BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 61, No. 4, 2013
DOI: 10.2478/bpasts-2013-0090

Dedicated to Professor M.P. Kaźmierkowski
on the occasion of his 70th birthday

Unified approach to the sliding-mode control and state estimation – application to the induction motor drive

T. ORLOWSKA-KOWALSKA* and G. TARCHALA

Institute of Electrical Machines, Drives and Measurements, Wroclaw University of Technology,
27 Wybrzeże Wyspianskiego St., 50-370 Wroclaw, Poland

Abstract. In this paper a generalized design procedure of the sliding mode systems is described. A unified approach is applied to control and state variables estimation algorithms. Selected solutions are then applied in the induction motor drive system. An identical design procedure is used to design the speed control and MRAS-type speed estimator. Presented algorithms are verified using experimental tests performed on the 3 kW laboratory setup.

Key words: induction motor, sliding mode control, motor torque control, chattering, state estimation, model reference adaptive system (MRAS).

Notation

e	–	estimation error,
f_s	–	stator voltage frequency, [Hz],
$\mathbf{i}_r = i_{r\alpha} + ji_{r\beta}$	–	rotor current vector,
$\mathbf{i}_s = i_{s\alpha} + ji_{s\beta}$	–	stator current vector,
$\mathbf{k}$	–	control signals / estimated variables vector,
L	–	Lyapunov function,
m_e, m_o	–	electromagnetic and load torques,
p	–	Laplace operator,
r_s, r_r	–	stator and rotor winding resistances,
$\mathbf{s}$	–	switching functions vector,
T	–	time constant, [s],
T_M	–	mechanical time constant of drive, [s],
$T_N = 1/(2\pi f_{sN})$	–	time constant, connected with the per unit system introduction, [s],
$\mathbf{u}_s = u_{s\alpha} + ju_{s\beta}$	–	stator voltage vector,
$\boldsymbol{x}$	–	state variables vector,
x_m	–	main reactance ,
$x_r = x_m + x_{r\sigma}$	–	rotor reactance,
$x_s = x_m + x_{s\sigma}$	–	stator reactance,
$x_{s\sigma}, x_{r\sigma}$	–	stator and rotor winding leakage reactance,
Γ	–	gain matrix,
$\gamma_{\psi s,r}$	–	angle of the stator/rotor flux vector,
$\sigma = 1 - x_m^2/(x_s x_r)$	–	leakage coefficient,
$\psi_r = \psi_{r\alpha} + j\psi_{r\beta}$	–	rotor flux vector,
$\psi_s = \psi_{s\alpha} + j\psi_{s\beta}$	–	stator flux vector,
$\psi_{s,r}$	–	amplitude of stator/rotor flux vector,
ω_m	–	mechanical speed.

All variables, written in small letters, are transferred to the per-unit system [p.u.]. Base values, necessary for the transition, can be found in the Appendix. Bold font indicates vector values.

Sub- and superscripts

Subscript	N	–	nominal value,
Superscript	max	–	maximum value,
Superscript	ref	–	reference value,
Superscript	^	–	estimated value,
Superscript	dyn	–	desired dynamics.

1. Introduction

Induction motor (IM) drives, due to their operational reliability, small size and low cost, are continuously becoming more and more popular among the electrical motors applied in the industry [1–3]. However, the most complicated mathematical model entails that the most complex control structure must be used.

One of the control method groups, which ensure excellent dynamics of the IM drive, is the Sliding Mode Control (SMC) [4]. It can be ranked as the vector control method. These control methods require that the nonmeasurable variables, such as the rotor flux vector, electromagnetic torque or stator current vector components in a synchronous frame, are estimated. Similarly, the sliding modes can be successfully used to design the state estimator, insensitive to some motor parametric changes.

The sliding mode control of the IM main three variables: torque, speed and position was proposed for the first time in [5]. The relay method is natural for the inverter – sliding mode algorithm defines the transistor on/off signals directly. Unfortunately, the proposed method introduce large acoustic noise and mechanical oscillations of the drive, caused by the chattering in regulated variables. Therefore, many papers tried

*e-mail: teresa.orlowska-kowalska@pwr.wroc.pl

to eliminate the above mentioned, negative effects. One of the first simple methods was to replace the sign function with its continuous approximation [6]. For this purpose the load estimator was proposed as well [7]. Many other techniques have also been introduced, such as adaptive [8, 9] or second-order control [10] methods.

Quite an extensive group of the SMC methods are the solutions, which take the advantage of the artificial intelligence methods. They were used to calculate the maximum value of parametric and external disturbances: a fuzzy-neural network in [11] and a genetic algorithm in [12]. Fuzzy logic operating simultaneously with the sliding mode controller is presented in [13].

The sliding mode control system is robust against the disturbances only after it reaches the switching surface. Therefore, there appeared solutions with the object of extending the robustness on the reaching phase. One of the most investigated methods is changing the position of the switching surface (or line) in time, changing the parallel position or the angle. This method was applied in the position control of the IM in [14]. The time-varying switching surface was used for the sliding mode linear feedback control with switched gains.

The sliding mode technique can be effectively applied to design the estimation system. Its first application to the IM drives was a speed and flux estimators [15]. Many of the solutions published afterwards dealt with the speed estimation. Two different approaches can be distinguished. In the first one speed is filtered from the high-frequency signal (obtained with the sign function) [16] and in the second one, it is calculated as a difference between estimated synchronous and slip speeds [17, 18]. Model Reference Adaptive System (MRAS) estimators belong to the first group. The voltage model as the reference model, and the current model as an adaptive system is presented in [19] and [20]. The first of these solutions [19] uses the relay technique (sign function instead of the PI controller), while the second one [20] is based on the concept of the equivalent control method.

The second major group of the sliding mode estimators are solutions incorporating the parameter estimation. Due to the difficult identification of the rotor parameters and the IFOC control structure usage, the most often estimated parameter was the rotor resistance (or rotor time constant) [21—23]. An estimator, insensitive to the rotor parameters is proposed in [24]. Other parameters of the IM have been also estimated using the sliding modes. The estimation of both stator and rotor resistances is introduced in [25]; an estimator intended to operate in a wide speed range, with the magnetizing reactance estimation is presented in [26].

The sliding mode control and estimation algorithms have been combined together in [27], also to create the so-called sensorless algorithms [28–29].

Most of the above mentioned algorithms are designed in a similar way. Additionally, this process can be unified for all sliding mode systems, i.e. both for control and estimation algorithms. The purpose of this paper is to systematise the designing procedure and to divide it into several successive stages.

The designing procedure is first described for a general plant, and next applied to the induction motor drive control and estimation. In the first place, the speed control structure, taking the advantage of the equivalent control method is presented. Then, the sliding mode estimator, designed using the Model Reference Adaptive System technique is described [30]. The estimator utilizes the relay method.

