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Continuous-time dynamic system identification with multisine random excitation revisited

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The paper presents a new, revisited and unified approach to a linear continuous-time dynamic single-input single-output system identification using input and output signal samples acquired with a deterministic constant or random sampling interval. The approach is based on a specially designed identification experiment with excitation of the form of a continuous-time multisine random excitation and digital processing of the corresponding signal samples obtained without analogue antialiasing filtration in the case of disturbances satisfying or not satisfying the Shannon's sampling theorem. Properties of the proposed approach are discussed taking into account nonlinearity of the excitation generation and data acquisition systems with a focus on model identification in the case of input and output signal levels comparable with data acquisition system accuracy. Methods reducing influence of the disturbances (including aliasing) as well as nonlinearities of the excitation generation and data acquisition systems on identification results are proposed, too.

Key words: continuous-time dynamic system identification, multisine random process, system excitation, signal sampling and reconstruction

1. Introduction

Models of linear continuous-time dynamic systems are important in many areas of research and engineering activities. Basic principles of their identification ([1], [2], [6], [31], [32], [33], [34], [35], [39], [41], [42], [43], [47], [56], [57], [58], [62], [63]) are not very much different from these for the discrete-time linear dynamic system identification case ([46], [55], [59]). In the literature of linear continuous-time dynamic system identification based on input and output signal samples a necessity of additional low-pass analogue antialiasing filtration of continuous-time dynamic system input as well as the corresponding output signals prior to their sampling with the constant sampling interval is stated. It is argued that such analogue filtration allows to reduce an influence

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of disturbances not satisfying the Shannon's sampling theorem on identification results. It is also emphasized that these filters should be identical taking into account dynamic characteristics to remove their influence on identification results (see e.g. [54], [55]). When this very restrictive and unrealistic condition is not satisfied, like it takes place in real-world applications of identification techniques in which additionally the same initial conditions of these filters are necessary, there is no possibility to remove the influence of these filters on identification results - a bias in obtained estimates may appear and identification errors implied by aliasing may be amplified.

The main aim of this paper is to present a new, revisited and unified approach to non-parametric and parametric linear continuous-time dynamic system identification with a specially designed excitation of the form of a continuous-time multisine random excitation ([13], [15], [17]) and data processing based on input and output signal samples acquired with a deterministic constant or random sampling interval ([3], [4], [10]) without prior analogue antialiasing input and output signal filtration ([27]) taking into account excitation signals generated directly from D/A converter equipped only with zero-order hold filter. A focus on model identification in the case of input and output signal levels comparable with data acquisition system accuracy is given. Different methods reducing influence of the disturbances (including aliasing) as well as nonlinearities of the excitation generation and data acquisition systems on obtained identification results are proposed, too.

The paper is organized as follows: (1) the linear continuous-time dynamic single-input single-output system identification problem with disturbances satisfying or not satisfying the Shannon's sampling theorem as well as nonlinear excitation generation and data acquisition systems is stated; (2) a new look at aliasing and an approach to non-linear dynamic system modeling with excitations being wide-sense stationary random processes are introduced; (3) the continuous-time multisine random excitation is defined and generation of its real-world realizations is described; (4) identification experiment with this excitation and initial processing of the corresponding continuous-time dynamic system input and output signal samples are proposed; (5) the linear continuous-time dynamic system frequency response and the corresponding transfer function parameter estimation are discussed.

