subseteq \Omega. \tag{9}$$

Let's notice that configuration of area $\widetilde{\Omega}_m$ is known priori or found using methods of the design of experiment on discrete set of an experimental area [7].

Geometrically, this task is reduced to the construction of m-dimensional parallelepiped with maximal volume, which is found on the basis of criterion (8) that is inscribed in the parameters area from ISLAE (3).

Let's consider in detail the method of finding the solution of task (8) in conditions (9).

For a search $\widetilde{\Omega}_m$ from the solution of tasks (8) and (9) let's consider the iteration procedure on each $k+1$ step on which it is to be searched a tolerance area $\Omega_m(k+1)$ with adding next one interval equation from $N-m$ that remained in the system after a choice of the base of m equations. Moreover, $\Omega_m(0)$ is a parallelepiped that in a space of parameters is formed by the base equations of the system.

Then it is possible to rewrite the task (8)–(9) for $k+1$ step in the following form:

$$V_{\Omega_m(k+1)} \xrightarrow{\Omega_m(k+1)} \max, \quad k = 0, ..., N-m-1, \tag{10}$$

on condition of including

$$\Omega_m(k+1) \subseteq \Omega \subseteq \{\Omega_m(k) \cap \breve{\Omega}(k+1)\}, \quad k = 0, ..., N-m-1, \tag{11}$$

where $\Omega_m(k+1)$ is m-dimensional parallelepiped which is got on $k+1$ iteration; $\breve{\Omega}(k+1)$ is hyperplane which is determined by $k+1$ equation from those which remained in the system (3) after a choice of m base equations.

Actually, the procedure for obtaining the tolerance estimation $\Omega_m(k+1)$ on $k+1$ step consists in moving the proper facets of parallelepiped $\Omega_m(k)$ in such way that the top, which on step k placed on the largest distance to hypherplane, must appear on this hyperplane. This hyperplane is set by an active limitation as a certain part of the interval equation in ISLAE (3) from $N-m$.

The procedure of borders moving for two steps in the case $m=2$ is depicted in Fig. 2.

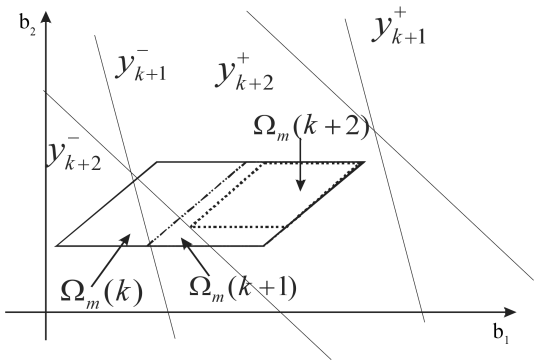

Fig. 2. Illustration of procedure of tolerance estimation of parameters area for a case $m=2$

As a result of this procedure realization for $N-m$ steps the tolerance area $\widetilde{\Omega}_m = \Omega_m(k=N-m)$ is obtained.

Let's conduct equivalent transformations for task (10) and for conditions (11).

As it is higher marked, the solution to this task on $k+1$ step is to be obtained as a result of moving the proper facets of parallelepiped $\Omega_m(k)$ on a condition of an implementation the including (11). Let's consider more detailed providing of this including. Let's use results from [8], where for every top of hyperparallelepiped it is proposed to use scalar functions

$L_s(k)$ and $L'_s(k)$ which characterize a distance between the proper top and a border of hyperplane $\breve{\Omega}(k+1)$:

$$L_s(k) = y^-_{k+1} - \vec{\varphi}^T(\vec{x}_{k+1}) \cdot \vec{b}_s(k), \tag{12}$$

$$L'_s(k) = \vec{\varphi}^T(\vec{x}_{k+1}) \cdot \vec{b}_s(k) - y^+_{k+1} = -L_s(k) - \Delta_{k+1}, \tag{13}$$

where $\vec{x}_{k+1}$ is a vector of input values in $k+1$ observation which determines $k+1$ equations in the system (3); y^-_{k+1}, y^+_{k+1} are lower and upper bounds of intervals the "outputs" in $k+1$ observation; $\Delta_{k+1} = y^+_{k+1} - y^-_{k+1}$.

