BULLETIN OF THE POLISH ACADEMY OF SCIENCES
TECHNICAL SCIENCES, Vol. 62, No. 3, 2014
*DOI: 10.2478/bpasts-2014-0056*

# A robust fixed-lag smoothing algorithm for dynamic systems with correlated sensor malfunctions

Yu.P. GRISHIN and D. JANCZAK*

Faculty of Electrical Engineering, Bialystok Technical University, 45D Wiejska St., 15-351 Bialystok, Poland

**Abstract.** A new robust fixed-lag smoothing algorithm for fault-tolerant signal processing in stochastic dynamic systems in the presence of correlated sensor malfunctions has been developed. The algorithm is developed using a state vector augmentation method and the Gaussian approximation of the current estimate probability density function. The algorithm can be used in the real-time fault-tolerant control systems as well as in radar tracking systems working in the complex interference environment. The performance of the developed algorithm are evaluated by simulations and compared with smoothing and nonlinear filtering algorithms.

**Key words:** outliers, robust signal processing, nonlinear filtering, fault detection, systems with changing structures.

## 1. Introduction

Fault diagnosis (FD) is defined as the detection and identification of a faulty element or a malfunctioning component causing the whole control system failure or malfunction [1–3]. Existing approaches for the FD can be categorized into the model-based and model-free methods [3]. The model-free approaches are based on a physical redundancy, qualitative models and various computational intelligence techniques, such as neural networks, fuzzy logic and genetic algorithms [1–6]. The model-based approaches use the state and parameter estimation techniques as well as the parity equations. The model-based algorithms are derived mainly for the linear systems [4, 7-10]. However, the algorithm may be developed using the theory of systems with abrupt changes of parameters value or structure changes. At present, very many methods are used for the description of models of failures which can be classified as abrupt changes in system structure or parameters [3, 7, 8, 11–13]. The sensors malfunctions can be modelled as an abrupt substantial increase of the observation noise level which are caused by changes of the observation conditions or by temporal measurement device failures [8, 14-16]. In practice they are correlated in time. In the fault-tolerant control systems these malfunctions must not cause considerable changes in the controlled process. In a case of the undisturbed linear control system the estimation of the object state vector can be found using the Kalman filter algorithm [9, 12, 17–20]. When these conditions cease to be valid it is necessary to find solution of the problem by means of non-linear filtering algorithms. At present such algorithms are based on different non-linear modifications of the Kalman filter or on using the particle filters [15]. Earlier in [7, 8] a non-linear filtering algorithm robust with respect to independent jumps of noise in the measurement channel was developed.

The paper deals with the problem of state vector estimation and the sensor failure detection in dynamic systems working in conditions of frequent correlated sensor interferences or malfunctions. An application of a fixed-lag smoothing filter with outlier suppression (FLSOS) [10] with a relatively small lag time is proposed. These conditions are acceptable if a system sample time is small enough. Thus, the objective of the paper is to present a new sub-optimal fix-lag smoothing estimation algorithm which would be robust with respect to the presence of the correlated sensor malfunctions under moderate computational burden. The approach which has been used is based on the methods of a system state vector augmentation and the Gaussian approximation of the current estimate probability density function [21].

The developed algorithm can be used in the real-time fault-tolerant control systems such as aerospace systems as well as in radar tracking systems working in the changing interference environment [8, 14, 17, 18].

## 2. The problem formulation

The system and measurement equations of a discrete-time stochastic control system are supposed to be described by the following equations:

$$x(k+1) = \Phi(k+1,k)x(k) + G(k)w(k), \tag{1}$$

$$y(k) = H(k)x(k) + \gamma(k)v(k), \tag{2}$$

where $x(k)$ is the object state vector, $\Phi(k+1,k)$ is the transition matrix, $w(k)$, $\nu(k)$ are white Gaussian sequences with zero mean and covariance matrixes $Q(k)$ and $R(k)$ respectively, $y(k)$ is the measurement vector and $H(k)$ is the observation matrix, $G(k)$ is $n \times q$ matrix, $n$, $q$, $s$ are sizes of $x(k)$, $w(k)$, $y(k)$ vectors.

