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ANALYSIS OF RELIABILITY AND STABILITY OF BAR STRUCTURES

U. RADOŃ[1]

In the paper, the Hasofer-Lind index is applied for determining the probability of stability loss of truss structure under random load. In 1974 Hasofer-Lind proposed a modified reliability index that did not exhibit the invariance problem. The "correction" is the evaluation the limit state function at a point known as the "design point", instead of the mean values. The design point is generally not known a priori, an iteration technique must be used to find out the reliability index. The paper shows how the reliability index changes under the influence of different variables mean value, standard deviation, and probability density function.

Key words: probability density function, reliability index, design point, stability loss, node snapping.

1. Introduction

The presented study considers the problems of stability and reliability of bar structure. A special attention is given to truss structure subjected to considerable displacements and susceptible to stability loss from the condition of node snapping. Nonlinear geometrical relations are defined in the Lagrangian description. Stability analysis of structure is executed by means of the finite element method. In the paper the current stiffness parameter method and the constant arc length method are used for the determination of equilibrium path. Let us now ask a question: what is the advantage of the inclusion of reliability analysis methods to the analysis of stability? The answer is following: using the methods of reliability analysis, moving along the equilibrium path of structure, we can determine the level of probability of failure when we approach the critical point. Load parameters are assumed as random variables. Probability distributions of random variables are assumed from among several ones most often applied in practical solutions. The condition of non exceeding admissible displacements of structure nodes is considered.

The first part of the study defines concepts connected with the stability of structure: equilibrium path and numerical techniques. Then Cornell's and Hasofer-Lind's indexes, which are measures of structure reliability, will be described. The Hasofer-

[1] PhD. Faculty of Civil and Environmental Engineering, Kielce University of Technology Al. 1000-lecia PP7, 25-314 Kielce, Poland, e-mail: zmbur@tu.kielce.pl

-Lind's index will be used in the FORM method to the analysis of the behaviour of reliability index while moving along the equilibrium path. The examples provide an analysis how the Hasofer-Lind reliability index changes under the influence of different variables of mean value, standard deviation, and probability density function when it approaches the limit value of displacement.

2. The equilibrium path

We can present the potential energy of the system as a function of a displacement vector $\boldsymbol{q}$ and a load vector $\boldsymbol{Q}$

$$V = V(q, Q) \tag{2.1}$$

Assuming the vector $\boldsymbol{Q}$ as a proportional, conservative, one-parameter load, we can write

$$Q = \mu \cdot P \tag{2.2}$$

where:

μ – load multiplier,

$\boldsymbol{P}$ – comparative load vector.

Change of μ parameter leads to changes of the displacement state which is described by the vector $\boldsymbol{q}$. The successive solutions generate a curve called an equilibrium path in an (N+1) dimensional space $(q_1, q_2, ..., q_N, \mu)$.

We are considering a point $A(\tilde{q}, \tilde{\mu})$ on the equilibrium path. This does not refer to a singular point nor to its vicinity. Then we assume that we are allowed to describe the change of the state system by the change of μ parameter. We look for a new point $B(\tilde{q} + \Delta q, \tilde{\mu} + \Delta\tilde{\mu})$ on the equilibrium path.

The value of potential energy at B point close to A point has the form:

$$\begin{aligned} V_B = V(\tilde{q} + \Delta q, \tilde{\mu} + \Delta\tilde{\mu}) = V(\tilde{q}, \tilde{\mu}) + \frac{\partial V(\tilde{q}, \tilde{\mu})}{\partial q}\Delta q + \frac{\partial V(\tilde{q}, \tilde{\mu})}{\partial \mu}\Delta\mu + \\ + \frac{1}{2!}\left[\frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q^2}(\Delta q)^2 + 2\frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q \partial \mu}\Delta q \Delta\mu + \frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial \mu^2}(\Delta\mu)^2\right] + ... \\ + \frac{1}{n!}\left[\sum \begin{pmatrix} n \\ k \end{pmatrix} \frac{\partial^n V(\tilde{q}, \tilde{\mu})}{\partial q^{n-k} \partial \mu^k}(\Delta q)^{n-k}(\Delta\mu)^k\right] \end{aligned} \tag{2.3}$$

Let us expand into the Taylor series the first derivative of potential energy to expand around a point $\dfrac{\partial V}{\partial q}$

(2.4)
$$\frac{\partial V_B}{\partial q} = \frac{\partial V(\tilde{q} + \Delta q, \tilde{\mu} + \Delta\mu)}{\partial q} = \left[\frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q^2}\Delta q + \frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q \partial \mu}\Delta\mu\right] + \\ + \frac{1}{2!}\left[\frac{\partial^3 V(\tilde{q}, \tilde{\mu})}{\partial q \partial \mu^2}(\Delta\mu)^2 + 2\frac{\partial^3 V(\tilde{q}, \tilde{\mu})}{\partial q^2 \partial \mu}\Delta q \Delta\mu + \frac{\partial^3 V(\tilde{q}, \tilde{\mu})}{\partial q^3}(\Delta q)^2\right] + ...$$

Neglecting in the above expression the nonlinear term of the increments Δq, $\Delta\mu$ and assuming the configuration $(\tilde{q} + \Delta q)$ and $(\tilde{\mu} + \Delta\mu)$ to be also an equilibrium state, the linearized incremental equation set can be written in the form

(2.5)
$$\frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q^2}\Delta q + \frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q \partial \mu}\Delta\mu = 0$$

If an external load $\boldsymbol{Q}$ is conservative, then according to Castigliano's theorem it is

(2.6)
$$-P = \frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q \partial \mu}$$

The above relations bring us to the equation set (5) in the matrix form

(2.7)
$$K_T(q, \mu) \cdot \Delta q - \Delta\mu P = 0$$

where:

$\frac{\partial^2 V(\tilde{q}, \tilde{\mu})}{\partial q^2} = K_T$ – tangent stiffness matrix of the system.