In Eqs. (12) and (13) $\vec{b}_s(k)(s = 1, ..., 2^m)$ there is a vector of coordinates the top s of hyperparallelepiped $\Omega_m(k)$ on $k+1$ iteration which is calculated by Eq. (14) [9]

$$\vec{b}_s(k) = F_m^{-1} \cdot \vec{Y}_s(k), \tag{14}$$

where $\vec{Y}_s(k)$ is a vector which is composed from combinations of lower y^-_k and upper y^+_k interval values for each base m equations; F_m^{-1} is a matrix of values of the base functions.

On the basis of the statement from [8]: if $L_s(k) > 0$ ($L'_s(k) > 0$), than top s of hyperparallelepiped is placed outside of hyperplane which is obtained from an active limitation that is cut off by this hyperplane; else if $L_s(k) < 0$ ($L'_s(k) < 0$), than limitation is not active; else if $L_s(k) = 0$ ($L'_s(k) = 0$), than top s belongs to "hyperplane" which is obtained from this active limitation.

Then for a top which is placed on the largest distance to the bound of a hyperplane which is set by an active limitation it is correct to state that

$$L_{s^*}(k) = \max_{s=1,...,2^m} L_s(k) > 0,$$

$$L'_{s^*}(k) = \max_{s=1,...,2^m} L'_s(k) > 0.$$

So for a case $L_{s^*}(k) > 0$ ($L'_{s^*}(k) > 0$) the condition (10) on $k+1$ step can be possibly written in the following form:

$$L_{s^*}(k+1) = 0, \tag{15}$$

$$L'_{s^*}(k+1) = 0. \tag{16}$$

Let's put to conditions (15) and (16) the values of the proper functions from Eqs. (12)–(14), it is obtained

$$y^-_{k+1} - \vec{\varphi}^T(\vec{x}_{k+1}) \cdot F_m^{-1} \cdot \vec{Y}_{s^*}(k+1) = 0, \tag{17}$$

$$\vec{\varphi}^T(\vec{x}_{k+1}) \cdot F_m^{-1} \cdot \vec{Y}_{s^*}(k+1) - y^+_{k+1} = 0, \tag{18}$$

where $\vec{Y}_{s^*}(k+1)$ is a vector which comes from combinations

$$y^+_i(k+1) = y^+_i(k) - \delta^+_i(k+1), \tag{19}$$

$$y^-_i(k+1) = y^-_i(k) + \delta^-_i(k+1). \tag{20}$$

For example $\vec{Y}_{s^*}(k+1)$ can be:

$$\vec{Y}_{s^*}(k+1) = (y^+_1(k) - \delta^+_1(k+1), ...,$$
$$y^-_i(k) - \delta^-_i(k+1), ..., y^+_m(k) - \delta^+_m(k+1))^T.$$

Moreover, $0 \le \delta^+_i(k+1) \le y^+_i(k) - y^-_i(k)$, $0 \le \delta^-_i(k+1) \le y^+_i(k) - y^-_i(k)$.

Taking into account the aforementioned it is evident that the iterative procedure (8) can be rewritten in the following equivalent form:

$$\prod_{i=1}^{m} (y^+_i(k+1) - y^-_i(k+1))^2 \times \det(F_m F_m^T)^{-1} \xrightarrow{y^+_i(k+1), y^-_i(k+1),\, i=1,...,m} \max. \tag{21}$$