The malfunctions of the measurement sensors in the observation equation can be described by a random multiplier $\gamma(k)$, which can take on values of 1 when the sensors are in normal operation and $\gamma_0(k) \gg 1$ when they are in the failure state. Correlation in time of the random sequence $\gamma(k)$,

*e-mail: djanczak@pb.edu.pl

can be described by the stationary Markov chain, of which initial probability vector $p^{\gamma}(0)$ and transition matrix $p_{\gamma}^{ij}$ can be known or not known to the designer [17]. For the second order Markov chain they have the following form:

$$p^{\gamma}(0) = \begin{bmatrix} p_1(0) \\ p_{\gamma_0}(0) \end{bmatrix}, \qquad p_{\gamma}^{ij} = \begin{bmatrix} p_{11} & p_{1\gamma_0} \\ p_{\gamma_0 1} & p_{\gamma_0\gamma_0} \end{bmatrix}. \tag{3}$$

The Markov chain approach makes it possible to introduce into consideration correlated outliers acting on the system.

The controlled object state vector can be treated as a conditional mean of the following form:

$$\begin{aligned} \widehat{x}(k/k+N) &= E(x(k)/Y_1^{k+N}) \\ &= E(x(k)/y(1),\ ...,\ y(k),\ ...,\ y(k+N)) \end{aligned} \tag{4}$$

which is a fixed-lag smoothing estimate of the state vector $x(k)$ with $N$ step lag.

The problem can be solved by augmenting the state vector $x(k)$ and reformulating the problem of smoothing into the problem of filtering:

$$\begin{aligned} x_a(k+N) &= [x^T(k+N)x^T(k+N-1)...x^T(k)]^T \\ &= [x_0^T(k+N)x_1^T(k+N)...x_N^T(k+N)]^T \\ &= x_a(j) = [x_0^T(j)x_1^T(j)...x_N^T(j)]^T, \\ & j = k+N, \end{aligned} \tag{5}$$

where $x_a(j)$ is the augmented state vector and

$$x_i(j) = x_i(k+N) = x(k+N-i), \qquad i = 0,\ 1,\ ...,\ N. \tag{6}$$

In new notations the state and observation equations for the augmented system can be written as:

$$x_a(j+1) = \Phi_a(j+1,j)x_a(j) + G_a w(j), \tag{7}$$

$$y(j) = H_a(j)x_a(j) + \gamma(j)v(j). \tag{8}$$

where

$$\Phi_a\ (j+1,j) = \begin{bmatrix} \Phi(j+1,j) & 0_{n\times N\cdot n} \\ I_{N\cdot n\times N\cdot n} & 0_{N\cdot n\times n} \end{bmatrix},$$

$$G_a = \begin{bmatrix} G_{n\times q} \\ 0_{N\cdot n\times q} \end{bmatrix},$$

$$H_a(j) = \begin{bmatrix} H(j) & 0\ _{s\times N\cdot n} \end{bmatrix}.$$

Now it is easy to notice that the estimate of augmented state vector $\widehat{x}_a(j/j)$ contains as its $N+1$ component the unknown estimate of fixed-lag smoothing

$$\widehat{x}_N(j/j) = \widehat{x}_N(k+N/k+N) = \widehat{x}(k/k+N), \tag{9}$$

that immediately follows from (5). Thus for developing the robust FLSOS we can use the procedure of a robust filtering synthesis presented in [8]. However in [3, 9] the robust filtering algorithm was developed and analysed only for independent outliers at the input of the system. Further this algorithm will be derived for correlated outliers.

## 3. Main results

The procedure of synthesis of the robust filtering algorithm is the following. First of all it is necessary to calculate the estimates of the state vector in known observation channel conditions, which can be founded using the Kalman filter algorithm [10, 17]:

$$\begin{aligned} \widehat{x}_a(j/j) &= \widehat{x}_a(j/j-1) \\ &+ K_a^1(j)\,[y(j) - H_a(j)\,\widehat{x}_a(j/j-1), \end{aligned} \tag{10}$$

where index $a$ concerns the augmented system and $K_a^1(j)$ is the Kalman gain matrix in the nominal state.

When the results of observations contain the malfunctions described by (2) for calculating the system estimation it is necessary to use a general approach. In this case the dynamic system state vector estimation can be found as a conditional mean of the following form [8, 17]:

$$\widehat{x}(k/k) = E[x(k)/Y_1^k] = \sum_{i\in 2^k} \widehat{x}^i(k/k)P(\overline{\Gamma}_k^i/Y_1^k), \tag{11}$$

where $Y_1^k = \{y(1), y(2), ...y(k)\}$ is the sequence of input data, $P(\circ)$ is conditional probability of the measurement channel state, $\overline{\Gamma}_k^i = \{\gamma(1), \gamma(2), ...\gamma(k)\}$ denotes the measurement channel state sequence and

$$\widehat{x}^i(k/k) = E[x(k)/Y_1^k, \overline{\Gamma}_k^i] \tag{12}$$

are the partial estimates of the augmented state vector (5) that are calculated in correspondence with the equation (10) (for a simplicity of designations the subscript $a$ is eliminated).