2. Problem statement

In the presented discussion an asymptotically stable, linear, rational, time-invariant continuous-time dynamic single-input single-output (SISO) system is considered, described by the following differential equation:

$$\frac{d^p y(t)}{dt^p} + \sum_{\nu=0}^{p-1} a_\nu \frac{d^\nu y(t)}{dt^\nu} = \sum_{\nu=0}^{r} b_\nu \frac{d^\nu u(t)}{dt^\nu} + \gamma(t), \tag{1}$$

where $a_0, a_1, \ldots, a_{p-1}$, $b_0, b_1, \ldots, b_r$ $(p \geqslant r)$ are the linear continuous-time dynamic SISO system parameters, t denotes continuous time, $u(t)$ is the linear continuous-time dynamic SISO system input, $y(t)$ is the corresponding linear continuous-time dynamic SISO system output and $\gamma(t)$ is the disturbance being a continuous-time random process with the expected value $E\{\gamma(t)\}$ equal to 0 and finite variance ($E\left\{\gamma^2(t)\right\} < \infty$). It is also assumed that the disturbance $\gamma(t)$ is uncorrelated with the input $u(t)$.

The aim of identification is to determine estimates of the linear continuous-time dynamic SISO system frequency response

$$K(j\omega) = \frac{\sum_{\nu=0}^{r} b_\nu (j\omega)^\nu}{(j\omega)^p + \sum_{\nu=0}^{p-1} a_\nu (j\omega)^\nu} \tag{2}$$

for frequencies ω from the range $[0, \omega_{max}]$ or (and) estimates of the linear continuous-time dynamic SISO system parameters $a_0, a_1, \ldots, a_{p-1}$, $b_0, b_1, \ldots, b_r$ on the basis of linear continuous-time dynamic SISO system input and output signal samples acquired during specially designed identification experiments ([11], [12], [15], [29], [30]) in which a continuous-time multisine random process ([13], [15], [17]) is used as the excitation. Additionally, it is assumed that:

- the linear continuous-time dynamic SISO system input and output signals are additionally disturbed prior to their sampling by additive disturbances being continuous-time random processes with expected values equal to 0 and finite variances. These disturbances can be mutually correlated but they are uncorrelated with the input signal $u(t)$;
- there is no analogue antialiasing signal filtration prior to signal sampling;
- the linear continuous-time dynamic SISO system input and output signals are sampled with the sampling interval being a deterministic constant value or a set of a random variable realizations;
- all disturbances may satisfy or not satisfy the Shannon's sampling theorem for the expected value of sampling interval chosen;
- the excitation is generated using D/A converter equipped only with zero-order hold filter - there is no special analogue reconstruction filter;
- nonlinearities of the excitation generation and data acquisition systems are taken into account. These nonlinearities are especially important in the case of input and output signal levels comparable with the data acquisition system accuracy.

The presented in the sequel new, revisited and unified approach to nonparametric and parametric linear continuous-time dynamic system identification is based on a new look at aliasing and an approach to nonlinear system modeling with excitations being wide-sense stationary random processes ([26]).

3. A new look at aliasing

Let $\xi(t)$ be a wide-sense stationary continuous-time random process with the expected value $E\{\xi(t)\}$ equal to 0, finite variance ($E\left\{\xi^2(t)\right\} < 0$) and power spectral density $\Phi_{\xi\xi}(\omega)$ ($\omega \in [0,\infty)$). Its realizations are denoted by the superscript ^{r} - i.e. $\xi^r(t)$ is the r-th ($r = 1,2,\ldots$) realization of $\xi(t)$. It is assumed for discussion in this section that these realizations are sampled with the deterministic constant sampling interval μ.

The power spectral density $\Phi_{\xi\xi}(\omega)$ may be decomposed into the two components $\Phi_{\xi_1\xi_1}(\omega)$ and $\Phi_{\xi_2\xi_2}(\omega)$ such that $\Phi_{\xi_1\xi_1}(\omega) = 0$ (for $\omega > \omega_N$), $\Phi_{\xi_2\xi_2}(\omega) = 0$ (for $\omega \leqslant \omega_N$) and:

$$\Phi_{\xi\xi}(\omega) = \Phi_{\xi_1\xi_1}(\omega) + \Phi_{\xi_2\xi_2}(\omega), \tag{3}$$

where ω_N denotes the Nyquist frequency for sampling interval μ. The corresponding time-domain decomposition is the following:

$$\xi(t) = \xi_1(t) + \xi_2(t). \tag{4}$$

It is obvious that the components $\xi_1(t)$ and $\xi_2(t)$ are uncorrelated.

