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Application of agent cooperation by means of DES supervision to manufacturing system

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The supervision of complex manufacturing systems is handled in this paper. Production lines of manufacturing systems are understood here to be hybrid agents. Such an agent expresses continuous material flow together with a set of imperative discrete operations. In other words, the hybrid agent makes a cooperation of continuous and discrete event devices possible. The results are applied to complex continuous manufacturing system. First Order Hybrid Petri Nets (FOHPN) are utilized in order to model the elementary autonomous hybrid agents. The cooperation of hybrid agents is based on the DES (discrete-event systems) control theory. It is realized by means of DES supervision methods and the agent negotiation is based on place/transition Petri nets (P/T PN). The proposed approach is illustrated in details on the cooperation of hybrid production lines in the real complex manufacturing systems recycling waste plastics into plastic bags. The usefulness and applicability of the approach consists especially in the fact that the procedures in analytical terms can be employed. In such a way the approach turns lucid also in complicated cases.

Key words: agents, control, cooperation, discrete-event systems, continuous systems, hybrid Petri nets, supervisor, synthesis.

1. Introduction and preliminaries

There are many kinds of complex manufacturing systems in present technology. The complexity consists in the fact that production lines contain very complicated devices and they often have a hybrid character. Namely, they contain both the continuously working devices and the discretely working ones which are complex in themselves. Mathematical modeling such devices is very difficult and many times it is impossible. There are many reasons for this. Continuous systems (CS) are usually described by means of differential equations describing processes at respecting physical laws. However, there

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F. ČAPKOVIČ

are certain complex CS where it is very difficult or even practically impossible to obtain a model corresponding to the real system. Two main difficulties occur on that way. The first of them is how to determine the kind of differential equations describing the particular CS. In case of linear systems it is necessary to guess the most suitable order of the linear differential equations, while in case of nonlinear systems it is necessary to choose the most suitable kind of nonlinear differential equations. Even, in case when we contrive the suitable equations, the second problem arises, namely, how to identify all parameters of the chosen kind of differential equations by means of measuring (if any such parameters are measurable). In case of models describing mathematically the real complex system many parameters cannot be found. Moreover, any system (including CS) has minimally two discrete states. Namely, the system can be either *idle* (passive, switched off) or *working* (active, switched on). The mutual transitions between these states are, as matter of fact, the discrete events.

On that account, other methods of modeling the complex manufacturing systems have to be found. Hybrid Petri nets (HPN) [9] are frequently used for modeling complex hybrid systems (HS). However, especially the First Order Hybrid Petri Nets (FOHPN) [1, 10, 17] seem to be very suitable for HS modeling. HPN in general combine continuous Petri nets (CPS) [9, 10] with different kinds of Petri nets (PN) e.g. like place/transition Petri nets (P/T PN), deterministic timed PN, stochastic PN, etc. Thus, the following idea arose. In the first step the model of hybrid autonomous agents (e.g. production lines in complex manufacturing systems) by means of FOHPN will be created. Then, in the second step the cooperation of the agents by means of discrete-event systems (DES) supervision will be synthesized. Namely, DES are driven by discrete events and the cooperation of hybrid agents exhibits attributes of DES.

The paper consists of four main sections. In the current section first of all the DES will be defined. Then P/T PN as a tool for mathematical and graphical modeling DES will be presented. Finally, FOHPN as a tool for mathematical and graphical modeling HS will be introduced. The second section is devoted to FOHPN-based modeling one of the production lines of real manufacturing system producing the plastic bags from recycled waste plastics. The third section describes needs of cooperation among the production lines being the hybrid agents. It points out the possibility of the negotiation of two agents in general. In the fourth section general principles of agent cooperation are presented and the cooperation of six production lines of the real manufacturing process is synthesized.

This paper is the extension of the conference paper [8].

1.1. Discrete event systems

Discrete Event Systems (DES) are systems discrete in nature, i.e. driven by discrete events. Namely, the course of a DES variable evolves in response to certain discrete qualitative changes, called events. A simple illustration is given in Fig. 1. Formally, DES can be described by the triplet

$$\langle X, \mathcal{U}, \mathcal{A} \rangle$$

where:





- $\begin{aligned} &\mathcal{X} = \{X_0, X_1, ..., X_N\} \text{ is the set of the DES states (the state space)} \\ &X_k = \{{}^kx_1, ..., {}^kx_n\}, k = 0, 1, ..., N, {}^kx_i \in \{0, 1, ..., {}^kc_i\}, i = 1, ..., n \text{ is the set of the states of DES elementary (atomic) subsystems} \\ &\mathcal{U} = \{U_0, U_1, ..., U_{N-1}\} \text{ is the set of the DES discrete events} \\ &U_k = \{{}^ku_1, ..., {}^ku_m\}, k = 0, 1, ..., N-1, {}^ku_j \in \{0, 1\}, j = 1, ..., m \text{ is the set of DES elementary (atomic) discrete events} \end{aligned}$
- $\mathcal{A} \subseteq \mathcal{X} \times \mathcal{U} \to \mathcal{X}$ is the set of mutual causal relations among the states and the discrete events.