The probability density function of the estimates (11) can not be defined exactly because of infinitely growing memory. That is why for calculating the probability density function of $f(x(k)/Y_1^k)$ it is worthwhile to use the Gaussian approximation approach [10, 21]. In such an approach the state vector estimates $\widehat{x}(k/k)$ can be expressed as the weighted sum only of two partial estimates $\widehat{x}^i(k/k)$ corresponding to presence and absence of the outliers in the current measurement [8]:

$$\widehat{x}(k/k) = \sum_{i\in 1,\gamma_o} \widehat{x}(k/k, \gamma_k = i)P(\gamma(k) = i/Y_1^k). \tag{13}$$

The a posteriori probability of the measurement device state $P(\gamma(k) = 1/Y_1^k) = p_{1/k}$ depends on the outlier stochastic characteristics. In general case of the Markov chain when there exists time correlation between outliers these probabilities can be found as the following:

$$p_{1/k} = \frac{f(y(k)/\gamma(k) = 1, Y_1^{k-1})p_{1/k-1}}{\sum\limits_{i=1,\gamma_o} f(y(k)/\gamma(k) = i, Y_1^{k-1})p_{i/k-1}}, \tag{14}$$

where

$$\begin{aligned} & f(y(k)/\gamma(k) = i, Y_1^{k-1}) \\ &= N\{H\widehat{x}(k/k-1), H\widetilde{P}(k/k-1)H^T + i^2R(k)\} \\ & i = 1,\ \gamma_0 \end{aligned} \tag{15}$$

denotes the Gaussian density function of the predicted estimates and $\widetilde{P}(k/k-1)$ is the corresponding covariance matrix.

Recurrent calculations of the a posterior probability of the measurement device state can be found using the elements of the Markov chain transition matrix:

$$\begin{aligned} p_{1/k-1} &= p_{1\gamma_0} P[\gamma(k-1) = \gamma_0 / Y_1^{k-1}] \\ &+ p_{11} P[\gamma(k-1) = 1 / Y_1^{k-1}]. \end{aligned} \tag{16}$$

The state estimation equation with taking into account Eqs. (10), (13) and (14) can be written as the following:

$$\begin{aligned} &\hat{x}_a(j/j) = \hat{x}_a(j/j-1) \\ &+ [p_{1/j} K_a^1(j) + (1 - p_{1/j}) K_a^{\gamma}(j)] \\ &\cdot [y(j) - H_a(j)\, \hat{x}_a(j/j-1)], \end{aligned} \tag{17}$$

where prediction estimate is:

$$\hat{x}_a(j/j-1) = \Phi_a(j,j-1)\hat{x}_a(j-1/j-1). \tag{18}$$

The prediction error covariance matrix can be presented in the following form:

$$P_a(j/j-1) = \begin{bmatrix} P(j,j/j-1) & ... & P(j,j-N/j-1) \\ P(j-1,j/j-1) & ... & P(j-1,j-N/j-1) \\ ... & ... & ... \\ P(j-N,j/j-1) & ... & P(j-N,j-N/j-1) \end{bmatrix}, \tag{19}$$

where

$$\begin{aligned} &P(j-i, j-m/j-1) \\ &= E\{[x(j-i) - \hat{x}(j-i/j-1)] \\ &\cdot [x(j-m) - \hat{x}(j-m/j-1)]^T\} \\ &i = 0, 1, ...N. \end{aligned} \tag{20}$$

State estimate update can be written as:

$$\begin{cases} \hat{x}(j/j) = \Phi(j,j-1)\hat{x}(j-1/j-1) + [p(1/j)\cdot \\ \cdot K_0^{(1)}(j) + (1-p(1/j))K_0^{(\gamma)}(j)] \cdot \tilde{z}(j/j-1) \\ \hat{x}(j-1/j) = \hat{x}(j-1/j-1) + [p(1/j) \\ \cdot K_1^{(1)}(j) + (1-p(1/j))K_1^{(\gamma)}(j)] \cdot \tilde{z}(j/j-1) \\ ...................... \\ \hat{x}(j-N/j) = \hat{x}(j-N/j-1) + [p(1/j) \\ \cdot K_N^{(1)}(j) + (1-p(1/j))K_N^{(\gamma)}(j)] \cdot \tilde{z}(j/j-1) \\ i = 1, ..., N \end{cases} \tag{21}$$

The last element of the state estimate update $\hat{x}_a(j/j)$ is the fixed-lag estimate.