This paper is organized as follows: first, the whole designing procedure of the sliding mode system is presented and its particular steps are listed. Then, these steps are applied to the induction motor speed control and speed estimation. The presented algorithms are verified using experimental results. Finally, there are some conclusions at the end of the paper.

2. Sliding mode systems design

Sliding mode systems design can be divided into several steps. Every step of the procedure is strictly connected with all the other steps. These steps are identical for both control and estimation.

2.1. Determination of the mathematical model of the plant and identification of its parameters. To design the sliding mode system, a mathematical model of the plant, together with its parameters must be determined. This phase of the design process is important especially when the state variables estimation is taken into consideration. However, the parameters of the plant are also used in the control algorithm.

2.2. Choice of the control signals / vector of estimated variables. On the basis of the mathematical model and the topology of the structure of the control system or the state estimator, the following vector has to be defined:

$$\mathbf{k} = \begin{bmatrix} k_1 & k_2 & \dots & k_n \end{bmatrix}^T, \tag{1}$$

where n – number of available signals.

Considering the induction motor drives, this vector (1) can contain control signals, such as on/off signals of the voltage source inverter transistors or can become a scalar when a cascade control structure is taken into account. On the other hand, it can consist of estimated signals, such as speed and stator current vector components.

2.3. Definition of the switching functions vector. The next step is to select so-called switching functions – the purpose of the sliding mode system is to ensure a zero value of these functions. They can be gathered in the following vector:

$$\mathbf{s} = \begin{bmatrix} s_1 & s_2 & \dots & s_n \end{bmatrix}^T. \tag{2}$$

The maximum dimension of the above vector (2) corresponds with the size of the vector (1). The form of switching functions depends strictly on the application. The switching functions can be both stationary in time [4] and time-varying [31]. Most often they are linear, however, they can be nonlinear as well: parabolic [32], elliptical [33] or the so-called terminal attractor [34], which ensures a finite reaching time of the switching function. The terminal attractor can also be time-varying [35].

The forms of the switching functions, in the estimator application, are most often the estimation errors. However, they can be also a combination of state variables and estimation errors; it will be presented in the case of the induction motor speed estimation.

Knowledge of the derivative of the switching functions vector is required in the design process:

$$\dot{\mathbf{s}} = \begin{bmatrix} \dot{s}_1 & \dot{s}_2 & \dots & \dot{s}_n \end{bmatrix}^T. \tag{3}$$

The switching functions vector (2) must be chosen in a way that allows the decomposition of (3), described by the following equation:

$$\dot{\mathbf{s}} = \mathbf{f} + \mathbf{D}\mathbf{k}, \tag{4}$$

where $\mathbf{D}$ – matrix dependent on the control signal vector $\mathbf{k}$ and $\mathbf{f}$ – column vector independent on $\mathbf{k}$. Additionally, vector $\mathbf{f}$ can be decomposed into:

$$\mathbf{f} = \mathbf{f_1} + \mathbf{f_2}, \tag{5}$$

where $\mathbf{f}_1$ – part that can be calculated using measured or estimated signals, $\mathbf{f}_2$ – part which depends on the nonmeasurable variables, such as external disturbances.

2.4. Sliding mode control / estimation law. The most important part of the design process is the choice of the sliding mode algorithm, which ensures the zero value of all switching functions (2). Three chosen structures are presented in Fig. 1 [36]. This stage of the design is strictly connected with the chosen control and switching functions vectors. All of the presented methods can be applied in control and estimation systems as well.

The relay method (Fig. 1a) is most often used sliding mode control/estimation method. In this method, the control signals take only two opposite values:

$$\mathbf{k} = -\mathbf{\Gamma}\mathrm{sign}(\mathbf{s}^*)^T, \tag{6}$$

where Γ – gain matrix:

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \gamma_n \end{bmatrix} \tag{7}$$

with positive parameters $\gamma_1, \gamma_2, \dots, \gamma_n > 0$. Modified switching functions vector:

$$\mathbf{s}^* = \mathbf{s}^T\mathbf{D} = \begin{bmatrix} s_1^* & s_2^* & \dots & s_n^* \end{bmatrix}. \tag{8}$$

In order to reduce the oscillations in the control and estimated variables signals **the equivalent control method** (Fig. 1b) can be applied. In this method, vector $\mathbf{k}$ consists of two different parts: continuous $\mathbf{k}^{eq}$ and discontinuous $\mathbf{k}^d$:

$$\mathbf{k} = \mathbf{k^{eq}} + \mathbf{k^{d}}, \tag{9}$$

where $\mathbf{k}^{eq}$ can be calculated from the following expression:

$$\dot{\mathbf{s}} = \mathbf{f} + \mathbf{D}\mathbf{k^{eq}} = \mathbf{0} \tag{10}$$

that yields:

$$\mathbf{k^{eq}} = -\mathbf{D}^{-1}\mathbf{f}. \tag{11}$$

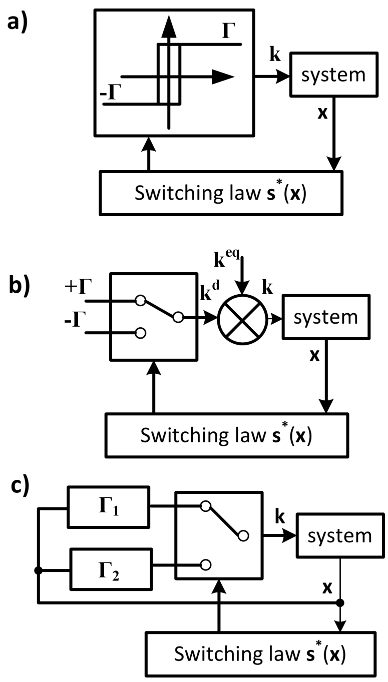

Fig. 1. Three chosen sliding mode control topologies: a) relay control, b) equivalent control method, c) linear feedback control with switched gains

However, since vector $\mathbf{f}$ consists of the part, which is nonmeasurable, the continuous component of the control signal becomes:

$$\mathbf{k^{eq}} = -\mathbf{D}^{-1}\mathbf{f_1}. \tag{12}$$

The discontinuous component is to overcome the nonmeasurable and unknown signals in $\mathbf{f}_2$:

$$\mathbf{k^d} = -\mathbf{\Gamma^d} sign\left(\mathbf{s}^*\right)^T, \tag{13}$$

where, similarly, the gain matrix:

$$\mathbf{\Gamma^d} = \begin{bmatrix} \gamma_1^d & 0 & \dots & 0 \\ 0 & \gamma_2^d & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \gamma_n^d \end{bmatrix} \tag{14}$$

with positive parameters $\gamma_1^d, \gamma_2^d, ..., \gamma_n^d > 0$.