For the solution of the set of nonlinear equations the incremental-iterative method of the current stiffness parameter and constant arc length are applied.

3. Description of numerical techniques

In the first method a certain indicator proposed by Bergan and Soreide [1, 2] and referred as the current stiffness parameter, is very useful in the solution of the set of nonlinear equations.

The current stiffness parameter (CSP) is the ratio between the scaled quadratic forms of the incremental stiffness in initial and current steps, respectively.

(3.1)
$$CSP = \frac{\Delta q^{oT} \cdot K_T^o \cdot \Delta q^o}{\Delta q^{iT} \cdot K_T^i \cdot \Delta q^i}$$

It is a measure of changes of stiffness matrix $\boldsymbol{K}_T$ of the system during motion in N-dimensional displacement space of solutions. The current stiffness parameter can have many different applications:

- estimation of the system stiffness by a changing variable value,
- estimation of stability of the investigated segment of an equilibrium path by checking the changing sign,
- selection of effective step length,
- control near limit points.

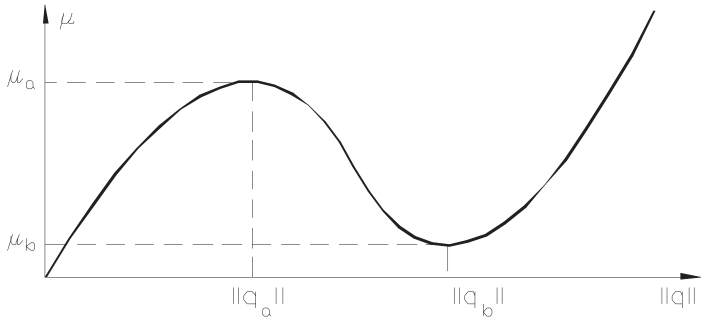


Fig. 1. Dependence of load parameter μ of the norm $||\mathbf{q}||$.
Rys. 1. Zależność parametru obciążenia μ od normy $||\mathbf{q}||$

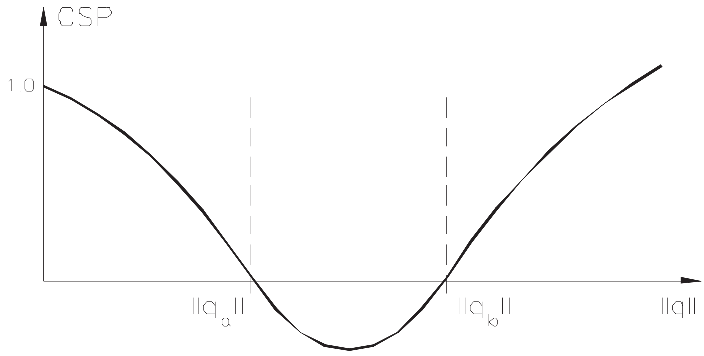


Fig. 2. Dependence of CSP parameter on the norm $||\mathbf{q}||$.
Rys. 2. Zależność parametru CSP od normy $||\mathbf{q}||$

Figure 1 shows a typical snap-through problem (load parameters μ versus some norm of displacement vector $||\mathbf{q}||$). The associated curve for CSP as a function of $||\mathbf{q}||$ is traced in Fig. 2. It is noticeable that at the extreme points of the load-displacement curve CSP has the value zero. In this situation the incremental stiffness matrix $\boldsymbol{K}_T$ is singular. CSP is positive for the stable branches of the load-displacement curve. The instable configurations are characterized by negative values of CSP. The current stiffness parameter may be actively used in the selection of effective step length. The basic idea is that the change in CSP should be close to the same for all load steps. This implies that the incremental stiffness should be allowed to change by a prescribed magnitude for each new step. Figure 3 gives an illustration of the process. The anticipated change in CSP per step is denoted ΔCSPc. This quantity is given as input to the computer program.

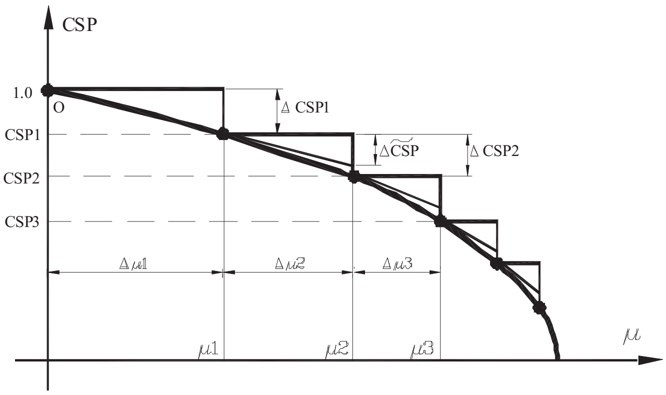


Fig. 3. Estimation of effective step length.
Rys. 3. Oszacowanie efektywnej długości kroku przyrostowego

The current stiffness parameter may actively be used in controlling of iteration around extreme points (Fig. 4).