In order to model DES Petri nets (PN) are frequently used. Especially, place/transition Petri nets (P/T PN) are utilized.

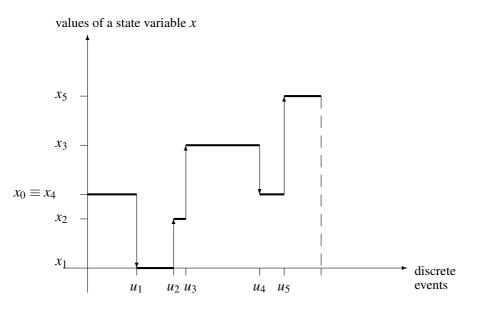


Figure 1. The development of a state variable *x* of DES.

1.2. Place/transition Petri nets

The P/T PN were defined by Carl Adam Petri in sixtieth of the last century, more precisely in his PhD thesis [16]. Later, the mathematical background was extended by Peterson [15] and Murata [14]. Since many thousands papers about PN have been written by many authors. In general, P/T PN are bipartite directed graphs

$$\langle \mathcal{P}, \mathcal{T}, \mathcal{F}, \mathcal{G} \rangle; \qquad \mathcal{P} \cap \mathcal{T} = \emptyset, \qquad \qquad \mathcal{F} \cap \mathcal{G} = \emptyset$$

where:

 $\mathcal{P} = \{p_1, p_2, ..., p_n\}$ is set of PN places; $p_i \in \mathcal{P}, i = 1, ..., n$ represents the states



of elementary (atomic) activities of DES,

- $\mathcal{T} = \{t_1, t_2, ..., t_m\}$ is set of PN transitions; $t_j \in \mathcal{T}, j = 1, ..., m$ represents the discrete events,
- $\mathcal{F} \subseteq \mathcal{P} \times \mathcal{T}$ is the set of interconnections (causal relations) from place to transitions $(\mathcal{P} \to \mathcal{T})$,
- $\mathcal{G} \subseteq \mathcal{T} \times \mathcal{P}$ is the set of interconnections (causal relations) from transitions to places $(\mathcal{T} \to \mathcal{P})$.

However, P/T PN have also their dynamics represented by movement of marks assigned to PN places.

$$\langle X, \mathcal{U}, \delta, \mathbf{x}_0 \rangle$$
 (1)

where:

 $\mathcal{X} = {\mathbf{x}_0, \mathbf{x}_1, ..., \mathbf{x}_N}$ is the set of state vectors; $\mathbf{x}_k \in \mathcal{X}, k = 0, 1, ..., N$ represents the state vector of elementary places; $\mathbf{x}_k = (\mathbf{\sigma}_{p_1}^k, ..., \mathbf{\sigma}_{p_n}^k)^T$; $\mathbf{\sigma}_{p_i}^k \in \{0, 1, ..., c_{p_i}\}$, $i = 1, ..., n; 0 \le \sigma_{p_i}^k \le c_{p_i}; c_{p_i}$ is the capacity, $\mathcal{U} = \{\mathbf{u}_0, \mathbf{u}_1, ..., \mathbf{u}_M\}$ is the set of control vectors; $\mathbf{u}_k \in \mathcal{U}, k = 0, 1, ..., M$ represents the state vector of elementary transitions; $\mathbf{u}_k = (\gamma_{t_1}^k, ..., \gamma_{t_m}^k)^T$; $\gamma_{t_j}^k \in \{0, 1\}, j = 1, ..., m;$ $\delta : X \times \mathcal{U} \to X$ is the transition function

 \mathbf{x}_0 is the initial state vector.