Control signal in the third control method, the **linear feedback control with switched gains** (Fig. 1c) is as follows:

$$\mathbf{k} = \mathbf{\Gamma}(\mathbf{x})\,\mathbf{x}. \tag{15}$$

The gain matrix is switched depending on the state variables vector:

$$\mathbf{\Gamma}(\mathbf{x}) = \begin{cases} \mathbf{\Gamma_1} & \text{when } \mathbf{s}(\mathbf{x})\mathbf{x} \geq 0 \\ \mathbf{\Gamma_2} & \text{when } \mathbf{s}(\mathbf{x})\mathbf{x} < 0 \end{cases}, \tag{16}$$

and two different gain matrices:

$$\mathbf{\Gamma^1} = \begin{bmatrix} \gamma_1^1 & 0 & ... & 0 \\ 0 & \gamma_2^1 & ... & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & ... & \gamma_n^1 \end{bmatrix}, \quad \mathbf{\Gamma^2} = \begin{bmatrix} \gamma_1^2 & 0 & ... & 0 \\ 0 & \gamma_2^2 & ... & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & ... & \gamma_n^2 \end{bmatrix}. \tag{17}$$

Due to the large number of the parameters in (17) this control method and its application to the induction drive control will not be considered in the following part of the paper. However, this control method has been applied with the time-varying switching line in the position control of the induction motor drive [14].

2.5. Selection of control / estimation gains. All parameters included in the gain matrices: $\mathbf{\Gamma}$, $\mathbf{\Gamma}^d$, $\mathbf{\Gamma}^1$, $\mathbf{\Gamma}^2$ must be determined properly. The parameters in the $\mathbf{\Gamma}$ matrix (7), in the relay control, are often assumed as maximum available values, e.g. ± 1, which indicate the on/off state, or a maximum acceptable torque value in the cascade speed control. However, this method and its large gains leads to an undesired phenomenon called chattering, which induces oscillations in controlled variables, acoustic noise and mechanical vibrations of the real system.

As it will be shown, the parameters included in the $\mathbf{\Gamma}^d$ matrix can be set to much lower value in comparison with the relay control method. This helps to reduce the oscillations effectively.

This point of the design process is strictly connected with the following stage – the parameters in every method must be large enough to ensure the stability of the proposed system.

2.6. Stability analysis of the proposed system. The last step of the design should be the verification of the proposed system stability. This step proves that the designed algorithm ensures a zero value of the switching functions. This task can be made using the Lyapunov method. The positive-defined Lyapunov function can be defined as:

$$L = \frac{1}{2}\mathbf{s}^T\mathbf{s} = \frac{1}{2}\left(s_1^2 + s_s^2 + ... + s_n^2\right) > 0. \tag{18}$$

The system is asymptotically stable if the derivative of the function (18):

$$\dot{L} = \mathbf{s}^T\dot{\mathbf{s}} = s_1\dot{s}_1 + s_2\dot{s}_2 + ... + s_n\dot{s}_n \tag{19}$$

is negative.

In the case of the **relay method**, equation (19) becomes:

$$\begin{aligned} \dot{L} &= \mathbf{s}^T\dot{\mathbf{s}} = \mathbf{s}^T\left(\mathbf{f} + \mathbf{Dk}\right) \\ &= \mathbf{s}^T\left(\mathbf{f} - \mathbf{D\Gamma}sign(\mathbf{s}^*)^T\right) = \mathbf{s}^T\mathbf{f} - \mathbf{s}^*\mathbf{\Gamma}sign(\mathbf{s}^*)^T. \end{aligned} \tag{20}$$

There are defined two vectors:

vector $|\mathbf{a}|$, containing absolute values of its elements a_i:

$$|\mathbf{a}| = \begin{bmatrix} |a_1| & |a_2| & ... & |a_n| \end{bmatrix} \tag{21}$$

and a unit vector:

$$\mathbf{I} = \begin{bmatrix} 1 & 1 & ... & 1 \end{bmatrix}. \tag{22}$$

Taking into account (21) and (22) equation (20) yields:

$$\dot{L} = \mathbf{s}^T\mathbf{f} - \mathbf{I\Gamma}\left|\mathbf{s}^*\right|^T. \tag{23}$$

Negative value of (23) is given if:

$$\mathbf{s}^T\mathbf{f} < \mathbf{I\Gamma}\left|\mathbf{s}^T\mathbf{D}\right|^T \tag{24}$$

that is equivalent to:

$$\mathbf{\Gamma I}^T > \left|\mathbf{D}^{-1}\mathbf{f}\right|, \tag{25}$$

for each element of the above vectors, respectively.

The parameters included in the $\mathbf{\Gamma}$ matrix must be positive and large enough to fulfil the above inequality (25).

Similar methodology can be applied to the **equivalent control method.** The derivative of the Lyapunov function becomes:

$$\begin{aligned} \dot{L} &= \mathbf{s}^T\dot{\mathbf{s}} = \mathbf{s}^T\left(\mathbf{f_1} + \mathbf{f_2} + \mathbf{Dk}\right) \\ &= \mathbf{s}^T\left(\mathbf{f_1} + \mathbf{f_2} - \mathbf{DD^{-1}f_1} - \mathbf{D\Gamma^d}\mathrm{sign}(\mathbf{s}^*)^T\right) \\ &= \mathbf{s}^T\mathbf{f_2} - \mathbf{s}^*\,\mathbf{\Gamma^d}\mathrm{sign}(\mathbf{s}^*)^T. \end{aligned} \tag{26}$$

Its negative value is ensured when:

$$\mathbf{\Gamma^d I}^T > \left|\mathbf{D}^{-1}\mathbf{f}_2\right|, \tag{27}$$

similarly as for (25).

Comparing (25) and (27) it is clear that the values of the parameters in the $\mathbf{\Gamma}^d$ matrix can be remarkably reduced, in relation to the relay control method.

3. Application of the sliding modes to the induction motor control and state estimation

3.1. Mathematical model of the controlled object. The design process presented in the previous chapter can be successfully applied to create both control and estimation algorithms. In the following part of the article, the mentioned steps will be taken to design the equivalent speed control method and relay speed estimator.

The first stage of the designing process should be the **determination of the mathematical model of the plant and identification of its parameters**. Both control and estimation algorithms utilize the same mathematical model of the squirrel-cage induction motor. The model can be derived using commonly known simplifications, in the $\alpha - \beta$ frame and per unit system [1, 3]:

– Voltage equations:

$$\mathbf{u_s} = r_s\mathbf{i_s} + T_N\frac{d}{dt}\boldsymbol{\psi}_\mathbf{s}, \tag{28}$$

$$\mathbf{0} = r_r\mathbf{i_r} + T_N\frac{d}{dt}\boldsymbol{\psi}_\mathbf{r} - j\omega_m\boldsymbol{\psi}_\mathbf{r}, \tag{29}$$

– Current-flux equations:

$$\boldsymbol{\psi}_\mathbf{s} = x_s\mathbf{i_s} + x_m\mathbf{i_r}, \tag{30}$$

$$\boldsymbol{\psi}_\mathbf{r} = x_r \mathbf{i}_\mathbf{r} + x_m \mathbf{i}_\mathbf{s}, \tag{31}$$

– Equation of motion and electromagnetic torque:

$$\frac{d\omega_m}{dt} = \frac{1}{T_M}\left(m_e - m_o\right), \tag{32}$$

$$m_e = \text{Im}\left(\boldsymbol{\psi}_\mathbf{s}^* \mathbf{i}_\mathbf{s}\right) = \psi_{s\alpha} i_{s\beta} - \psi_{s\beta} i_{s\alpha}, \tag{33}$$

where Im($\mathbf{x}$) – imaginary part of a complex value.