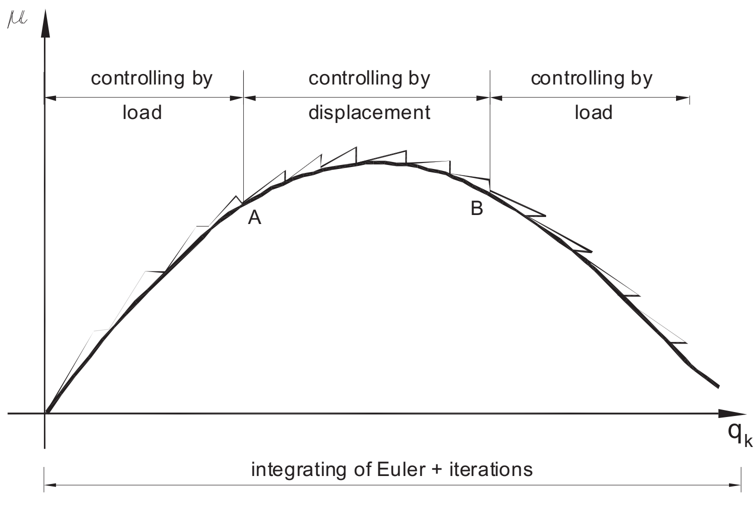


Fig. 4. Control of iteration around extreme points.
Rys. 4. Sterowanie procesem iteracyjnym w otoczeniu punktów typu maksimum

In the second method the solution of nonlinear equations requires a starting configuration (or initial iterate) which is "close" to the solution to be determined. This requirement fits well into the scenario of an incremental procedure which is designed to solve an equilibrium path in terms of a sequence of successive but distinct points. Each point obtained offers means to construct the starting configuration for the next to be computed. The accuracy of the "initial" iterate can be controlled by keeping the distance between the known and the still unknown point within certain bounds.

Two well known strategies are shown in Fig. 5a and 5b. In the first case, the load parameter μ is used as the prescribed variable. In the second case one of the displacement parameters q is taken to fulfil this role. Each point computed by the first method is determined by the intersection of a surface $\mu = \eta$ and the equilibrium path. A point computed by the second method is determined by the intersection of a surface $q = \eta$ with the same solution curve. Both methods fail in the neighbourhood of the turning points Figs. 6a and 6b.

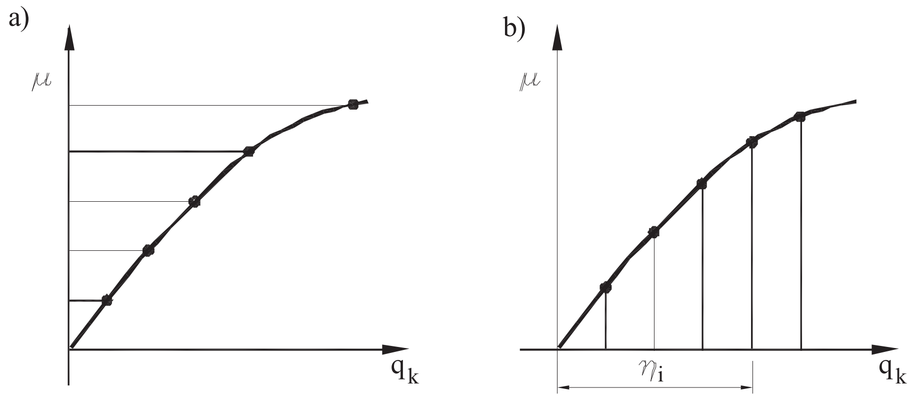


Fig. 5. Control of the incremental process by a load and displacement parameter.
Rys. 5. Sterowanie procesem przyrostowym przy pomocy parametru obciążenia i przemieszczenia

The breakdown of the procedures described does usually not occur suddenly, but is announced some time in advance by a marked increase in number of iterations necessary to obtain converged solutions. This phenomenon is coupled with the decrease in quality of the intersection of the surfaces with the equilibrium path, when the critical points are approached.

A measure of quality of intersection is given by θ, the angle between the tangent of the equilibrium curve and the normal to the intersecting surface at the point of intersection. The intersection is considered good if θ is close or equal to zero, and bad if it is close or equal to $\pi/2$. In this sense the ideal would be a family of surfaces which intersects the equilibrium curve everywhere according to the condition $\theta = 0$.

Of course, it is not possible to construct such a set of "ideal" surfaces, because this would require advance knowledge of the solution curve.