Error covariance update matrix has the same structure as Eq. (20) with elements:

$$\begin{aligned} &P(j-i, j-m/j) = P(j-i, j-m/j-1) \\ &- p(1/j)K_i^{(1)}(j)H(j)P(j, j-m/j-1) \\ &- (1-p(1/j))K_i^{(\gamma)}(j)H(j)P(j, j-m/j-1) \\ &+ (1-p(1/j))p(1/j) \\ &\cdot [K_i^{(1)}(j) - K_i^{(\gamma)}(j)]S(j)[K_m^{(1)}(j) - K_m^{(\gamma)}(j)]^T \\ &i, m = 0, 1, ..., N, \end{aligned} \tag{22}$$

where $S(j) = \tilde{z}(j/j-1)\tilde{z}^T(j/j-1)$ and $\tilde{z}(j/j-1)$ is the innovation process.

The filter gain matrix can be calculated as follows:

$$\begin{aligned} &K_a^{(1)}(j) = \left[\; K_0^{(1)^T}(j) \;\; ... \;\; K_N^{(1)^T}(j) \;\right]^T, \\ &K_a^{(\gamma)}(j) = \left[\; K_0^{(\gamma)^T}(j) \;\; ... \;\; K_N^{(\gamma)^T}(j) \;\right]^T, \\ &K_i^{(1)}(j) = P(j-i, j/j-1)H^T(j) \\ &\cdot \left[H(j)P(j,j/j-1)H^T(j) + R(j)\right]^{-1}, \\ &K_i^{(\gamma)}(j) = P(j-i, j/j-1)H^T(j) \\ &\cdot \left[H(j)P(j,j/j-1)H^T(j) + \gamma^2(j)R(j)\right]^{-1}. \end{aligned} \tag{23}$$

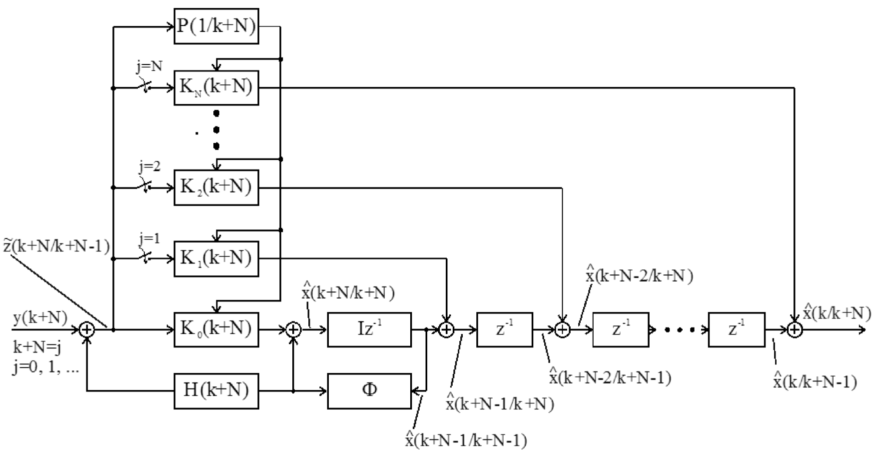


Fig. 1. Structure of the FLSOS

The expressions (18)–(23) describe the robust FLSOS. Its structure is presented in Fig. 1. It consists of the basic suboptimal non-linear filter of which the matrix gain is determined by *a posteriori* probabilities of the sensors state and it also contains a set of gains $K_1^{(1)}(j) - K_N^{(1)}(j)$, $N$delay elements and $N$ adding circuits. This filter estimates all components of the augmented state vector (5).

## 4. Simulation results

The efficiency of the proposed algorithm was investigated for dynamical systems of the first and the third orders. The system of the first order can be thought of as a model of the communication or measurement channels objected to abrupt changes of interferences. The main objective of simulations were investigation of accuracy of the proposed procedures, comparative analysis of theirs efficiency in condition of correlated changes of interference level, efficiency of malfunction detection and choice of the time-lag value.

The second example of simulations deals with the problem of target tracking in radar applications in situations when the level of interferences changes and delay of estimation results is acceptable. In this case the system of the third order was considered. The system of the second order was also investigated. The algorithm revealed similar properties and is not presented in this paper.

**Example 1.**

A first-order dynamic system with the following parameters was considered.

The performance of the proposed FLSOS was investigated by simulation of the first-order process (1) with $\Phi = 1$, $H = 1$, $Q(k) = 1$, $T_d = 0.1s$. The observation channel (2) is assumed to be described by $R(k) = 25$ when the outliers are absent and $R(k) = \gamma(k) \cdot 25$ when they are present. The random multiplier $\gamma$(k)=10 is supposed to be described by Markov chain with the following initial probability matrix $P_0$ and transition matrix $P_{ij}$:

$$P_0 = \begin{bmatrix} P_1 \\ P_\gamma \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix},$$

$$P_{ij} = \begin{bmatrix} P_{11} & P_{\gamma 1} \\ P_{1\gamma} & P_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.9 & 0.8 \\ 0.1 & 0.2 \end{bmatrix}.$$

The performance of the proposed method was investigated by using 1000 Monte Carlo runs.