The above decomposition implies that the corresponding discrete-time random process $\xi(i\mu)$ ($i = -\infty,\ldots,0,1,\ldots,\infty$) obtained by sampling the continuous-time random process $\xi(t)$ with the sampling interval μ is a sum of the two components $\xi_1(i\mu)$ and $\xi_2(i\mu)$, i.e.:

$$\xi(i\mu) = \xi_1(i\mu) + \xi_1(i\mu), \tag{5}$$

where:

- $\xi_1(i\mu)$ is the result of sampling the component $\xi_1(t)$ that satisfies the Shannon's sampling theorem for the sampling interval μ,
- $\xi_2(i\mu)$ is the result of sampling the component $\xi_2(t)$ that does not satisfy this theorem for the sampling interval μ - this component is called as aliasing.

The components $\xi_1(i\mu)$ and $\xi_2(i\mu)$ may have nonzero values of their power spectral densities for the same frequencies but it follows from the spectral representation theorem ([7], [17]) that taking into account ensemble averaging these two components are uncorrelated:

$$\mathcal{E}\{\xi_1(i\mu)\xi_2((i-\tau)\mu)\} = 0, \tag{6}$$

where $\tau = -\infty,\ldots,0,1\ldots,\infty$. Additionally, $E\{\xi_1(i\mu)\} = E\{\xi_2(i\mu)\} = 0$.

The above interpretation of aliasing implies that a digital orthogonal filtration ([50], [51], [64]) is a tool that allows to reduce influence of aliasing on digitally processed data, including identification results, in the case when there is no analogue antialiasing signal filtration prior to signal sampling. Its applications results in estimates of the realization $\xi_1^r(i\mu)$ values.

It is also worth to note that continuous-time dynamic system model identification without identical analogue antialiasing filters used prior to input and output signal sampling in the case of disturbances not satisfying the Shannon's sampling theorem for sampling interval chosen results in reduction of only signal to noise ratio while comparing with the corresponding model identification based on signal samples acquired with application of these analogue filters.

4. Nonlinear system modeling

The second tool used in the sequel is an approach to nonlinear dynamic system modelling with excitations being wide-sense stationary random processes ([22], [24], [26]). In this approach any nonlinear dynamic system (e.g. continuous-time excitation signal reconstruction system using D/A converter with zero-order filter, data acquisition system with the corresponding sensor and A/D processing unit) is approximated by a linear dynamic system in which nonlinearity is represented by a disturbance $\psi(t)$ at the linear dynamic system output that is uncorrelated with the system input $u(t)$ as well as with the undisturbed linear dynamic system output $y_f(t)$. Properties of the disturbance $\psi(t)$ and undisturbed linear dynamic system output $y_f(t)$ differs from the discussed, in the previous section, properties of components the $\xi_1(t)$ and $\xi_2(t)$. They may have nonzero values of their power spectral densities for the same frequencies but they are uncorrelated and after sampling with the sampling interval μ:

$$\mathcal{E}\left\{u(i\mu)\psi((i-\tau)\mu)\right\} = \mathcal{E}\left\{y_f(i\mu)\psi((i-\tau)\mu)\right\} = 0, \tag{7}$$

where $i = -\infty, \ldots, 0, 1, \ldots, \infty$ and $\tau = -\infty, \ldots, 0, 1 \ldots, \infty$.