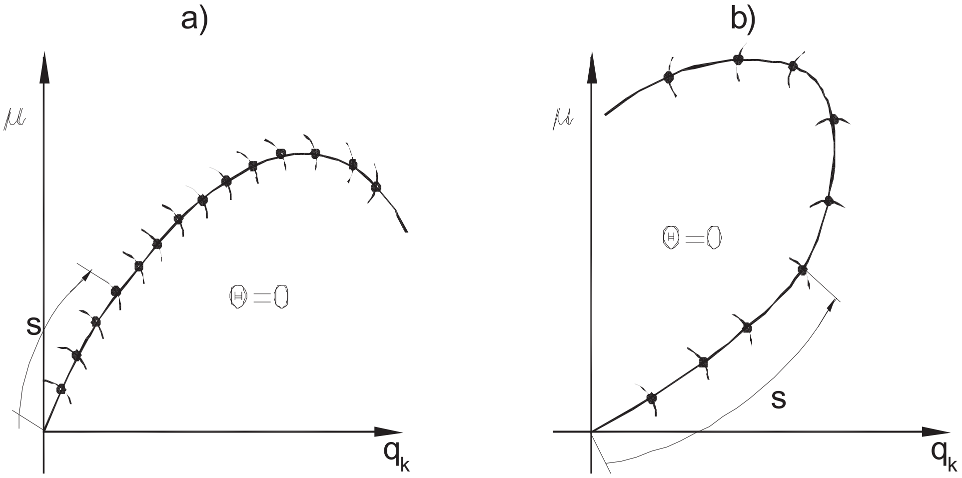


Fig. 6. The limit points of load (a) and displacement (b).
Rys. 6. Punkty graniczne obciążenia (a) i przemieszczenia (b)

We can be satisfied by a certain approximation which enables angle θ to be close to zero. Riks [3] proposed an additional equation, called the constraint equation in the form:

$$(\dot{q})^T_\alpha \cdot (q - q_\alpha) + \dot{\mu}_\alpha \cdot (\mu - \mu_\alpha) = (\eta - \eta_\alpha) \tag{3.2}$$

where:
dots denote derivatives with respect to the length of arc,

q – generalized coordinates vector,

$\eta - \eta_\alpha$– parameter approximating the arc length.

This equation defines a surface, which is normal to the tangent $[\dot{\mathbf{q}}_\alpha, \dot{\mu}_\alpha]$ and its distance to $[q_\alpha,\mu_\alpha]$ is $(\eta - \eta_\alpha)$. It is intersected by the equilibrium curve with a small angle θ if the distance $\eta - \eta_\alpha$ is kept small.

4. Cornell reliability index

After defining conceptions of stability analysis, we now show the basic assumptions and measures of reliability, Cornell index [4] and Hasofer-Lind index [5]. The problems of reliability analysis of structures considered in this study are based on the following premises. Firstly, it is assumed that structure can be in one of two admissible states: safe state or failure state. Failure is understood as non-fulfilment of a certain limitation imposed by the designer on the performance of structure. It is assumed that parameters describing the state of structure are treated as random variables (as contrasted with random processes). The considered problems of reliability analysis will be related to the so called element reliability. The element is understood here as all the structure or its part, whose state is determined by one limit state function (failure criterion). It is necessary to underline here that this study is not concerned with system reliability in which many limit state functions are taken into account, and the possibility of the sequence of failures of particular elements, which leads to the destruction of the whole structure.

In practical solutions we often do not have detailed information about the type of distribution for each random variable. For calculating Cornell reliability index we use only information about mean values and standard deviations of random variables. When the limit state function is nonlinear, we can obtain an approximate answer by linearization function using a Taylor series expansion. The linearization point is a point corresponding to the mean values of the random variables.

$$g(X) \approx \overline{g}(X) = g\left(X^0\right) + \sum_{i=1}^{n} \frac{\partial g(X)}{\partial x_i}\Big|_{x=X^0} \left(X_i - X_i^0\right) \tag{4.1}$$

Expectation value and variance function $\overline{g}(X)$ can be written as

$$\begin{aligned} g^0(X) &\approx \overline{g}^0(X) = g\left(X^0\right) \\ \sigma_g^2(X) &\approx \nabla g^T(X)\big|_{x=X^0} \, C_X \nabla g(X)\big|_{x=X^0} \end{aligned}, \tag{4.2}$$

where $\nabla g(X)|_{x=X^0}$ is a gradient function $g(X)$ computed for mean values vector $\boldsymbol{X}$ and $\boldsymbol{C}_X$ is a covariance matrix.

Probability of failure for the condition function $[\overline{g}(X) \leq 0]$ based on a linearized limit function has the form:

$$P[g(X) \leq 0] \approx P[\overline{g}(X) \leq 0] = P\left[\frac{\overline{g}(X) - \overline{g}^0(X)}{\sigma_{\overline{g}}} \leq \frac{\overline{-g}^0(X)}{\sigma_{\overline{g}}}\right] = \Phi(-\beta_C), \tag{4.3}$$

where β_C is Cornell reliability index

$$\beta_C = \frac{\overline{g}^0(X)}{\sigma_{\overline{g}}(X)} \tag{4.4}$$

The reliability index defined in equation (4.4) is called a first-order second-moment mean value reliability index. It is a long name, but the underlying meaning of each part of the name is very important. First order, because we use first-order terms in the Taylor series expansion. Second moment, because only means and variances are needed. Mean value, because the Taylor series expansion is around the mean values.

The method has both advantages and disadvantages in the structural reliability analysis. Its advantages include:
it is easy to use;
it does not require knowledge of the distributions of random variables.

Its disadvantages include:
results are inaccurate if the tails of the distribution functions cannot be approximated by a normal distribution;
the value of the reliability index depends on the specific form of the limit state function.

The invariance problem can be avoided using the Hasofer-Lind reliability index.

5. Hasofer-Lind reliability index

In 1974 Hasofer-Lind proposed a modified reliability index that did not exhibit the invariance problem. The "correction" is to evaluate the limit state function at a point known as the "design point" instead of the mean values. The design point is a point on the failure surface. The design point is generally not known *a priori*, an iteration technique must be used to solve for the reliability index.