The main results of the simulation are presented in Fig. 2–6. In Figs. 2 and 4 all the outlier sequences were the same (outliers arising at $k = 9$, 14 15 16 17 21 28 39 53 54) and are marked by arrows in the bottom part of figures. Results presented in the other figures are obtained for outlier sequences randomly generated in each simulation and results are averaged. The efficiency of the proposed fix-lag smoothing filter with outlier suppression (FLSOS) was compared with three algorithms: linear fixlag smoothing filter (FLS) [10], Kalman Filter (KF) [10], Kalman Filter with outlier suppression (KFOS) [8].

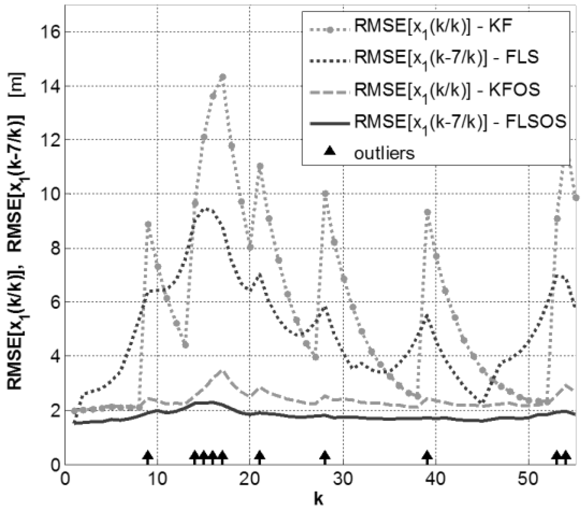


Fig. 2. Estimate RMS error for different filtering algorithms (fixed outliers position)

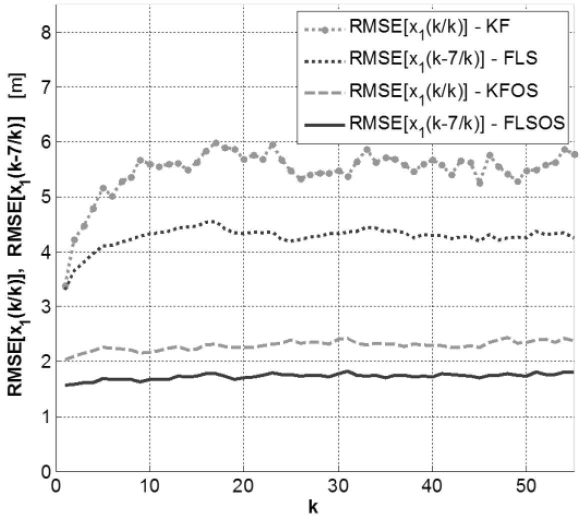


Fig. 3. Estimate RMS error for different filtering algorithms (time-averaged outliers position)