Let's conduct equivalent transformations of Eq. (21) on the basis of substitutions by Eqs. (19) and (20) and then find the logarithm from a result, and also, replacing a condition (9) by Eqs. (17) and (18), then the task of mathematical programming is to be obtained:

$$2 \cdot \sum_{i=1}^{m} \ln(y^+_i(k) - \delta^+_i(k+1) - y^-_i(k) - \delta^-_i(k+1)) + \ln(\det(F_m F_m^T)^{-1}) \xrightarrow{\delta^-_i(k+1),\, \delta^+_i(k+1),\, i=1,...,m} \max, \tag{22}$$

$$y^-_{k+1} - \vec{\varphi}^T(\vec{x}_{k+1}) \cdot F_m^{-1} \cdot \vec{Y}_{s^*}(k+1) = 0, \tag{23}$$

$$\vec{\varphi}^T(\vec{x}_{k+1}) \cdot F_m^{-1} \cdot \vec{Y}_{s^*}(k+1) - y^+_{k+1} = 0, \tag{24}$$

$$0 \le \delta^+_i(k+1) \le y^+_i(k) - y^-_i(k),$$
$$0 \le \delta^-_i(k+1) \le y^+_i(k) - y^-_i(k). \tag{25}$$

As a configuration of an area the estimations is given, then in Eq. (22) it is possible to omit an element $\ln(\det(F_m F_m^T)^{-1}$ that simplifies an optimization task

$$\sum_{i=1}^{m} \ln(y^+_i(k) - \delta^+_i(k+1) - y^-_i(k) - \delta^-_i(k+1)) \xrightarrow{\delta^-_i(k+1),\, \delta^+_i(k+1),\, i=1,...,m} \max. \tag{26}$$

Taking into account the monotonicity of the function $\ln(x)$, the mathematical programming task with conditions (22)–(25) can be solved by methods of linear programming and actually reduced on step $k+1$ to moving the no more m facets of polyhedron $\Omega_m(k)$. This means that on step $k+1$ for a given i the next rule is true: if $\delta^+_i(k+1) \neq 0$ then $\delta^-_i(k+1) = 0$, and conversely if $\delta^-_i(k+1) \neq 0$, then $\delta^+_i(k+1) = 0$.

Let's notice that in the process of the calculable procedure realization which consists in solving the task of mathematical programming (22)–(25) on $k+1$ step simultaneously in a space of tolerance parameters no more m facets of polyhedron $\Omega_m(k)$ is moved.

The realization of the tolerance estimation method foresees using the algorithm which on step 1 sets the tolerance area configuration and after that $N - m$ iteration steps.

Let's consider step-by-step of a numerical algorithm in detail.

Step 1. Setting the configuration matrix of estimation area.

Step 2. Calculation of the functions $L_s(k)$ or $L'_s(k)$ by Eqs. (12) or (13).

Step 3. Solving $L_{s^*}(k) = \max_{s=1,...,2^m} L_s(k)$, $L'_{s^*}(k) = \max_{s=1,...,2^m} L'_s(k)$.

Step 4. Solving the mathematical programming task with an goal function (22)–(25).

Step 5. Calculation of the interval bounds $y_i^-(k+1)$; $y_i^+(k+1)$ by Eqs. (19) and (20), accordingly.

Step 6. If $k \leq N-m$ then go to step 2 else end of the procedure.

The result of calculations by a numerical algorithm is a tolerance polyhedron $\widetilde{\Omega}_m$ which tops are determined by equation:

$$\vec{b}_s(k) = F_m^{-1} \cdot \vec{Y}_s(k = N-m). \tag{27}$$

An ellipsoid estimation of polyhedron $\widetilde{\Omega}_m$ is found according to the theorem from [10, 11] and has the following form:

$$Q_m = \{\vec{b} \in R^m \Big| (\vec{b} - \vec{\overline{b}})^T \cdot F_m^T \cdot E^{-2} \cdot F_m \cdot (\vec{b} - \vec{\overline{b}}) \leq 1\}, \tag{28}$$

where $\vec{\overline{b}} = F_m^{-1} \cdot (0.5 \cdot (y_1^+(N-m) + y_1^-(N-m)), ..., 0.5 \times \times (y_m^+(N-m) + y_m^-(N-m)))^T$ is a vector which sets the center of ellipsoid; $E = diag(y_1^+(N-m) - y_1^-(N-m), ..., y_i^+(N-m) - y_i^-(N-m), ..., y_m^+(N-m) - y_m^-(N-m))$ is a diagonal matrix of resultant interval errors.