Let us consider a limit state function $g(X_1, ..., X_n)$ where the random variables X_i are all uncorrelated. The limit state function is rewritten in terms of the standard form of the variables (reduced variables) using

$$Z_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad \text{and} \quad X_i = \mu_{X_i} + Z_i \sigma_{X_i}. \tag{5.1}$$

Replace$X_1,\ldots,X_n$ by $Z_1,\ldots,Z_n$, we obtain a new limit state function:

$$g'(Z_1, ..., Z_n). \tag{5.2}$$

The Hasofer-Lind reliability index is defined as the shortest distance from the origin of the reduced variable space to the limit state function $g'(Z_1, ..., Z_n) = 0$. It is the most probable point of failure from among all points in this area. If failure occurs, it is most likely to occur just at this point. Finding a design point is a task for non-linear programming with limitations. The accuracy of results obtained with the use of the Hasofer-Lind index is often sufficient for practical needs. The index gained a considerable popularity as a reliability measure, particularly in conjunction with transformation methods which use full information about random variable distributions, e.g. in the FORM method discussed in the next section.

6. METHOD OF FIRST-ORDER RELIABILITY ANALYSIS FORM

The FORM method is one of most effective approximate methods of the calculation of reliability measures. In a general case, when the probability distribution of vector $\boldsymbol{X}$ of base variables is not a vector with the Gaussian distribution, transformation is used to reduce this vector to the Gaussian vector whose coordinates are independent of standard normal variables. The existence of this type of transformation and the manner of its structure was shown for the first time by ROSENBLATT [6] for the case when coordinates of vector $\boldsymbol{X}$ have uniform distributions. HOHENBICHLER and RACKWITZ [7] adapted this transformation to reliability calculations. The transformation of basic random variables to the Gaussian standard space must ensure the equivalence of the formulation of the reliability problem. The probability of failure, defined in space $\boldsymbol{X}$, must be equal to the probability defined in space $\boldsymbol{Z}$.

$$P_f = \int_{\Omega_f} f_X(x)dx = \int_{\Delta_f} \prod_{i=1}^{n} \varphi(u_i)dz_1 dz_2 \dots dz_n \tag{6.1}$$

where:

$f_X(\boldsymbol{x})$ is a joint probability density function of basic random variables , Ω_f is failure region in space $\boldsymbol{X}$, Δ_f is failure region in space $\boldsymbol{Z}$.

The transformation of regions can be written as:

$$\Omega_f = \{x : g(x) \le 0\} \to \Delta_f = \{z : G(z) \le 0\}. \tag{6.2}$$

The limit state function is transformed in this way:

$$g(x) = 0 \quad \to \quad g\left[T^{-1}(z)\right] = G(z) = 0 \tag{6.3}$$

The dependent random vector $\boldsymbol{X}$ may be transformed to the independent uniformly distributed random vector $\boldsymbol{Z}$ through the Rosenblatt transformation given by:

$$\Phi(z_1) = H_1(x_1) = F_1(x_1) = \int_{-\infty}^{x_1} f_1(t)\mathrm{d}t, \tag{6.4}$$

$$\Phi(z_2) = H_2(x_2|\, x_1) = \int_{-\infty}^{x_2} \frac{f_2(x_1, t)}{f_1(x_1)}\, dt, \tag{6.5}$$

$$\Phi(z_i) = H_i(x_i|\, x_1, x_2, ..., x_{i-1}) = \int_{-\infty}^{x_i} \frac{f_i(x_1, x_2, ..., x_{i-1}, t)}{f_{i-1}(x_1, x_2, ..., x_{i-1})}\, dt, \tag{6.6}$$

where:

$f_i(x_1, x_2, ..., x_i)$is marginal probability density function.

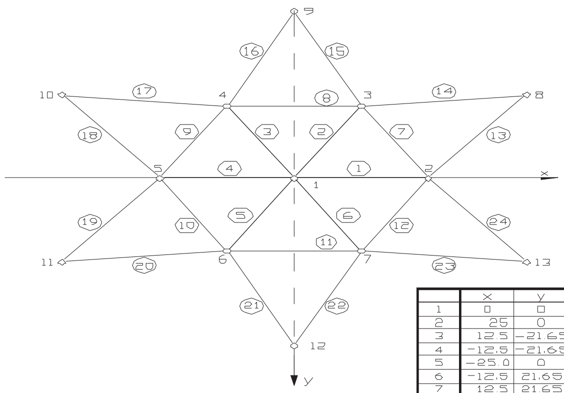
$$(6.7) \qquad f_i(x_1, x_2, ..., x_i) = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} f_X(x_1, x_2, ..., x_n) \mathrm{d}x_{i+1} \mathrm{d}x_{i+2} ... \mathrm{d}x_n.$$