Equations (28)–(31), after some algebraic operations, will be used to create a model of the sliding mode estimator:
– Dynamics of the stator current vector:

$$\begin{aligned} T_N \frac{d\,\mathbf{i}_\mathbf{s}}{dt} = \frac{1}{x_s \sigma}\Bigg(\mathbf{u}_\mathbf{s} - \left(r_s + \frac{r_r x_m^2}{x_r^2}\right)\mathbf{i}_\mathbf{s} \\ + \frac{x_m}{x_r}\frac{r_r}{x_r}\boldsymbol{\psi}_\mathbf{r} - j\frac{x_m}{x_r}\omega_m \boldsymbol{\psi}_\mathbf{r}\Bigg), \end{aligned} \tag{34}$$

– Dynamics of the rotor flux vector:

$$T_N \frac{d\boldsymbol{\psi}_\mathbf{r}}{dt} = -\frac{r_r}{x_r}\boldsymbol{\psi}_\mathbf{r} + \frac{x_m r_r}{x_r}\mathbf{i}_\mathbf{s} + j\omega_m \boldsymbol{\psi}_\mathbf{r}. \tag{35}$$

The following parameters of the mathematical model have to be identified:

- stator resistance, r_s,
- rotor resistance, r_r,
- main reactance, x_m,
- stator leakage reactance, $x_{s\sigma}$,
- rotor leakage reactance, $x_{r\sigma}$,
- mechanical time constant, T_M.

3.2. Sliding Mode Equivalent Speed Control. Induction motor speed can be controlled using two main topologies: direct and cascade. The first of them, using the speed switching function, defines on/off transistor signals of the voltage source inverter directly [37]. The direct method does not guarantee any supervision of the electromagnetic torque value and it can exceed the maximum value – this is its main disadvantage. However, it also produces large, undesired steady-state error [38].

In order to prevent the above mentioned negative effects, the cascade control structure (a series connection of two regulators) will be the point of interest of the following part of this paper. The outer speed regulator produces a reference torque value, hence **the control signals vector** (1) becomes a scalar:

$$\mathbf{k} = \left[m_e^{ref}\right]. \tag{36}$$

The inner torque controller ensures that the electromagnetic torque follows the reference value. It is assumed that the torque regulator is any regulator, which acts approximately to the first order inertial element:

$$\frac{m_e(p)}{m_e^{ref}(p)} = \frac{1}{T_{me}p + 1}, \tag{37}$$

where T_{me} – substitute time constant of the torque regulation loop.

Similarly, the **switching function vector** becomes a scalar as well:

$$\mathbf{s} = [s_\omega] = \omega_m^{ref} - \omega_m - T_c\dot{\omega}_m, \tag{38}$$

where T_c – a time constant that defines the desired dynamics of the speed. If the control purpose is met, i.e. zero value of the switching function (38), the speed control structure has the dynamics of the first order inertial block with T_c time constant.

Derivative of the switching function (38) can be calculated as:

$$\dot{s}_\omega = f_{1\omega} + f_{2\omega} + d_\omega m_e^{ref}, \tag{39}$$

where respectively:

$$f_{1\omega} = \dot{\omega}_m^{ref} + \frac{T_c - T_{me}}{T_M T_{me}} m_e, \tag{40}$$

$$f_{2\omega} = \frac{T_c}{T_M}\dot{m}_o + \frac{1}{T_M}m_o. \tag{41}$$

According to the methodology given in previous chapter, the **equivalent sliding mode control law** becomes as follows:

$$m_e^{ref} = m_e^{ref,\,eq} + m_e^{ref,\,d}, \tag{42}$$

where the continuous part:

$$m_e^{ref,\,eq} = \frac{T_M T_{me}}{T_c}\left(\dot{\omega}_m^{ref} + \frac{T_c - T_{me}}{T_M T_{me}} m_e\right), \tag{43}$$

and the discontinuous part:

$$m_e^{ref,\,d} = \Gamma_{me}^d \frac{T_M T_{me}}{T_c}\,\text{sign}(s_\omega). \tag{44}$$

There are two **parameters that have to be selected** – the time constant T_c and the discontinuous part gain Γ_{me}^d. The value of the parameters should differ according, among others, to the power of the motor.

As it was mentioned above, the control gain Γ_{me}^d should be determined taking into consideration the **stability analysis** of the proposed system. The Lyapunov function can be defined as:

$$L = \frac{1}{2}s_\omega^2 > 0 \tag{45}$$

and its derivative, *

$$\dot{L} = s_\omega \dot{s}_\omega = s_\omega f_{2\omega} - \Gamma_{me}^d\left|s_\omega\right|. \tag{46}$$

The derivative (46) is negative, and the system is asymptotically stable, when

$$\Gamma_{me}^d > \left|\frac{T_c}{T_M}\dot{m}_o + \frac{1}{T_M}m_o\right|. \tag{47}$$

According to (47), the gain Γ_{me}^d should be selected to compensate the load torque and its changes.

The block diagram of the sliding mode equivalent control method for induction motor drive is presented in Fig. 2. The inner torque regulator relies on the electromagnetic torque and rotor flux vector – they are estimated using the proper estimator. If it is necessary, the measured stator current vector and DC link voltage are used as well. The second purpose of the inner controller is to stabilize the amplitude of the chosen motor flux – stator or rotor. The control signal, i.e. reference torque, is limited to the maximum available value $m_e^{\max}$.

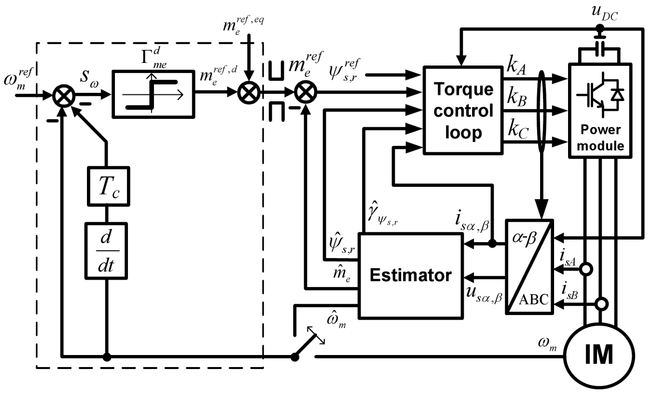

Fig. 2. The block diagram of the sliding mode equivalent control method in application to the induction motor drive speed control

3.3. Sliding Mode Model Reference Adaptive System speed estimator – SM-MRAS. The design method of the sliding mode estimator, presented below, will be analogical to the one described in chapter 2. This approach helps to generalize the sliding mode algorithms and indicates the universality of this technique.