On the whole the offered method is reduced to the iteration procedure of solving the linear programming tasks. However, as it is evident from the higher resulted algorithm the most complex calculable procedure is calculations on step 2, which enable to find limitations (23) and (24). Moreover, this procedure repeats oneself iteratively, and its realization has a high time complexity, especially in a case of the high dimension task.

Therefore, in the paper parallelization of the procedure of calculation of the functions $L_s(k)$ ($L'_s(k)$) is offered.

3. Parallelization of calculations of the tolerance ellipsoid estimations

The important property of hyperparallelepiped $\Omega_m(k)$ is that the tops $\vec{b}_s(k)$ belong to one plane if in Eq. (14) the proper vectors $\vec{Y}_s(k)$ have at least one joint component. Thus, iterative "moving" of bounds $y_i^-(k)$ and $y_i^+(k)$ by Eqs. (19), (20) in space of parameters is equivalent to moving of the proper borders of hyperparallelepiped $\Omega_m(k)$ in a direction of its size diminishing.

As it is shown above, the analysis of the values of functions $L_s(k)$ and $L'_s(k)$ allows to find the hyperparallelepiped tops location in relation to hyperplane which is formed by $k+1$ equation.

Let's find a recurrent formula for $L_s(k+1)$. For this purpose in Eq. (14) instead of i component $y_i^-(k+1)$ of vector $\vec{Y}_s(k+1)$ its value $y_i^-(k) + \delta_i^-(k+1)$ from Eq. (19) is put and the recurrent formula is achieved:

$$\vec{b}_s(k+1) = \vec{b}_s(k) + \vec{f}_i \cdot \delta_i^-(k+1), \tag{29}$$

where $\vec{f}_i$ is i column of matrix F_m^{-1}.

Considering (29) a calculation formula for values $L_s(k+1)$ of $k+1$ iteration is:

$$L_s(k+1) = L_s(k) - \vec{\varphi}^T(\vec{x}_{k+1}) \cdot \vec{f}_i \cdot \delta_i^-(k+1). \tag{30}$$

As follows from Eq. (30), that with positive value $\delta_i^-(k+1)$ the simultaneous implementation of conditions (15) and (16) is possible only in the case $\vec{\varphi}^T(\vec{x}_{k+1}) \cdot \vec{f}_i > 0$.

Using properties of hyperparallelepiped $\Omega_m(k)$ the following assertion is true.

Assertion 1. Let the vector of coordinates of the some top $\vec{b}_{s^*}(k)$ of hyperparallelepiped $\Omega_m(k)$ is known, then coordinates of tops $\vec{b}_s(k)$ $s = 1, ..., m$ which lie on the edges that come from this top, is calculated by such formula

$$\vec{b}_s(k) = \vec{b}_{s^*}(k) \pm \Delta_i(k) \cdot \vec{f}_i, \qquad i = 1, ..., m, \tag{31}$$

where $\Delta_i(k) = y_i^+(k) - y_i^-(k)$ and its sign is "+" if in Eq. (14) for calculation of top $\vec{b}_{s^*}(k)$ the component i of vector $\vec{Y}_{s^*}(k)$ is $y_i^-(k)$ and its sign is "-" if this component is $y_i^+(k)$.