The formally defined transition δ can be expressed by the following linear discrete system which is the effective P/T PN model

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{x}_k + \mathbf{B} \cdot \mathbf{u}_k \quad , \quad k = 0, \dots, K \\ \mathbf{B} &= \mathbf{G}^T - \mathbf{F} \\ \mathbf{F} \cdot \mathbf{u}_k &\leqslant \mathbf{x}_k \end{aligned}$$
 (2)

where:

k is the discrete step of the dynamics development

- B, F, G are structural matrices of constant elements
- $\mathbf{F} = \{f_{ij}\}_{n \times m}$, where $f_{ij} \in \{0, M_{f_{ij}}\}, i = 1, ..., n, j = 1, ..., m$ express the causal relations between the states and the discrete events
- $\mathbf{G} = \{g_{ij}\}_{m \times n}$, where $g_{ij} \in \{0, M_{g_{ij}}\}, i = 1, ..., m, j = 1, ..., n$ express the causal relations between the discrete events and the DEDS states.

First order hybrid Petri nets (FOHPN) 1.3.

FOHPN are a special kind of HPN, while HPN in general are an extension of standard PN. They are able to model the coexistence of discrete and continuous variables in complex systems. Usage of HPN and FOHPN brings several advantages like reducing the dimensionality of the state space, increasing the computational efficiency of the simulation process, defining optimization problems of polynomial complexity, and so



forth. Because HPN are a composition of discrete and continuous PN, their structure is a composition of two parts. Its places, transitions and oriented arcs consist of two groups - discrete and continuous. Moreover, because discrete and continuous parts of HPN cooperate each other, beside the arcs among discrete places and discrete transitions and the arcs among continuous places and continuous transitions there exist *cross* arcs. Namely, there are the arcs among discrete places and continuous transitions and the arcs among the continuous places and discrete transitions. HPN dynamics has its peculiarities too. Namely, the discrete places and transitions handle discrete tokens while the continuous places and transitions handle continuous variables being usually flow rates concerning different kinds of material flows (e.g. liquids, powdery materials, granules, rigid materials, etc.). Moreover, the mutual interaction between these groups is possible according to prescribed rules.

FOHPN were introduced by [1]. The set P of HPN places consists of two subsets $P = P_d \cup P_c$ where P_d is a set of discrete places and P_c is a set of continuous places. The discrete places are graphically represented by simple circles, while the continuous places are graphically represented by double concentric circles. Cardinalities of the sets are, respectively, n, n_d and n_c . Analogically, the set of transitions T consists of two subsets $T = T_d \cup T_c$, where T_d is a set of discrete transitions and T_c is a set of continuous transitions. The discrete transitions are graphically represented by simple rectangles, while the continuous places are graphically represented by double rectangles where a smaller rectangle is put inside of the bigger one. Their cardinalities are, respectively, q, q_d and q_c . Moreover, the set of discrete transitions T_d can contain two subsets - a subset of immediate transitions like those used in ordinary PN and/or a subset of timed transitions like those used in timed PN. The timed transitions express the behavior of discrete events in time. These transitions may be deterministic and/or stochastic. FOHPN have to be well-formed nets. Due to ensuring such qualitative properties of FOHPN the following rule has to be met: firing of continuous transitions must not influence marking of discrete places. FOHPN marking is a function assigning a non-negative integer number of tokens to each of the discrete places and an amount of fluid to each of the continuous places. An instantaneous firing speed (IFS) is assigned to each of the continuous transition t_i . It determines an amount of fluid per a time unit - i.e. a sort of the flow rate. The flow rate fires the continuous transition in a time instance τ . For all of the time instances τ holds $V_i^{min} \leq v_i(\tau) \leq V_i^{max}$, where min and max denote the minimal and maximal values of the speed $v_i(\tau)$. Consequently, IFS of any continuous transition is piecewise constant. An empty continuous place p_i is filled through its enabled input transition. In such a way the fluid can flow to the output transition of this place. The continuous transition t_i is enabled in the time τ if its input discrete places $p_k \in P_d$ have marking $m_k(\tau)$ at least equal to $Pre(p_k, t_i)$ and all of its input continuous places $p_i \in P_c$ satisfies the condition that either $m_i(\tau) \ge 0$ or the place p_i is filled. Pre(.) is the element of the incidence matrix Pre. The incidence matrices Pre and Post are well known in the PN theory - see e.g. [14]. For instance, in the P/T PN model of the discrete-event system (2) the incidence matrix **F** corresponds to **Pre** and the incidence matrix \mathbf{G}^T corresponds to **Post**. If all of the input continuous places of the transition t_i have non-zero marking then t_i is strongly enabled,



otherwise t_j is weakly enabled. The continuous transition t_j is disabled if some of its input places is not filled.