It follows from the above discussion that influence of unwanted nonlinearity on digitally processed data may be reduced using the digital orthogonal filtration. This reduction can also be obtained by a data processing based on Wiener model identification with instrumental variable estimation method ([22], [24], [26]). The data processing, calculation of undisturbed linear dynamic system output $y_f(t)$ value estimates based on the results of model identification, is a tool that allows to look from another point of view on many digital signal processing problems like for example:

- identification problems implied by nonlinearities of digital measurement systems, e.g. finite precision of quantizers used in D/A and A/D converters,
- continuous-time system identification with data coming from identification experiments in which plant is excited directly from continuous-time excitation signal reconstruction system based on D/A converter with zero-order filter without additional analogue signal reconstruction filtration,
- model identification in the case of input and output signal levels comparable with data acquisition system accuracy,

- model identification in the case of disturbances containing periodic components,
- estimation of input and output signal values in between multiplicities of D/A and A/D converter quant,
- controllers working without expensive analogue antialiasing filters.

It allows also to reduce influence of disturbances not satisfying the Shannon's sampling theorem on identification results.

5. System excitation

The proposed approach to linear continuous-time dynamic SISO system identification is based on an excitation of the form of a continuous-time multisine random process ([13], [15], [17]). This process is defined in the time-domain by a sum of $\frac{N}{2}+1$ (N is even) harmonic continuous-time sines including a constant component:

$$\upsilon(t)=\sum_{n=0}^{\frac{N}{2}}A_n\sin(\Omega nt+\phi_n), \tag{8}$$

where $\Omega=\frac{2\omega_{max}}{N}$ denotes the fundamental frequency for the frequency range $[0,\omega_{max}]$, $n=0,1,\ldots,\frac{N}{2}$ denotes consecutive harmonics of this frequency in the range $[0,\omega_{max}]$, A_n are deterministic amplitudes of sine components, ϕ_n are phase shifts of which ϕ_0 is deterministic and the remaining phase shifts are random, independent and:

- uniformly distributed on $[0,2\pi)$ for $n=1,2,\ldots,\frac{N}{2}-1$,
- Bernoulli distributed $\mathcal{B}\left(\frac{1}{2},\left\{\frac{\pi}{2},\frac{3\pi}{2}\right\}\right)$ for $n=\frac{N}{2}$, i.e.:

$$P\left\{\phi_{\frac{N}{2}}=\frac{\pi}{2}\right\}=P\left\{\phi_{\frac{N}{2}}=\frac{3\pi}{2}\right\}=\frac{1}{2}, \tag{9}$$

  where $P\{X\}$ denotes the probability of an event X.

Spectral properties of the continuous-time multisine random process $\upsilon(t)$ are defined by the power spectral density $\Phi_{\eta\eta}(\omega)$ ($\Phi_{\eta\eta}(\omega)<\infty$ for $\omega\leqslant\omega_{max}$) of a wide-sense stationary real-valued continuous-time random process $\eta(t)$. It means that the deterministic amplitudes A_n are chosen in a special way ([11], [14], [17]):

- for $n=1,2,\ldots,\frac{N}{2}-1$ as $A_n=2\sqrt{\frac{\Phi_{\eta\eta}(\Omega n)}{N\mu}}$,
- for $n=0,\frac{N}{2}$ as $A_n=\sqrt{\frac{\Phi_{\eta\eta}(\Omega n)}{N\mu}}$,

where μ is a parameter interpreted in the sequel as the sampling interval.

Realizations $\upsilon^r(t)$ $(r = 1,2,\ldots)$ of the continuous-time multisine random process $\upsilon(t)$ with defined spectral properties may be obtained numerically on the basis of recursive synthesis and simulation of N-sample multisine random time series realizations $\upsilon^r(\iota\mu)$ $(\iota = 0,1,\ldots,N-1)$ with the multisine transformation ([21], [23], [25]) aided via the classical *sinc* interpolation used to calculate $\upsilon^r(t)$ values in between sampling instants [28]. Such continuous-time random process reconstruction is useful for *off-line* signal processing, e.g. simulation.