The estimator will be designed as a Model Reference Adaptive System (MRAS) estimator. The reference model will be the induction motor itself. The adaptive part of the estimator will be both stator current and rotor flux vectors equations.

In order to start the designing process **the estimated variables vector must be selected**. The estimated speed and an additional variable μ, introduced to decrease the sensitivity of the estimator to the rotor time constant changes, form this vector:

$$\mathbf{k_o} = [\widehat{\omega}_m,\ \widehat{\mu}]^T. \tag{48}$$

Selecting such pair of the estimated variables (48), the mathematical model of the estimator is a modification of equations (34)–(35):

$$\begin{aligned} T_N\frac{d\widehat{\mathbf{i}}_\mathbf{s}}{dt} = \frac{1}{x_s\sigma}\left(\mathbf{u_s} - \left(r_s + \frac{r_r x_m^2}{x_r^2}\right)\widehat{\mathbf{i}}_\mathbf{s}\right.\\ \left. + \frac{x_m}{x_r}\left(\frac{r_r}{x_r} + \widehat{\mu}\right)\widehat{\psi}_\mathbf{r} - j\frac{x_m}{x_r}\widehat{\omega}_m\widehat{\psi}_\mathbf{r}\right), \end{aligned} \tag{49}$$

$$T_N\frac{d\widehat{\psi}_\mathbf{r}}{dt} = -\left(\frac{r_r}{x_r} + \widehat{\mu}\right)\widehat{\psi}_\mathbf{r} + \frac{x_m r_r}{x_r}\mathbf{i_s} + j\widehat{\omega}_m\widehat{\psi}_\mathbf{r}. \tag{50}$$

Next step should be the **determination of the switching functions vector**. This vector can be a combination of stator current vector estimation errors and rotor flux vector components [26]:

$$\mathbf{s_o} = \begin{bmatrix} s_\omega \\ s_\mu \end{bmatrix} = \begin{bmatrix} \left(\widehat{i}_{s\beta} - i_{s\beta}\right)\widehat{\psi}_{r\alpha} - \left(\widehat{i}_{s\alpha} - i_{s\alpha}\right)\widehat{\psi}_{r\beta} \\ \left(\widehat{i}_{s\beta} - i_{s\beta}\right)\widehat{\psi}_{r\beta} + \left(\widehat{i}_{s\alpha} - i_{s\alpha}\right)\widehat{\psi}_{r\alpha} \end{bmatrix}. \tag{51}$$

Derivative of the switching function vector (51) can be decomposed into:

$$\dot{\mathbf{s}}_\mathbf{o} = \mathbf{f_o} + \mathbf{D_o}\mathbf{k_o}, \tag{52}$$

where

$$\begin{aligned} \mathbf{f_o} &= \frac{1}{T_N}\begin{bmatrix} f_{o1} \\ f_{o2} \end{bmatrix}, \\ f_{o1} &= -\alpha_1 s_\omega - \frac{r_r}{x_r}s_\mu - x_m\frac{r_r}{x_r}\left(e_{is\beta}i_{s\alpha} + e_{is\alpha}i_{s\beta}\right) + \\ &\alpha_2\left(e_{\psi r\alpha}\widehat{\psi}_{r\beta} - e_{\psi r\beta}\widehat{\psi}_{r\alpha}\right) + \alpha_3\omega_m\left(\psi_{r\alpha}\widehat{\psi}_{r\alpha} + \psi_{r\beta}\widehat{\psi}_{r\beta}\right), \\ f_{o2} &= -\left(\alpha_1 + \frac{r_r}{x_r}\right)s_\mu - x_m\frac{r_r}{x_r}\left(e_{is\beta}i_{s\beta} + e_{is\beta}i_{s\beta}\right) + \\ &-\alpha_2\left(e_{\psi r\beta}\widehat{\psi}_{r\beta} + e_{\psi r\alpha}\widehat{\psi}_{r\alpha}\right) + \alpha_3\omega_m\left(\psi_{r\alpha}\widehat{\psi}_{r\beta} - \psi_{r\beta}\widehat{\psi}_{r\alpha}\right) \end{aligned} \tag{53}$$

and the estimation errors:

$$\begin{aligned} e_{is\alpha} = i_{s\alpha} - \widehat{i}_{s\alpha},\ e_{is\beta} = i_{s\beta} - \widehat{i}_{s\beta}, \\ e_{\psi r\alpha} = \psi_{r\alpha} - \widehat{\psi}_{r\alpha},\ e_{\psi r\beta} = \psi_{r\beta} - \widehat{\psi}_{r\beta}. \end{aligned} \tag{54}$$

Matrix $\mathbf{D}_o$ is as follows:

$$\begin{aligned} \mathbf{D_o} &= \frac{1}{T_N}\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, \\ d_{11} &= -\frac{x_m}{x_r x_s\sigma}\widehat{\psi}_r^2 - e_{is\beta}\widehat{\psi}_{r\beta} - e_{is\alpha}\widehat{\psi}_{r\alpha}, \\ d_{12} &= -e_{is\beta}\widehat{\psi}_{r\alpha} + e_{is\alpha}\widehat{\psi}_{r\beta}, \\ d_{21} &= e_{is\beta}\widehat{\psi}_{r\beta} - e_{is\alpha}\widehat{\psi}_{r\alpha}, \\ d_{22} &= \frac{x_m}{x_r x_s\sigma}\widehat{\psi}_r^2 - e_{is\beta}\widehat{\psi}_{r\beta} - e_{is\alpha}\widehat{\psi}_{r\alpha}. \end{aligned} \tag{55}$$

The relay technique (6)–(8) is applied to design the sliding mode **estimation law**:

$$\mathbf{k_o} = -\mathbf{\Gamma_o}sign\left(\mathbf{s_o^*}\right)^T,\ \mathbf{s_o^*} = \mathbf{s_o}^T\mathbf{D_o}, \tag{56}$$

where the gain matrix:

$$\mathbf{\Gamma_o} = \begin{bmatrix} \gamma_\omega & 0 \\ 0 & \gamma_\mu \end{bmatrix}. \tag{57}$$

Assuming that the estimation errors in (55) can be omitted, equation (56) simplifies to:

$$\mathbf{k_o} = \begin{bmatrix} \widehat{\omega}_m \\ \widehat{\mu} \end{bmatrix} = \begin{bmatrix} \gamma_\omega sign s_\omega \\ -\gamma_\mu sign s_\mu \end{bmatrix}. \tag{58}$$

The chosen estimator gains: γ_ω, γ_μ must be large enough to ensure the asymptotic **stability** of the estimator, which can be described by the following formula:

$$\dot{L} = \mathbf{s_o}^T\dot{\mathbf{s}}_\mathbf{o} < 0, \tag{59}$$

that is equivalent to:

$$\mathbf{\Gamma_o}\mathbf{I}^T > \left|\mathbf{D_o}^{-1}\mathbf{f_o}\right|, \tag{60}$$

similarly as for (25).