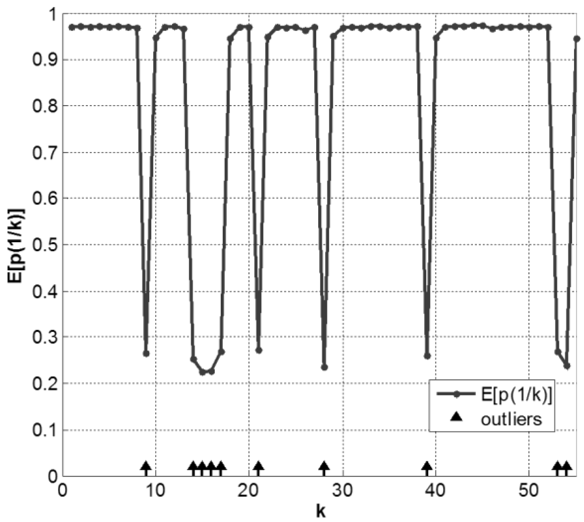


Fig. 4. A posteriori probability of the outlier absence

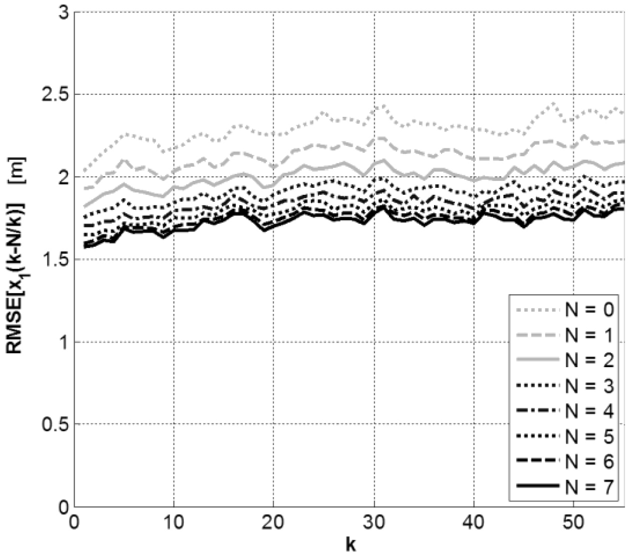


Fig. 5. RMSE of FLSOS for different time lag (time-averaged outliers position)

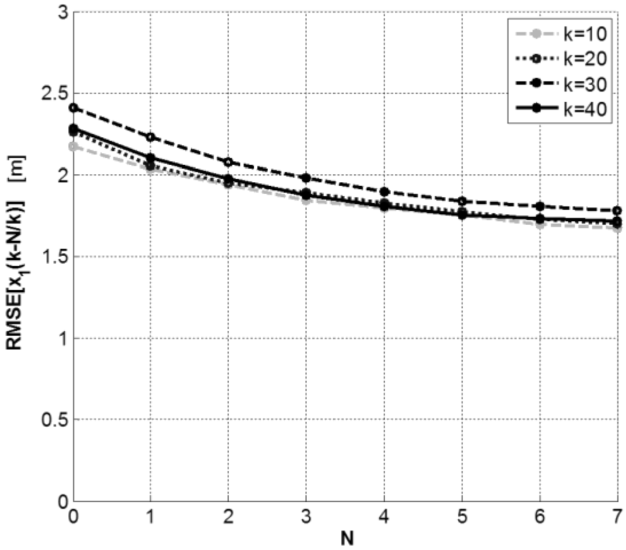


Fig. 6. RMS error as a function of time lag $N$

The estimation root mean square (RMS) errors for different filtering algorithms are presented in Fig. 2 for fixed outliers position and in Fig. 3 for time-averaged outliers position. This results show that efficiency of outlier elimination procedure is about 2.4 for both KF and FLS.

The Fig. 4 shows a posteriori probability of the outlier absence calculated according to (10). These results show that the procedure of outlier detection is rather reliable.

Figure 5 presents root mean square errors of the FLSOS for different time lag. The final dependences of estimation accuracy from time lag $N$ at the given time steps ($k$ = 10, 20, 30, 40) are presented in Fig. 6. As can be seen in this case smoothing gives the accuracy gain of state estimation of about 30%.

**Example 2.**

A third order dynamic system with the following parameters is considered.

$$\Phi = \begin{bmatrix} 1 & T_d & T_d^2/2 \\ 0 & 1 & T_d \\ 0 & 0 & 1 \end{bmatrix}, \qquad T_d = 0.1,$$

$$G_w = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \qquad Q(k) = 1,$$

$$H = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad R(k) = 25.$$

The observation channel noise is assumed to be described by covariance $R(k) = 25$ when the outliers are absent and $R(k) = \gamma(k) \cdot 25$ when they are present. The random multiplier $\gamma$(k) is described by Markov chain with the same parameters as in Example 1.

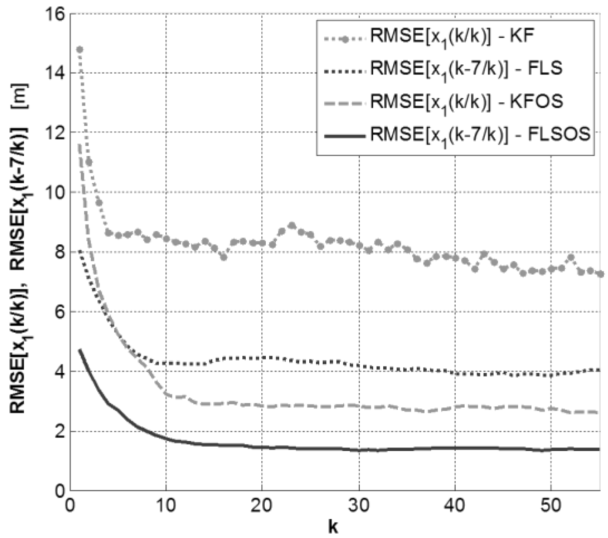


Fig. 7. RMS range estimation errors (time-averaged outliers position)