In the case of model identification the corresponding real-world realization $u^r(t)$ (multisine random excitation - the linear continuous-time dynamic SISO system input) of the wide-sense stationary multisine random process $\upsilon(t)$ may be reconstructed using D/A converter working with the period T $(T = \alpha\mu,\ \alpha = 1,2,\ldots)$ that is equipped only with zero-order hold filter. The reconstruction procedure is the following:

$$u^r(t) = u^r(iT) \tag{10}$$

for $t \in [iT,(i+1)T)$, $i = 0,1,\ldots$, and

$$u^r(iT) = \upsilon^r(i\alpha\mu) + \beta\Psi^r(i\alpha\mu), \tag{11}$$

where:

- $\upsilon^r(\iota\alpha\mu + qN) = \upsilon^r(i\alpha\mu)$ for $q = 0,1,\ldots$,
- $\beta \in \{1,2,\ldots\}$ is a parameter,
- $\Psi^r(t)$ is a realization of the continuous-time random process $\Psi(t) = \Delta$ and Δ is the uniformly distributed on $[0,\Delta)$ random variable being the D/A converter quant.

It is also worth to mention that such continuous-time multisine random process reconstruction may be aided by quatization of the multisine random excitation mean value, oversampling ([8], [9]) or (and) a band-limited interpolation [49] but in the proposed approach to linear continuous-time dynamic system identification they are not necessary for precise model estimation and in the identification experiment discussed below these methods are not used.

6. Identification experiment and initial data processing

Identification experiment is designed based on a classical approach for multisine excitations ([11], [12], [15], [29], [30]): the start of data acquisition is delayed with respect to the instant of putting the excitation at the linear continuous-time dynamic SISO system input. It starts after all transients implied by initial conditions have decayed. Under continuous-time dynamic SISO system steady-state conditions the multisine random excitation real-world realization $u^r(t)$ is repeated m times. To estimate models, the linear continuous-time dynamic SISO system input and output signals are represented by the following sets of their samples:

$$\left\{\breve{u}^r(0), \breve{u}^r(t_1^r), \ldots, \breve{u}^r(t_{mN-1}^r)\right\}, \tag{12}$$

$$\left\{\breve{y}^r(0), \breve{y}^r(t_1^r), \ldots, \breve{y}^r(t_{mN-1}^r)\right\} \tag{13}$$

collected at discrete-time instants t_i^r:

$$t_i^r = t_{i-1}^r + T_i^r, \tag{14}$$

where for $i = 0, 1, \ldots, mN-1$ the corresponding T_i^r is a deterministic constant value ($T_i^r = \mu$) or the mN-sample set of realisations of a random variable with the expected value μ and variance σ^2 ([3], [4], [10], [27]). It is worth to note that the ratio $\frac{\sigma}{\mu}$ is a parameter allowing to control continuous-time signal sampling and aliasing - in the case of $\frac{\sigma}{\mu} = 0$ the signals are sampled with the deterministic constant interval $T_i^r = \mu$.

During identification experiment the sets (12) and (13) of input and output signal samples are collected for P real-world realizations $u^r(t)$ ($r = 1, 2, \ldots, P$) of the continuous-time multisine random process $\upsilon(t)$. It is worth to start inputting the consecutive realizations $u^r(t)$ at the linear continuous-time dynamic SISO system input with delays being realizations of a uniformly distributed random variable and add to each sampled signal, prior to sampling, independent realizations of the random variable that is uniformly distributed in the range covering the quant of A/D converter used.