Aside from Rosenblatt's transformation, Hermite's transformation and the transformation resulting from the so called Nataf's model are also applied (A. Der Kiureghian, P.L. Liu[8]). The transformation carries the area of limit state $g(\boldsymbol{X}) = 0$ to another area $G(\boldsymbol{Z}) = 0$. It should be noted that the effective calculation of the integral of the function of the density of the n-dimension of a standard normal distribution along the failure area is still a complex problem, except for the case when $G(\mathbf{Z}) = 0$ is a hyperplane in space. However, two essential properties of the density of the standard normal distribution cause the transformation effective in the calculation of failure probability. The first of these properties is a rotary symmetry around the beginning of the coordinate system. The second property is exponential disappearance of this function together with the square of distance from the beginning of the coordinate system. Therefore, the greatest failure probability originates from the area in the neighbourhood of the point on the limit state area whose distance from the beginning of the coordinate system is the smallest. Therefore, in the FORM method the area of limit state $G(\mathbf{Z}) = 0$ is approximated by a hyperplane tangential to it at the point nearest to the beginning of the coordinate system. This leads to the following approximate formula of failure probability: $p_F = \Phi(-\beta)$. The minimum distance point is called a design point , while β is a reliability index. As the value of the density of normal distribution is highest at this point in the whole failure area, then this point is the highest reliability point. Finding the design point is thus reduced to the solution of the optimization problem. A number of algorithms have been developed to this end. The earliest ones, which originated from the works of HASOFER and LIND [5], as well as those of RACKWITZ and FIESSLER [9], are based on gradient procedures. The subsequent SCHITTKOWSKI [10, 11] and ARORA [12] algorithms made use of the method of sequence square programming. Comprehensive discussion of problems reliability may be found in papers by: NOWAK, COLLINS [13], MADSEN, KRENK, LIND [14] MELCHERS [15], DITLEVSEN, MADSEN(A) [16], THOFT-CHRISTENSEN, Baker, [17] AUGUSTI, BARATTA, CASCIATI [18] and HARRA [19].

7. EXAMPLES

Example 1

The steel truss structure, represented in Fig. 7 is a structure susceptible to stability loss from the condition of node snapping. Using the method of constant arc length and the method of the current stiffness parameter the, the equilibrium path, and consequently, coordinates of the limit point: $q = 0.783$, $\mu = 207.4$ were determined. On the basis of these coordinates, the limit function as the condition of the non exceeding the admissible vertical displacement of node one was formulated. Approaching to the critical point by a changing variable value, the current stiffness parameter CSP is shown in Fig.8.



	x	y	z
1	0	0	0
2	25	0	2
3	12.5	−21.65	2
4	−12.5	−21.65	2
5	−25.0	0	2
6	−12.5	21.65	2
7	12.5	21.65	2
8	43.3	−25	8.216
9	0	50	8.216
10	−43.3	−25	8.216
11	−43.3	25	8.216
12	0	50	8.216
13	43.3	25	8.216

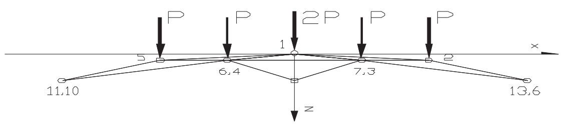


Fig. 7. The elements mesh and geometry of space truss.
Rys. 7. Siatka elementów i geometria kratownicy przestrzennej

Probabilistic independence acting on the structure of loads was assumed in the analysis. The truss is loaded in nodes by concentrated forces whose probability density functions are well-known.

The example provided an analysis how the Hasofer-Lind reliability index changes under the influence of different variables of mean value, standard deviation, and probability density function when it approaches the limit value of displacement node one.

Numerical calculations after reduction of value of standard deviation for normal distribution gave an increase in the value of reliability index of 100%. The value of reliability index for Gumbel distribution increased from 35% (for displacement equal 0.782) to 55% (for displacement equal 0.753). This is illustrated in Fig.10 and in Fig.11. In Fig.9 we can see effect of the accepted type of probability distribution on the value of the Hasofer-Lind reliability index.

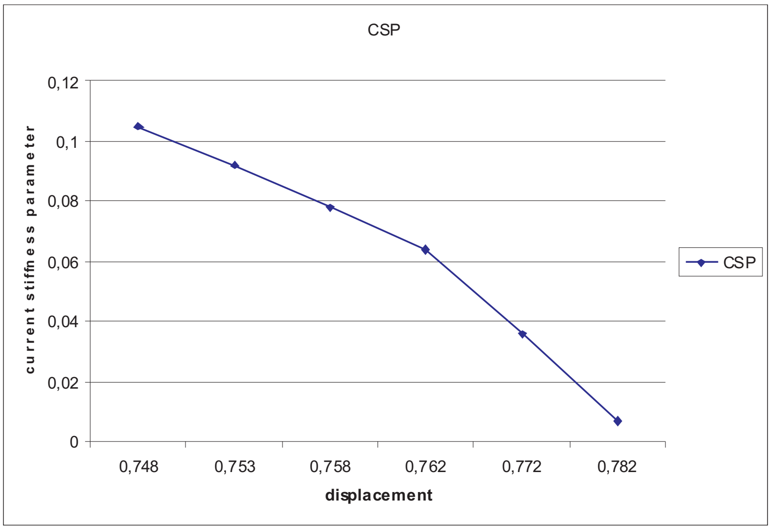


Fig. 8. Dependence of current stiffness parameter CSP on the displacement.
Rys. 8. Zależność skalarnego parametru sztywności od przemieszczenia