The block diagram of the sliding mode speed estimator is presented in Fig. 3. The estimated speed, obtained using (57), has only two values $\pm\gamma_\omega$ – this signal is useless in the control structure – therefore it must be filtered. The simplest low-pass filter and its transfer function are shown in Fig. 3.

Stator flux vector and electromagnetic torque can be calculated using directly the induction motor model:

$$\widehat{\psi}_{\mathbf{s}} = \frac{x_m}{x_r}\widehat{\psi}_{\mathbf{r}} + x_s\sigma\,\mathbf{i}_{\mathbf{s}}, \tag{61}$$

$$\widehat{m}_e = \widehat{\psi}_{s\alpha} i_{s\beta} - \widehat{\psi}_{s\beta} i_{s\alpha}. \tag{62}$$

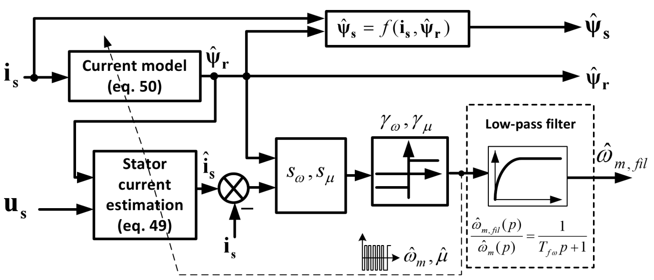

Fig. 3. The block diagram of the sliding mode speed estimator (SM-MRAS)

4. Experimental verification

In order to verify the presented sliding mode algorithms, both simulation and experimental tests were performed. Due to the limited capacity of the paper, only the experimental verification is shown in this chapter. The laboratory setup consisted of 3 kW induction motor and 5 kW DC-motor, acting as a load torque. Parameters of the induction motor are shown in Appendix. Data acquisition, measurement, estimation and control were executed using a digital signal processor – dSpace DS 1103. Integration step was equal to 100 μs (frequency 10 kHz) and data downsampling 1.

4.1. Equivalent sliding mode speed control method. The performance of the sliding mode equivalent control method applied to the induction motor speed control is shown in Fig. 4. It is assumed that the inner torque regulator is the Sliding-Mode Direct Torque Control structure, described in [26].

During the presented test, the reference speed was a reverse function, from 0.5 to -0.5 of the nominal value; the load torque was set to the nominal value ($m_N = 0.67$ [p.u.], see Appendix). The reference dynamics (marked grey in Fig. 4a) is the response of the first order inertial block with time constant $T_c = 0.1$ (95% settling time $T_s = 3T_c = 0.3$ s). The real speed follows the reference dynamics very well, without visible oscillations.

Continuous (eq) and discontinuous (d) components of the control signal are placed in Fig. 4b. In order to further reduce the oscillations of the discontinuous part, the sign function in (44) was replaced by its approximation – a saturation function [30]. The resultant control signal, the reference torque and its estimated value are presented in Fig. 4c. It can be seen that the estimated torque follows the reference value, and is constrained on the maximum value. Stator phase currents are limited as well (Fig. 4d). The interior torque controller performs also the stabilization of the stator flux – it can be noticed in Fig. 4e.

4.2. Sliding mode speed estimator. The sliding mode relay technique can be successfully applied to design the speed estimator, as it was described in the previous chapter, and thus create a speed-sensorless drive. Such operation is presented in Fig. 5. The estimated speed, torque and stator flux vector are used directly in the control structure. Again, the reverse speed operation is illustrated in the figure. Estimated speed (Fig. 5a) follows the real speed with a slight error, which is shown in Fig. 5b. This error becomes larger during the reversions – the dynamical delay (introduced by the low-pass filter) can be seen. However, despite this error, the drive works properly.

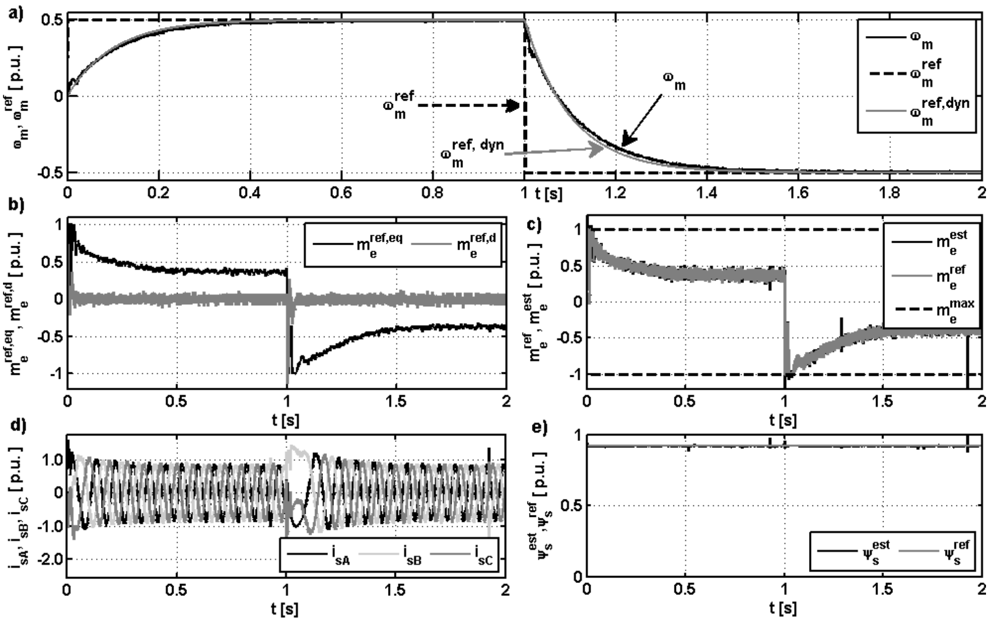

Fig. 4. Performance of the sliding mode equivalent control method in application to the induction motor speed control: a) reference and real mechanical speed, b) continuous and discontinuous parts of the control signal, c) reference, estimated and maximum electromagnetic torque, d) phase currents, e) reference and estimated amplitude of stator flux