Processing of signals sampled with the random sampling interval is not a simple task especially in the case when a parametric model of linear continuous-time dynamic SISO system identification is considered. In the sequel, to identify nonparametric and parametric models of the linear continuous-time dynamic SISO system efficiently, the mN-sample data sets (12) and (13) obtained with the deterministic constant or random sampling interval are transformed into the following N-sample data sets:

$$\{\widetilde{u}^r(0), \widetilde{u}^r(\mu), \ldots, \widetilde{u}^r((N-1)\mu)\}, \tag{15}$$

$$\{\widetilde{y}^r(0), \widetilde{y}^r(\mu), \ldots, \widetilde{y}^r((N-1)\mu)\}. \tag{16}$$

These data sets contain estimates of the linear continuous-time dynamic SISO system input $u(t)$ and output $y(t)$ signal samples obtained with the deterministic constant sampling interval μ for one repetition of the continuous-time multisine random process realization. In the proposed revisited and unified approach to linear continuous-time dynamic SISO system identification these N-sample data sets are calculated directly from the corresponding mN-sample data sets (12) and (13) by using a transformation based on the nonuniform finite discrete Fourier transform ([10]). For the given mN-sample data set (12) or (13) this transformation consists of the following two steps:

- in the first step a spectrum estimate of the continuous-time signal (input or output) for frequencies Ωm ($m = 0, 1, \ldots, \frac{N}{2}$) based on the corresponding mN-sample data sequence obtained with the random sampling interval is calculated using the nonuniform finite discrete Fourier transform,

- in the second step an estimate of N-sample sequence of the continuous-time signal samples obtained with the deterministic constant sampling interval μ is calculated using the inverse finite discrete Fourier transform of a discrete-time signal spectrum synthesized on the basis of the calculated spectrum estimate from the first step. This step uses numerical efficiency of fast Fourier transform algorithms ([5]).

In the case when the linear continuous-time dynamic SISO system input and output signals are sampled with the deterministic constant interval $T_i^r = \mu$ ($i = 0, 1, \ldots, mN-1$ and $r = 1, 2, \ldots, P$) the above transformation is the corresponding mean value calculation, e.g.:

$$\widetilde{u}^r(i\mu) = \frac{1}{m}\sum_{l=0}^{m-1} \breve{u}^r(t^r_{i+lN}). \tag{17}$$

When the disturbances do not contain components periodic in the time window of length $N\mu$ [38], the obtained N-sample data sequences (15) and (16) for each realization υ^r ($r = 1, 2, \ldots, P$) are unbiased and consistent estimators of the corresponding noise- and alias-free continuous-time input $u(t)$ and output $y(t)$ signal samples taken with the deterministic constant sampling interval μ. Their variances decline with the increase of the number m of processed N-sample data segments. It implies in this case that the above transformation is also a tool that allows to reduce influence of aliasing on identification results. It is also worth to emphasize that in the case of disturbances containing components periodic in the time window of length $N\mu$ this transformation may be aided by the mentioned additional data processing based on Wiener model identification with instrumental variable estimation method as well as active noise control techniques [18], [19], [20], [24], [37], [45], [52].

Though the discussion in this section concerns a case of the period of D/A converter being multiplicity of the sampling interval μ results presented are also true for the case of A/D converter working with the random sampling interval characterized by expected value that may be greater than μ.

7. Frequency response estimation

It is assumed in this section that the data sets (15) and (16) are used directly without additional processing based on Wiener model identification, orthogonal filtration and active noise control to estimate frequency response of the linear continuous-time dynamic SISO system.

The linear continuous-time dynamic SISO system frequency response can be estimated for the r-th data sets (15) and (16) by using, for example, the following empirical transfer function estimator ([11], [29], [30], [46]):

$$\hat{K}^r(j\Omega n) = \frac{\widetilde{Y}^r(j\Omega n)}{\widetilde{U}^r(j\Omega n)}, \tag{18}$$

where frequencies $\Omega n \in [0, \omega_{max}]$ ($n \in \left\{0, 1, \ldots, \frac{N}{2}\right\}$) and:

- $\widetilde{Y}^r(j\Omega n)$ is N-sample the finite discrete Fourier transform of $\widetilde{y}^r(i\mu)$:

$$\widetilde{Y}^r(j\Omega n) = \mu \sum_{i=0}^{N-1} \widetilde{y}^r(i\mu) e^{-j\Omega\mu ni}, \tag{19}$$