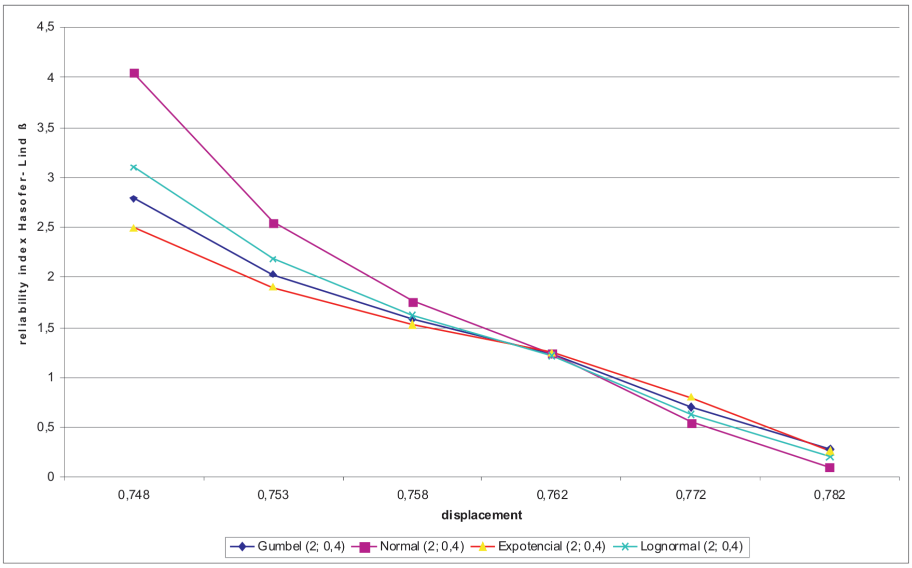


Fig. 9. Effect of the accepted type of probability distribution on the value of the Hasofer-Lind reliability index.
Rys. 9. Wpływ przyjętego typu rozkładu prawdopodobieństwa na wartość wskaźnika niezawodności Hasofera- Linda

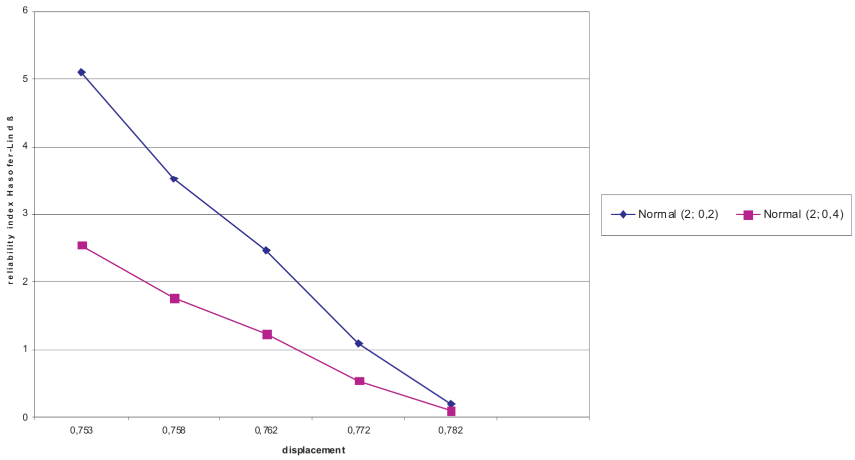


Fig. 10. Effect of the assumed description of normal distribution on the value of the Hasofer-Lind reliability index.
Rys. 10. Wpływ opisu parametrów rozkładu normalnego na wartość wskaźnika niezawodności Hasofera- Linda

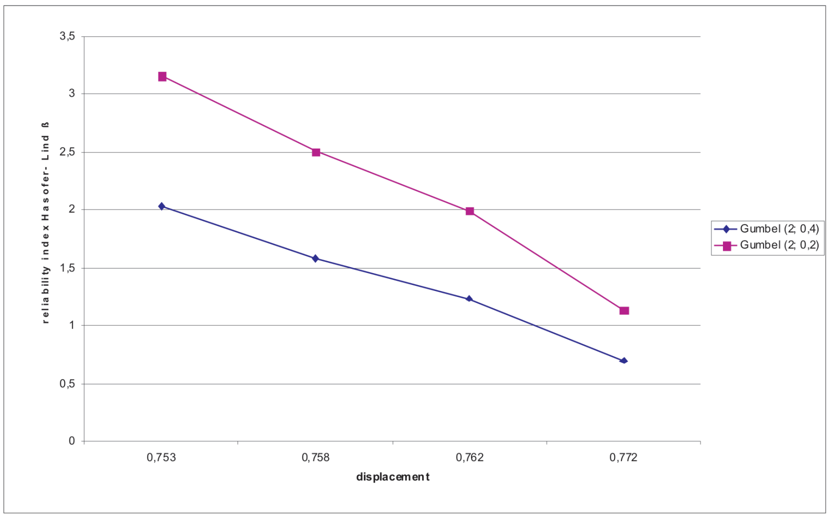


Fig. 11. Effect of the assumed description of Gumbel distribution parameters on the value of the Hasofer-Lind reliability index.
Rys. 11. Wpływ opisu parametrów rozkładu Gumbela na wartość wskaźnika niezawodności Hasofera- Linda

Example 2

The second example is focused on the analysis of the sensitivity of reliability index to the parameters of probability distributions and coordinates of the design point. Knowledge of the sensitivity of reliability index is essential in dealing with the problems of reliability optimization, as well as for a better understanding of the performance of structure. If sensitivity of reliability index due to variable X is small in comparison with other variables, we can assume that the effect of this random variable on the value of failure probability is small too, and we can treat it in subsequent calculations as a deterministic parameter. This statement also concerns the parameters of the distribution of the random variable, such as mean value and standard deviation. In the analyzed example the sensitivity of reliability index for displacement u= 0.777 for normal and Gumbel probability distributions were used in calculations. Forces which loaded structure nodes represented in Fig.7, were random variables in the problem under consideration. Reliability index sensitivities proves the correctness of the stochastic model. No noticeable differences between individual random variables were observed.