In general, the marking development of the continuous place $p_i \in P_c$ in time can be described by the differential equation

$$\frac{dm_i}{d\tau} = \sum_{t_j \in T_c} C(p_i, t_j) . v_j(\tau)$$
(3)

where $v_j(\tau)$ are entries of the IFS vector $\mathbf{v}(\tau) = (v_j(\tau), \dots, v_{q_c}(\tau))^T$ in the time τ and \mathbf{C} is the incidence matrix of the continuous part of FOHPN. Here, the matrix $\mathbf{C} = \mathbf{Post} - \mathbf{Pre}$. The differential equation holds provided that no discrete transition is fired in the time τ and all of the speeds $v_j(\tau)$ are continuous in the time τ . The IFS $v_j(\tau), j = 1, \dots, n_c$, defines enabling the continuous transition t_j . If t_j is strongly enabled then it can be fired with an arbitrary firing speed $v_j(\tau) \in [V_j^{min}, V_j^{max}]$. If t_j is weakly enabled then it can be fired with an arbitrary firing speed $v_j(\tau) \in [V_j^{min}, V_j]$, where $V_j \leq V_j^{max}$. Namely, t_j cannot take more fluid from any empty input continuous place than the amount entering the place from other transitions. It corresponds to the principle of conservation of matter.

The P/T PN are used here in the process of synthesizing the hybrid agent cooperation. On that way the earlier results concerning the agents cooperation based on supervision presented in [2, 3, 4, 5, 6] can be utilized as well as the theory of supervision [11, 12, 13]. However, the agent negotiation based on P/T PN [7] seems to be more sophisticated.

2. Modeling the real manufacturing system

Consider the manufacturing system consisting of six production lines. It is the recycling manufacturing system. Four lines represent the drawing lines producing the bales of the plastic double foil of different thickness and width from the granulate prepared from the waste plastics. The bales have a prescribed weight. The plastic foil is used for producing plastic bags. Other two lines represent the rolling lines which process the foil in such a way that they produce the belt of bags and roll them into rolls with a prescribed number of bags of different length. From the point of view of technology the first group of lines is more complex. Namely, each line contain the principle device - exhauster - in which the granules are fused into a soft mass and then a big bubble (balloon) of foil is exhausted. Then the bubble is drawn into the form of double foil. To model the exhauster by means of classical methods is practically impossible. Therefore the FOHPN-based modelling is used.

2.1. The rough FOHPN-based model of the recycling line

Consider the recycling line producing the plastic double foil from the granulate prepared from the waste plastic. The rough FOHPN model is displayed in Fig. 2. To distinguish continuous and discrete places as well as the continuous and discrete transitions, the continuous items are denoted by capitals.





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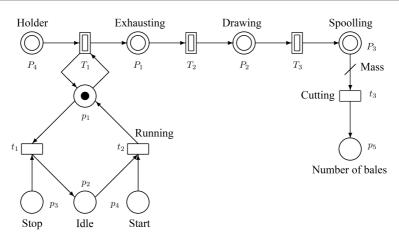


Figure 2. The rough FOHPN-based model of the production line.

The granulate is collocated in the Holder represented by the continuous place P_4 . Thence, the fluid flows through the continuous transition T_1 to the Exhausting Machine (Exhauster) represented by the place P_1 where a big bubble is blow (in order to make producing the double foil possible). Subsequently, the double foil is drawing into the prescribed width and thickness on the Drawing Line P_2 and proceeds to the Spooling Machine P_3 where the bales of a prescribed Mass are prepared. Here, after achieving the prescribe Mass, the foil is aborted by cutting, the completed bale is withdrawn and the new bale starts to be spooled on a new spool. The completed bale proceeds to another production line where the bags are performed, and the belt of bags is rolled into rolls with a uniform number of bags in each roll. The marking of the discrete place p_5 expresses the number of bales produced by the line.

2.2. The detailed FOHPN-based model of the recycling line

In the real machines time delays occur. The same is valid for the Exhausting Machine as well as for the Drawing Line. Namely, a time is necessary for producing the suitable bubble in the Exhausting Machine as well as there exist a transport delay of the Drawing Line (while the drawn foil achieve the spooling device). Consequently, it is necessary to build these time delays into the FOHPN model. Hence, we obtain the model given in Fig. 3 which is more detailed than that in Fig. 2. Here, the $\{p_6 - p_9, t_4, t_5\}$, $\{p_{10} - p_{13}, t_6, t_7\}$ model, respectively, the delays of exhausting and drawing processes. While $t_4 - t_7$ are timed transitions modeling delays, p_8 , p_9 , p_{12} , p_{13} appropriately affect their firing. Moreover, it is necessary to ensure regular supplying the granulate in order to avoid breakdown of the line caused by the lack of the granulate. This is done by means of the feedback $T_1 \rightarrow P_5 \rightarrow t_9 \rightarrow p_{15} \rightarrow t_8$. Here, M_{fb} denotes the multiplicity of the arc due to added amount of the granulate in one batch. The marking N_g of the discrete