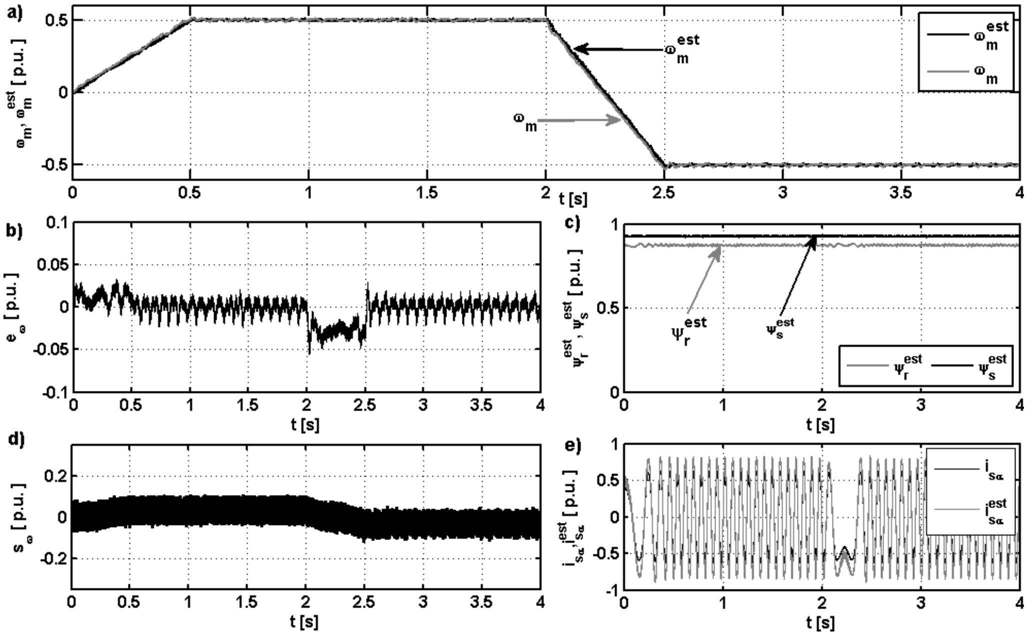

Fig. 5. Performance of the sliding mode speed estimator SM-MRAS in application to induction motor drive: a) real and estimated speed, b) speed estimation error, c) estimated amplitudes of stator and rotor fluxes, d) speed switching function, e) measured and estimated α-component of the stator current vector

The estimated amplitude of the stator flux is stabilized by the control structure (Fig. 5c) on the nominal value. The estimated value of the rotor flux amplitude is also constant.

The speed switching function value is almost zero (Fig. 5d) – it was the purpose of the sliding mode algorithm. Since the switching function equation is based on the estimation error of the stator current vector components, the real and estimated component of the α-axis is shown in Fig. 5e. As it can be seen, the difference between them is quite small, and increases during reversions.

5. Conclusions

This paper deals with the sliding mode systems design and its application to the induction motor drives. The design procedure is unified for both the control system and the estimator. The paper tries to divide the design process into several steps, from the determination of the mathematical model of the plant to the stability analysis of the designed system. Three different sliding mode techniques: relay method, equivalent control method and linear control with switched gains are described.

The equivalent control method is then applied to the induction motor speed control. This method gives an excellent speed control and ensures the required dynamics of the speed.

Finally, the relay technique is used to design the speed estimator. The estimator is the Model Reference Adaptive System type estimator. The speed is estimated properly, with small dynamic error during speed reversions.

The presented algorithms are illustrated with the experimental results obtained using a 3 kW induction motor drive.

Appendix – parameters and nominal values of the induction motor, base values

In the following tables parameters of tested induction motor (Table 1), its nominal parameters (Table 2) and base values, necessary during the transition to the per unit system (Table 3) are presented. The moment of inertia of the drive is $J = 0.0292$ kg m^2, that corresponds to the mechanical time constant $T_M = J\Omega_b/(p_b M_b) = 0.15$ s.

Table 1
Parameters of the induction motor

	Symbol	Physical values [Ω]	Per unit values [p.u.]
Stator resistance	R_s	7.073	0.071
Rotor resistance	R_r	7.372	0.074
Main reactance	X_m	187.8	1.88
Stator leakage reactance	$X_{s\sigma}$	9.80	0.098
Rotor leakage reactance	$X_{r\sigma}$	9.80	0.098

Table 2
Nominal parameters of the induction motor

	Symbol	Physical values		Per unit values [p.u.]
Power	P_N	3.0	[kW]	0.625
Torque	M_N	20.46	[Nm]	0.67
Rotational speed	N_N	1400	[rpm]	0.93
Stator voltage	U_{sN}	400	[V]	0.707
Stator current	I_{sN}	4.0	[A]	0.707
Frequency	f_{sN}	50	[Hz]	1
Stator flux	Ψ_{sN}	1.65	[Wb]	0.9188
Rotor flux	Ψ_{rN}	1.54	[Wb]	0.8605
Pole pairs	p_b	2	[–]	2

Table 3
Base values

	Equation	Physical values	
Power	$S_b = 3/2U_b I_b$	4.8	[kW]
Torque	$M_b = p_b S_b/\Omega_b$	30.56	[Nm]
Rotational speed	$N_b = 60 f_{sN}/p_b$	1500	[rpm]
Stator voltage	$U_{sb} = \sqrt{2}U_{sN}$	565.7	[V]
Stator current	$I_{sb} = \sqrt{2}I_{sN}$	5.66	[A]
Frequency	$f_{sb} = f_{sN}$	50	[Hz]
Angular velocity	$\Omega_b = 2\pi f_{sN}$	100π	[rad/s]
Flux	$\Psi_b = U_b/\Omega_b$	1.80	[Wb]

Acknowledgements. This work was supported by the National Science Centre (Poland) under the grant UMO-2011/03/N/ST7/03508 (2012–2014).

REFERENCES

[1] M.P. Kazmierkowski and H. Tunia, *Automatic Control of Converter Fed Driver,* Elseevier, Amsterdam, 1994.

[2] M.P. Kazmierkowski, R. Krishnan, and F. Blaabjerg, *Control in Power Electronics,* Academic Press, London, 2002.

[3] T. Orlowska-Kowalska, *Sensorless Induction Motor Drive,* Wroclaw University of Technology Press, Wroclaw, 2003.

[4] V.I. Utkin, J. Guldner, and J.X. Shi, *Sliding Mode Control in Electromechanical Systems*, Taylor and Francis, New York, 1999.

[5] D.B. Izosimov, B. Matic, V.I. Utkin, and A. Sabanovic, "Application of sliding modes in problems of electrical machine control", *Doklady Akademii Nauk SSSR* 241, 769–772 (1978), (in Russian).

[6] M.W. Dunnigan, S. Wade, B.W. Williams, and X. Yu, "Position control of a vector controlled induction machine using Slotine's sliding mode control approach", *IEE Proc.-Electric Power Appl.* 145, 231–238 (1998).