- $\widetilde{U}^r(j\Omega n)$ is the N-sample finite discrete Fourier transform of $\widetilde{u}^r(i\mu)$:

$$\widetilde{U}^r(j\Omega n) = \mu \sum_{i=0}^{N-1} \widetilde{u}^r(i\mu) e^{-j\Omega\mu ni}. \tag{20}$$

Properties of the estimator (18) follow directly from the definition of empirical transfer estimator used for the case of continuous-time dynamic system identification ([15]), properties of multisine random processes [17] and the fact that N-sample finite discrete Fourier transforms of $\widetilde{u}^r(i\mu)$ and $\widetilde{y}^r(i\mu)$ inherit statistical properties of the corresponding N-sample data sequences (15) and (16). It can be proven that:

- in the case of disturbances not containing components periodic in the time window of length $N\mu$ (e.g. $\Delta = 0$) the estimator (18) is consistent one under $m \to \infty$ and:

$$\lim_{m\to\infty} \hat{K}^r(j\Omega n) = K(j\Omega n) \quad a.s.; \tag{21}$$

- in the case of disturbances containing components periodic in the time window of length $N\mu$ the estimator (18) is not consistent one but the following estimator

$$\overline{\hat{K}}(j\Omega n) = \frac{1}{P} \sum_{r=1}^{P} \hat{K}^r(j\Omega n), \tag{22}$$

have the property:

$$\lim_{P\to\infty} \overline{\hat{K}}(j\Omega n) = K(j\Omega n) \quad a.s. \tag{23}$$

- in the case of input and output signal levels less than quant of the A/D converter used the above estimators are not consistent ones but

$$\lim_{\beta\to\infty, P\to\infty} \overline{\hat{K}}(j\Omega n) = K(j\Omega n) \quad a.s. \tag{24}$$

Application of correlation method of data processing ([12], [16], [18]) allows to calculate the linear continuous-time dynamic SISO system frequency response estimate $\hat{K}^r(j\omega)$ for frequencies $\omega \in [0, \omega_{max}]$ as:

$$\hat{K}^r(j\omega) = \frac{\hat{\Phi}_{\widetilde{y}^r \upsilon^r}(j\omega)}{\hat{\Phi}_{\widetilde{u}^r \upsilon^r}(j\omega)} \tag{25}$$

or

$$\hat{K}^r(j\omega) = \frac{\hat{\Phi}_{\widetilde{y}^r\widetilde{u}^r}(j\omega)}{\hat{\Phi}_{\widetilde{u}^r\widetilde{u}^r}(j\omega)}, \tag{26}$$

where $\hat{\Phi}_{\widetilde{u}^r\widetilde{u}^r}(j\omega)$ is the power spectral density estimate obtained using $\widetilde{u}^r(i\mu)$ and $\hat{\Phi}_{\widetilde{y}^r\upsilon^r}(j\omega)$, $\hat{\Phi}_{\widetilde{u}^r\upsilon^r}(j\omega)$, $\hat{\Phi}_{\widetilde{y}^r\widetilde{u}^r}(j\omega)$ are cross power spectral density estimates obtained using N-sample sequences $\widetilde{y}^r(i\mu)$, $\upsilon^r(i\mu)$ and $\widetilde{y}^r(i\mu)$, respectively. They are calculated using the corresponding auto ($\hat{R}_{\widetilde{u}^r\widetilde{u}^r}(\tau\mu)$) and cross correlation function estimates ($\hat{R}_{\widetilde{y}^r\upsilon^r}(\tau\mu)$, $\hat{R}_{\widetilde{u}^r\upsilon^r}(\tau\mu)$, $\hat{R}_{\widetilde{y}^r\widetilde{u}^r}(\tau\mu)$), e.g.:

$$\hat{\Phi}_{\widetilde{y}^r\upsilon^r}(j\omega) = \mu \sum_{\tau=-M}^{\tau=M} \hat{R}_{\widetilde{y}^r\upsilon^r}(\tau\mu)e^{-j\omega\mu\tau}, \tag{27}$$

where M is a parameter. If $M \to \infty$ and $N \to \infty$ in such a way that $\frac{M}{N} \to 0$ then estimator (25) is consistent and

$$\lim_{M,N\to\infty} \hat{K}^r(j\omega) = K(j\omega) \quad a.s. \tag{28}$$

for all frequencies $\omega \in [0, \omega_{max}]$. This property have also estimators (25) and (26) in the case when disturbance influencing measurements of $u(t)$ have no components periodic in the time window of length $N\mu$. It is obvious that in the case of correlation method of data processing the properties (23) and (24) also hold.

It is worth to mention that increase of the number m of each continuous-time multisine random excitation realization repetitions or (and) increase of the number P of collected data sequences for different continuous-time multisine random excitation realizations are the next tools that allow to reduce influence of disturbances (including aliasing) on identification results.

The additional processing of the data sets (15) and (16) based on Wiener model identification, orthogonal filtration or active noise control not influence the properties of discussed above frequency response estimators. It is also worth to emphasize in this place that results presented in this section for frequency response estimation of rational systems are also true for nonrational plants [53].

8. Transfer function estimation

The identified frequency response $\overline{\hat{K}}(j\omega)$ is a starting point to calculate the corresponding transfer function estimate $\hat{K}(s)$ using approximation methods. Properties of the parameter estimates $(\hat{a}_0, \hat{a}_1, \ldots, \hat{a}_{p-1}, \hat{b}_0, \hat{b}_1, \ldots, \hat{b}_r)$ obtained depend highly on properties of approximation algorithm used (see e.g. [62], [61]). Without estimation of the frequency response the N-sample linear continuous-time SISO system input and output signal estimates $\widetilde{u}^r(i\mu)$ and $\widetilde{y}^r(i\mu)$ ($i = 0, 1, \ldots, N-1$ and $r = 1, 2, \ldots P$) may be used to calculate the corresponding values of input and output signals derivatives or integrals at time instants $i\mu$ ([15]) that are necessary to estimate the unknown system parameters using for example instrumental variable estimation method ([56], [62]). A good tool to control an influence of disturbances on results of the parameters estimation is a disturbance adjustment ([11], [29], [30]). Obtained identification results may be also enhanced using a method of identification results variance reduction ([36]).

9. Conclusions

In the paper, a new, revisited and unified approach to linear continuous-time dynamic system identification with a specially designed excitation of the form of a continuous-time multisine random excitation and digital data processing based on signal sampling with a deterministic constant or random sampling interval was proposed. Properties of the excitation and data processing algorithms allow to identify precisely nonparametric as well as parametric models of linear continuous-time dynamic systems without analogue antialiasing input and output signal filtration nevertheless disturbances satisfying or not satisfying the Shannon's sampling theorem and to reduce an influence of the excitation generation and data acquisition system nonlinearities on identification results. A focus on model identification in the case of input and output signal levels comparable with data acquisition system accuracy was given.

It is also worth to emphasize that in the presented discussion only finite variance of disturbances is assumed. Its power spectral density may be not bounded – periodic disturbances are taken into account. It is a class of disturbances that is more general that this discussed in classical books on system identification (e.g.: [1], [2], [40], [44], [46], [48], [59], [60], [62]) and the corresponding publications based on them.

The approach presented in this paper is a tool that allows to enhance properties of existing measurement systems and introduce superposition principle in a nonlinear world. It is also a background for a new generation of digital measurement systems, like for example very precise spectrum and system analyzers, as well as discrete-time controllers working without expensive analogue antialiasing filters. Special cases of the presented discussion are a closed-loop system identification and the discrete-time model identification.

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