After substituting recurrent Eq. (31) with the sign "+" and "-" before $\Delta_i(k)$ in proper Eqs. (12), (13) which are used for calculation $L_s(k)$ and $L'_s(k)$ get

$$\begin{aligned} L_s(k) &= L_{s^*}(k) - \Delta_i(k) \cdot \xi_i, \\ L'_s(k) &= L'_{s^*}(k) - \Delta_i(k) \cdot \xi_i, \\ & i = 1, ..., m, \end{aligned} \tag{32}$$

where $L_{s^*}(k)$, $L'_{s^*}(k)$ are scalar functions that calculated for fixed top $\vec{b}_{s^*}(k)$; $\xi_i = \vec{\varphi}^T(\vec{x}_{k+1}) \cdot \vec{f}_i$.

Let's notice that values of all ξ_i, $i = 1, ..., m$ can be calculated before beginning the k iteration.

Let's define an optimum, that is without reiterations, sequence of recurrent calculations of values $L_s(k)$ and $L'_s(k)$. For this purpose conditionally all tops of hyperparallelepiped $\Omega_m(k)$ are divided into two groups of 2^{m-1} tops, so that for tops of the first group it is possible to calculate values $L_s(k)$ by a proper part of recurrent Eq. from (32) and then $L'_s(k)$ by an equation:

$$L'_s(k) = -L_s(k) - \Delta_{k+1}.$$

For the second group at first also a proper part of recurrent Eq. from (32) for a calculation $L'_s(k)$ is used and then a value $L_s(k)$ is calculated by equation:

$$L_s(k) = -L'_s(k) - \Delta_{k+1}.$$

Under such calculation conditions in the first group of tops should be the top $\vec{b}_s(k)$ which coordinates are calculated by Eq. (14) that includes vector $\vec{Y}_s(k)$ consisting only of components $y_i^-(k)$, $\forall\ i = 1, ..., m$. Similarly, the top of the second group includes vector $\vec{Y}'_s(k)$ which contains only the component $y_i^+(k)$, $\forall\ i = 1, ..., m$. Let these tops in the groups have number $s = 1$. Then, using Eqs. (12) and (13) with replacement of appropriate vectors $\vec{b}_s(k)$ by Eq. (14) on $F_m^{-1} \cdot \vec{Y}_{s=1}(k)$ and $F_m^{-1} \cdot \vec{Y}'_{s=1}(k)$, the ratio for calculating the function values $L_{s=1}(k)$ and $L'_{s=1}(k)$ for top $s = 1$ of the first and second groups is get

$$L_{s=1}(k) = y_{k+1}^- - \vec{\varphi}^T(\vec{x}_{k+1}) \cdot F_m^{-1} \cdot \vec{Y}_{s=1}(k), \tag{33}$$

$$L'_{s=1}(k) = \vec{\varphi}^T(\vec{x}_{k+1}) \cdot F_m^{-1} \cdot \vec{Y}'_{s=1}(k) - y_{k+1}^+, \tag{34}$$

where $\vec{Y}_{s=1}(k)$ and $\vec{Y}'_{s=1}(k)$ are vectors which include only components $y_i^-(k)$ and $y_i^+(k)$ $\forall\ i = 1, ..., m$ accordingly.

Considering Eqs. (33) and (34) the diagram of relationship between given recurrent ratios (32) is represented by two parallel calculable graphs.

The parallel calculable graphs for the case $m = 5$ are showed in Fig. 3.

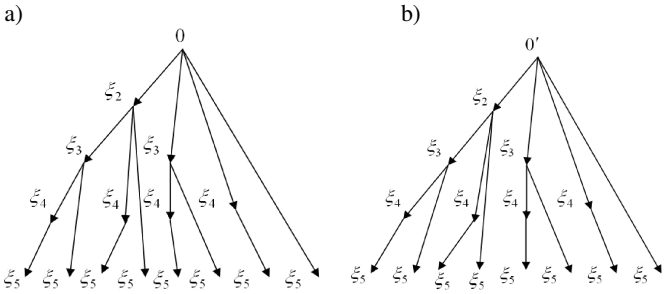

Fig. 3. Computational graphs on iteration k of localization method

The top of the first graph (see Fig. 3a) marked "0" is in accordance with the calculation Eq. (33) and Eq. (34) is in accord with a top "0′" of second graph (see Fig. 3b). Other tops of both graphs marked by characters ξ_i $(i = 2, ..., m)$ are in accord with recurrent Eqs. (32). Let's fix any top on the first graph (Fig. 3a) for which the known value of function $L_{s^*}(k)$ corresponds. Then it moves downward on the edges of graph to the contiguous tops ξ_i. For calculation of the values $L_s(k)$ in these tops Eq. from (32) is used, in which a value ξ_i is chosen on the basis of marking the received tops of a graph.