[7] W. J. Wang and J.Y. Chen, "A new sliding mode position controller with adaptive load torque estimator for an induction motor", *IEEE Trans. Energy Conversion* 14, 413–418 (1999).

[8] O. Barambones and A.J. Garrido, "Adaptive sensorless robust control of AC drives based on sliding mode control theory", *Int. J. Robust and Nonlinear Control* 17, 862–879 (2007).

[9] C.M. Lin and C.F. Hsu, "Adaptive fuzzy sliding-mode control for induction servomotor systems", *IEEE Trans. Energy Conversion* 19, 362–368 (2004).

[10] M. Rashed, K.B. Goh, M.W. Dunnigan, P. MacConnell, A. Stronach, and B. W. Williams, "Sensorless second-order sliding-mode speed control of a voltage-fed induction-motor drive using nonlinear state feedback", *IEE Proc.-Electric Power Appl.* 152, 1127–1136 (2005).

[11] R.J. Wai and F.J. Lin, "Fuzzy neural network sliding-mode position controller for induction servo motor drive", *IEE Proc.-Electric Power Appl.* 146, 297–308 (1999).

[12] F.J. Lin, W.D. Chou, and P.K. Huang, "Adaptive sliding-mode controller based on real-time genetic algorithm for induction motor servo drive", *IEE Proc.-Electric Power Appl.* 150, 1–13 (2003).

[13] F. Barrero, A. Gonzalez, A. Torralba, E. Galvan, and L.G. Franquelo, "Speed control of induction motors using a novel fuzzy sliding-mode structure", *IEEE Trans. Fuzzy Systems* 10, 375–383 (2002).

[14] F. Betin and G.A. Capolino, "Sliding mode control for an induction machine submitted to large variations of mechanical configuration", *Int. J. Adaptive Control and Signal Processing* 21, 745–763 (2007).

[15] D.B. Izosimov, "Sliding-mode nonlinear state observer of an induction motor", in *Control of Multiconnected Systems*, Nauka, Moscow, 1983.

[16] V.I. Utkin, "Sliding mode control design principles and application to electric drives", *IEEE Trans. Ind. Electronics* 40, 23–36 (1993).

[17] H.U. Rehman, A. Derdiyok, M.K. Guven, and L.Y. Xu, "A new current model flux observer for wide speed range sensorless control of an induction machine", *IEEE Trans. on Power Electronics* 17, 1041–1048 (2002).

[18] C. Lascu, I. Boldea, and F. Blaabjerg, "A class of speed-sensorless sliding-mode observers for high-performance induction motor drives", *IEEE Trans. on Ind. Electronics*. 56, 3394–3403 (2009).

[19] M. Comanescu and L.Y. Xu, "Sliding-mode MRAS speed estimators for sensorless vector control of induction machine", *IEEE Trans. on Ind. Electronics* 53, 146–153 (2006).

[20] S.M. Gadoue, D. Giaouris, and J.W. Finch, "MRAS sensorless vector control of an induction motor using new sliding-mode and fuzzy-logic adaptation mechanisms", *IEEE Trans. on Energy Conversion* 25, 394–402 (2010).

[21] R.J. Wai, D.C. Liu, and F.J. Lin, "Rotor time-constant estimation approaches based on energy function and sliding mode for induction motor drive", *Electric Power Systems Research* 52, 229–239 (1999).

[22] A. Derdiyok, M.K. Guven, H. Rehman, N. Inanc, and L.Y. Xu, "Design and implementation of a new sliding-mode observer for speed-sensorless control of induction machine", *IEEE Trans. on Ind. Electronics* 49, 1177–1182 (2002).

[23] S. Rao, M. Buss, and V.I. Utkin, "An adaptive sliding mode observer for induction machines", *Proc. American Control Conf.* 1, CD-ROM (2008).

[24] A.B. Proca and A. Keyhani, "Sliding-mode flux observer with online rotor parameter estimation for induction motors", *IEEE Trans. on Ind. Electronics* 54, 716–723 (2007).

[25] C. Picardi and F. Scibilia, "Sliding-mode observer with resistances or speed adaptation for field-oriented induction motor drives", *Prof. 32nd IEEE Annual Conf. on Ind. Electronics (IECON)* 1, 1481–1486 (2006).

[26] T. Orlowska-Kowalska, G. Tarchala, and M. Dybkowski, "Sliding-mode direct torque control and sliding-mode observer with a magnetizing reactance estimator for the field-weakening of the induction motor drive", *Trans. IMACS–Mathematics and Computers in Simulation*, http://dx.doi.org/10.1016/j.matcom.2013.05.012 (2013).

[27] A. Benchaib, A. Rachid, E. Audrezet, and M. Tadjine, "Real-time sliding-mode observer and control of an induction motor", *IEEE Trans. Ind. Electronics* 46, 128–138 (1999).

[28] Z. Yan, C.X. Jin, and V.I. Utkin, "Sensorless sliding-mode control of induction motors", *IEEE Trans. Ind. Electronics*. 47, 1286–1297 (2000).

[29] K. Jezernik, M. Rodic, and A. Sabanovic, "Speed sensorless variable structure torque control of induction motor", *Automatika* 51, 33–40 (2010).

[30] G. Tarchala, T. Orlowska-Kowalska, "Sliding mode speed observer for the induction motor drive with different sign function approximation forms and gain adaptation", *Electrical Engineering Review* 89, 1–6 (2013), (in Polish).

[31] A. Bartoszewicz and A. Nowacka, "Reaching phase elimination in variable structure control of the third order system with state constraints", *Kybernetika* 42, 111–126 (2006).

[32] A. Jabbari, M. Tomisuka, and T. Sakaguchi, "Robust Nonlinear Control of Positioning Systems with Stiction", *Proc. American Control Conf.* 1, 1097–1102 (1990).

[33] R.H. Takahashi, P.L. Peres, and L.L. Barbosa, "A sliding mode controlled sinusoidal voltage source with ellipsoidal switching surface", *IEEE Trans. Circuits and Systems I-Fundamental Theory and Appl.* 46, 714–721 (1999).

[34] M. Zak, "Terminal attractors for addressable memory in neural networks", *Physics Letters* A 133, 18–22 (1988).

[35] A. Bartoszewicz, "Time-varying sliding modes for second-order systems", *IEE Proc. – Control Theory and Appl.* 143, 455–462 (1996).

[36] F. Betin, D. Pinchon, and G.A. Capolino, "A time-varying sliding surface for robust position control of a DC motor drive", *IEEE Trans. Ind. Electronics* 49, 462–473 (2002).

[37] T. Orlowska-Kowalska, G. Tarchala, "Sliding mode speed and torque control of the induction motor", *Electrical Engineering Review* 87, 245–248, 2011 (in Polish).

[38] G. Tarchala, "Sliding modes application to the control and state variables estimation of the drive system with induction motor", *Ph.D. Thesis*, Wroclaw University of Technology, Wroclaw, 2013, (in Polish).