NORMAL DISTRIBUTION
Displacement u = 0.777
Probability of failure = 3.744774e-001
Reliability index beta = 0.320018
Beta sensitivity with respect to random variables

4.60608954e+000	-1.58798880e+000	-1.58842216e+000	-1.58842216e+000
-1.58798880e+000	-1.58842216e+000	-1.58842216e+000	

Beta sensitivity with respect to mean values

-4.60608954e+000	1.58798880e+000	1.58842216e+000	1.58842216e+000
1.58798880e+000	1.58842216e+000	1.58842216e+000	

Beta sensitivity with respect of standard deviations

-1.35790229e+000	-8.06990912e-002	-8.07431425e-002	-8.07431425e-002
-8.06990912e-002	-8.07431425e-002	-8.07431425e-002	

GUMBEL DISTRIBUTION
Displacement u = 0.777
Probability of failure = 3.689702e-001
Reliability index beta = 0.334582
Beta sensitivity with respect of random variables

4.66254919e+000	-1.61849706e+000	-1.61893934e+000	-1.61893934e+000
-1.61849706e+000	-1.61893934e+000	-1.61893934e+000	

Beta sensitivity with respect to mean values

-4.66255767e+000	1.61850028e+000	1.61894256e+000	1.61894256e+000
1.61850028e+000	1.61894256e+000	1.61894256e+000	

Beta sensitivity with respect to standard deviations

-6.21927496e-001	-3.34734229e-001	-3.34844051e-001	-3.34844051e-001
-3.34734229e-001	-3.34844051e-001	-3.34844051e-001	

8. Conclusion

Probabilistic calculations were carried out by applying the FORM method. The probability analysis program STAND built in IPPT PAN [20], [21] was used in calculations. It can be seen in the graphs that the sensitivity of the results obtained for an assumed type of probability distribution changes considerably depending on standard deviation. We can note that the adoption of the correct stochastic description is an essential problem in reliability analysis. Incomplete statistical data and improper assumptions concerning probability distributions may cause considerable differences in the evaluation of the safety of structure.

In the stochastic approach in structural mechanics the key problem is the collection of experimental data concerning parameters of random fields of structure and loads. Due to the fact that the accessible experimental results are in general insufficient to carry out the probabilistic analysis by engineers, therefore, there is an evident reluctance to use probabilistic methods. This also concerns the probabilistic numerical methods (as e.g. The Stochastic Finite Element Method), whose complexity is actually hidden inside computer programmes. An engineer's additional effort is necessary when he or she will characterize data by two parameters (expected value and variability coefficient) instead of one required parameter in deterministic methods. Thus, it is necessary to provide engineers with algorithms which make possible the estimation of statistical variable parameters occurring in analysis on the basis of quick reference data.

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ANALIZA NIEZAWODNOŚCI I STATECZNOŚCI KONSTRUKCJI PRĘTOWEJ

Streszczenie

W niniejszej pracy rozważane są zagadnienia stateczności i niezawodności konstrukcji kratowej podatnej na utratę stateczności z warunku przeskoku węzła. Podstawowym problemem w numerycznej analizie zagadnień nieliniowych jest występowanie na ścieżce równowagi punktów osobliwych. W punktach tych zawodzą standardowo stosowane algorytmy rozwiązywania układów równań liniowych. W pracy do określenia ścieżki równowagi konstrukcji wykorzystano metodę skalarnego parametru sztywności oraz metodę stałej długości łuku. Postawmy sobie teraz pytanie co daje nam włączenie do analizy stateczności metod analizy niezawodności. Odpowiedź jest następująca. Korzystając z metod analizy niezawodności możemy poruszając się po ścieżce równowagi konstrukcji określić z jakim poziomem prawdopodobieństwa awarii zbliżamy się do punktu krytycznego. W pracy jako zmienne losowe przyjęto obciążenie węzłów konstrukcji. Rozkłady prawdopodobieństwa zmiennych losowych przyjmowane są spośród kilku, najczęściej stosowanych w praktyce. Rozpatrywany jest warunek nieprzekroczenia dopuszczalnych przemieszczeń węzłów konstrukcji. W analizie niezawodności wykorzystano jako miarę niezawodności wskaźnik Hasofera-Linda. Dokładność wyników otrzymywanych przy użyciu wskaźnika Hasofera-Linda jest wystarczająca dla potrzeb praktycznych i dlatego też zyskał on dużą popularność jako miara niezawodności, szczególnie w połączeniu z metodami transformacji wykorzystującymi pełną informację o rozkładach zmiennych losowych. Obliczenia probabilistyczne przeprowadzono stosując metodę FORM. Do obliczeń wykorzystano program do analizy niezawodności STAND zbudowany w IPPT PAN [20], [21]. Z przedstawionych wykresów widać, że wrażliwość otrzymanych wyników na przyjęty typ rozkładu prawdopodobieństwa zmienia

się znacznie w zależności od odchylenia standardowego. Możemy zauważyć, jak istotnym zagadnieniem w analizie niezawodności jest przyjęcie prawidłowego opisu stochastycznego. Niekompletne dane statystyczne oraz niewłaściwie przyjęte założenia dotyczące rozkładów prawdopodobieństwa mogą prowadzić do poważnych różnic w ocenie bezpieczeństwa konstrukcji.

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