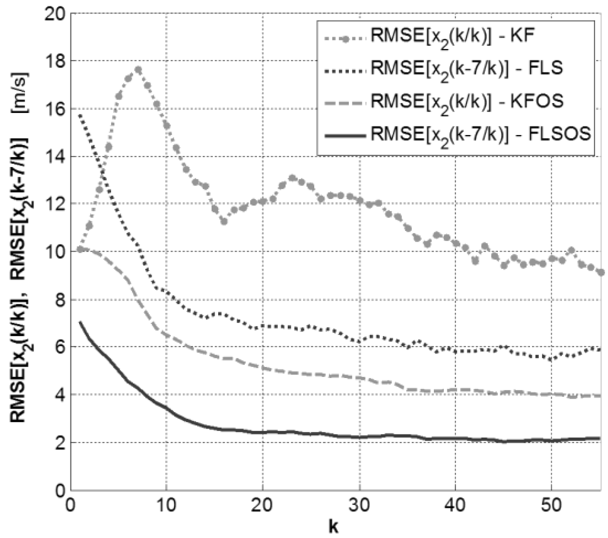


Fig. 8. RMS velocity estimation errors (time-averaged outliers position)

In Fig. 7–9 RMS errors of range, velocity and acceleration estimates for different filtering algorithms are presented (for time-averaged outliers position). This results show that efficiency gain of FLSOS procedure is about 22% to 50% in comparison with the KFOS.

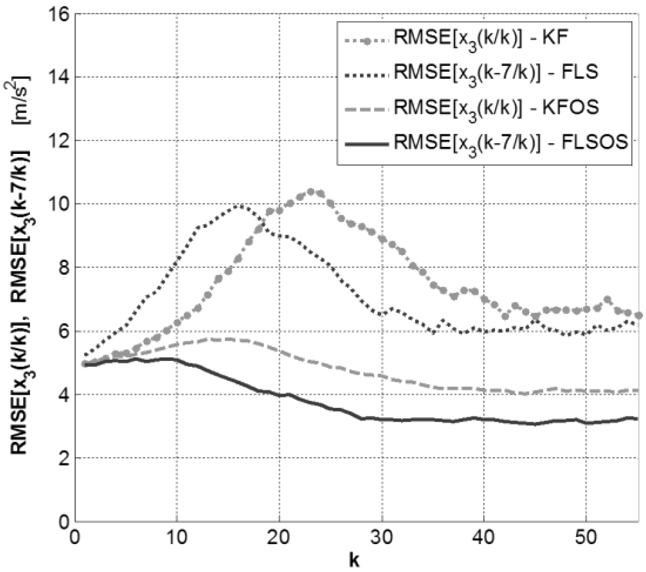


Fig. 9. RMS acceleration estimation errors (time-averaged outliers position)

The final dependences of range, velocity and acceleration estimation accuracy from time lag $N$ at the given time moments ($k = 10, 20, 30, 40$) are presented in Fig. 10. As can be seen in this case smoothing gives the accuracy gain of state estimation of about 50% for range, 50% for velocity and 30% for acceleration.

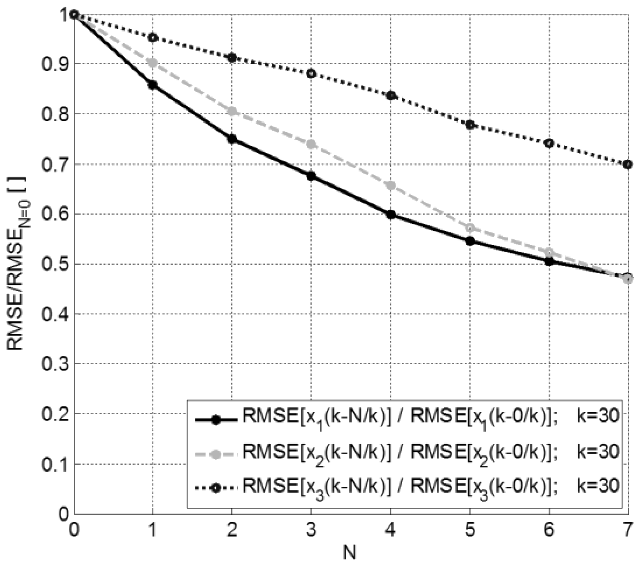


Fig. 10. Relative RMS estimation errors of range, velocity and acceleration for FLSOS as a function of time lag $N$

## 5. Conclusions

The paper presents a new suboptimal non-linear algorithm for state estimation in the presence of changing interferences and outliers. The algorithm is based on the non-linear smoothing fix-lag procedure with adaptive change of the filter matrix gain. It contains a mechanism of outliers or malfunction detection which makes it possible to employ it for sensors failure detection. The algorithm reveals a good performance and requires a moderate computational burden. As it follows from the simulation results the proposed algorithm reveals much better performance than traditional filters. The performance gain depends on interference level and filter bandwidth and practically does not change in time. It is worthy to emphasize that the efficiency of the proposed filter does not considerably differ from the optimal filter with known noise covariance matrix and at the same time it is robust with respect to observation conditions. But such an algorithm can be used only in the systems where time delay of estimates is acceptable.