On the second graph for a calculation $L'_s(k)$ similar procedures are applied.

The analysis of these graphs shows that a sequence of calculations which are implemented in order a "top-down" and "right-left" include the total number of intermediate values $L_{s^*}(k)$ $(L'_{s^*}(k))$ not more $m - 1$. These values must be remembered for calculations of the functions for the next tops by recurrent Eqs. (32). Let's denote an array of elements $L^*_j(k)$, $j = 1, ..., m-1$ which is designed to store intermediate values of calculations. Then using the graph (Fig. 3a) the order of calculations $L_s(k)$, $L'_s(k)$ $(s = 1, ..., 2^{m-1})$ for current top of hyperparallelepiped and values of elements $L^*_j(k)$, $j = 1, ..., m-1$ is determined.

If $s = 1$ then conduct next calculations:

1. Calculation of the function value $L_{s=1}(k)$ (for the top "0") by Eq. (33) and Eq.:
$$L'_s(k) = -L_s(k) - \Delta_{k+1}.$$
2. Setting $L^*_j(k) = L_{s=1}(k)\ \forall\ j = 1, ..., m-1$.
If number $s-1$ is odd then
$$L_s(k) = L^*_{m-1}(k) - \Delta_m(k) \cdot \xi_m,$$
$$L'_s(k) = -L_s(k) - \Delta_{k+1},$$
and values $L^*_j(k)$ remain unchanging for all $j = 1, ..., m-1$.
If $s-1$ is even then

a) determining the number p by a condition of simple division
$$(s-1)/\max_{p=1,..,m-2} 2^p.$$

The number p sets the amount of arcs in graph Fig. 3a which come out from this top. Accordingly, the calculated value $L_s(k)$ in this top is to be used for calculation of the p values of scalar functions of other tops;

b) calculating the values $L_s(k)$ and $L'_s(k)$ by Eqs. accordingly:
$$L_s(k) = L^*_{m-p}(k) - \Delta_{m-p}(k) \cdot \xi_{m-p},$$
$$L'_s(k) = -L_s(k) - \Delta_{k+1};$$

c) setting $L^*_{m-1}(k) = ... = L^*_{m-p-1}(k) = L_s(k)$.

The similar order of calculations must be applied for the graph Fig. 3b with replacement $L_s(k)$ on $L'_s(k)$ and Eq. $L'_s(k) = -L_s(k) - \Delta_{k+1}$ on an Eq. $L_s(k) = -L'_s(k) - \Delta_{k+1}$.

Graphs shown in Fig. 3, allow to perform recurrent procedures for the values calculation of scalar functions $L_s(k)$ and $L'_s(k)$ for all tops of hyperparallelepiped. However, a full implementation of the computational graph of the current iteration is undesirable due to considerable computational and time costs. For example the tests showed that the full implementation of both graphs for the task dimension $m = 30$ (2^{30} tops of hyperparallelepiped) in one iteration run take about 3.5 seconds of machine time. The tests conducted on a computer processor type CELERON, with a clock speed of 2.3 GHz and RAM 1 GB.

The above considerations allow to construct an efficient implementation of computations on each iteration of the localization method as a software module LOCNAS. The software interface of this module allows to set or to calculate the configuration of localization set and also to input the structure of interval model and observations in interval kind. A computational scheme built on the graphs (Fig. 3) allows for parallelization of a computing process. So, to implement this it is advisable to use the multiprocessor or multicomputer system.

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