REFERENCES

[1] J.J. Gertler, *Fault Detection and Diagnosis in Engineering Systems*, Marcel Dekker, NewYork, 1998.

[2] J. Korbicz, J.M. Koscielny, and Z. Kowalczuk, *Fault Diagnosis: Models, Artificial Intelligence, Applications,* Springer Verlag, Berlin, 2004.

[3] R.J. Patton, P.M. Frank, and R.N. Clark, *Issues of Fault Diagnosis for Dynamic Systems*, Springer-Verlag, Berlin, 2000.

[4] Tao Wei, Yufei Huang, and C.L.P. Chen, "Adaptive sensor fault detection and identification using particle filter algorithms", *IEEE Trans. on Systems, Man, and Cybernetics* 39, 201–213 (2009).

[5] K. Khemiri, F. Ben Hmida, J. Ragot, and M. Gossa, "Novel optimal recursive filter for state and fault estimation of linear stochastic systems with unknown disturbances", *Int. J. Applied Mathematics and Computer Science* 21 (4), 629–638 (2011).

[6] S. Wolinski, "Defining of the structural robustness", *Bull. Pol. Ac.: Tech.* 61 (1), 137–144 (2013).

[7] Yu.P. Grishin and D. Janczak, "State estimation of linear dynamic system with unknown input and uncertain observation with using dynamic programming", *Control and Cybernetics* 35 (4), 851–862 (2006).

[8] Yu.P. Grishin and D. Janczak, "A detection-estimation method for systems with random jumps with application to target tracking and fault diagnosis", *Nonlinear Dynamics, INTEH* 1, 343–366 (2010).

[9] D.G. Lainiotis and S.K. Park, "Joint detection, estimation and system identification: discrete data case", *Int. J. Control* 17 (3), 609–633 (1973).

[10] A.P. Sage and J.L. Melse, *Estimation Theory with Application to Communication and Control*, McGraw-Hill, New York, 1972.

[11] H. Dong, Z. Wang, and H. Gao, 'Fault detection for Markovian jump systems with sensor saturations and randomly varying nonlinearities", *IEEE Trans. on Circuits and Systems* 59 (10), 2354–2362 (2012).

[12] A.S. Willsky and H.L. Jones, "A generalized likelihood ratio approach to detection and estimation of jumps in linear systems", *IEEE Trans. on Automatic Control* 21 (1), 108–112 (1976).

[13] J. Klamka, A. Czornik, and M. Niezabitowski, "Stability and controllability of switched systems", *Bull. Pol. Ac.: Tech.* 61 (3), 547–555 (2013).

[14] E. Balaban, A. Saxena, P. Bansal, K.F. Goebel, and S. Curran, "Modeling, detection, and disambiguation of sensor faults for aerospace applications", *IEEE Sensors J.* 9 (12), 1907–1917 (2009).

[15] S. Rogers, "Sensor noise fault detection", *Proc. 2003 American Control Conf.* 5, 4267–4268 (2003).

[16] Xi. Zhang, "Sensor bias fault detection and isolation in a class of nonlinear uncertain systems using adaptive estimation", *IEEE Trans. on Automatic Control* 56 (5), 1220–1226 (2011).

[17] Y. Bar-Shalom, P.K. Willet, and X. Tian, *Tracking and Data Fusion*, YBS Publishing, London, 2011.

[18] B. Ristic, S. Arulampalam, and N. Gordon, *Beyond the Kalman Filter-Particle Filters for Tracking Applications*, Artech House, New York, 2004.

[19] M. Mrugalski, "An unscented Kalman filter in designing dynamic GMDH neural networks for robust fault detection", *Int. J. Applied Mathematics and Computer Science* 23 (1), 157–170 (2013).

[20] M. Mosallae and K. Salahshoor, "Sensor fault detection using adaptive modified extended Kalman filter based on data fusion technique", *Proc. $4^{th}$ Int. Conf. on Information and Automation for Sustainability* 1, 513–518 (2008).

[21] T. Katayma and S. Sugimoto, (ed), *Statistical Methods in Control and Signal Processing*, Marcel Dekker, New York, 1997.