F. ČAPKOVIČ

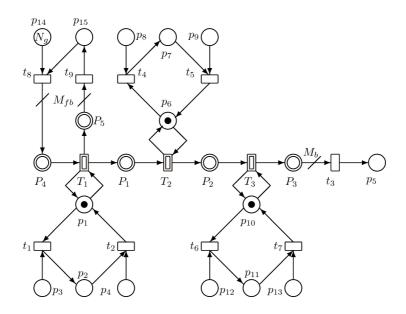


Figure 3. The FOHPN-based model of the production line with built time delays.

place p_{14} represents the number of the added batches of the granulate. In addition, M_b represents here the multiplicity of the arc corresponding to the prescribed mass of the bale.

2.2.1. The model parameters

The structure of the FOHPN model can be described by the following incidence matrices. There are four sorts of the structure: (i) the structure corresponding to the continuous part of the hybrid system, represented by the matrices \mathbf{Pre}_{cc} , \mathbf{Post}_{cc} ; (ii) the structure corresponding to the relation between the continuous and discrete parts of the hybrid system, represented by the matrices \mathbf{Pre}_{dc} , \mathbf{Post}_{dc} ; (iii) the structure corresponding to the relation between the discrete and continuous parts of the hybrid system, represented by the matrices \mathbf{Pre}_{cd} , \mathbf{Post}_{dc} ; (iv) the structure corresponding to the hybrid system, represented by the matrices \mathbf{Pre}_{cd} , \mathbf{Post}_{cd} ; (iv) the structure corresponding to the discrete part of the hybrid system, represented by the matrices \mathbf{Pre}_{dd} , \mathbf{Post}_{dd} . It is necessary to emphasize that the vertical dimensions of these matrices correspond to the incident transitions. The indices *cc*, *cd*, *dc*, *dd* denote, respectively, the incidences between places and transitions 'continuous-continuous', 'continuous-discrete', 'discrete-continuous', 'discrete-discrete'.



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F. ČAPKOVIČ

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The initial markings (initial state vectors) of the continuous and discrete part of the hybrid system are very important too. The initial marking of the continuous places is $\mathbf{M}_c = (0, 0, 0, M_{gr}, 0)^T$ where M_{gr} is the initial amount of the granulate in P_4 . The initial



marking of the discrete places is $\mathbf{m}_d = (1, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 1, 1, N_g, 0)^T$ where N_g is the number of the batches of the granulate to be added during the production.

Firing speeds of the continuous transitions $v_j(\tau)$, j = 1, 2, 3 are, respectively, from the intervals $[V_j^{min}, V_j^{max}]$ with the limits of intervals being $V_j^{min} = 0$, j = 1, 2, 3, $V_1^{max} = 1.8$, $V_2^{max} = 1.5$ and $V_3^{max} = 1.4$.

The discrete transitions are considered to be deterministic. Most of them is without any delay or with a transport delays mentioned above. However, in order to express the delays the discrete transitions $t_4 - t_7$ contain some delays. Hence, the delays of the discrete transitions are being the entries of the vector $(0, 0, 0, 0.01, 125, 0.01, 300, 0, 0)^T$. The other parameters of the line are the structural parameters $M_b = 270$, $M_{fb} = 3750$, the initial markings with $M_{gr} = 5000$ and $N_g = 4$. Here, M_b represents the prescribed weight of the bale of the drawn foil spooled at the end of the drawing process, M_{fb} represents the batch of the granulate added during the process in order to avoid the situation with empty Holder, and M_{gr} is initial content of granulate in the Holder, and N_g is number of added batches of granulate. Using the Matlab simulation tool HYPENS [17] we obtain the behavior of the simulated line. It is illustrated in Fig. 4 and Fig. 5. In Fig. 4 the course of the material flows is shown during the time segment when the granules are added four times.

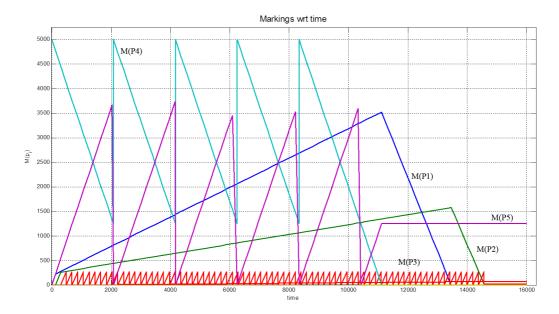


Figure 4. The dynamics behavior of the line material flows.



F. ČAPKOVIČ

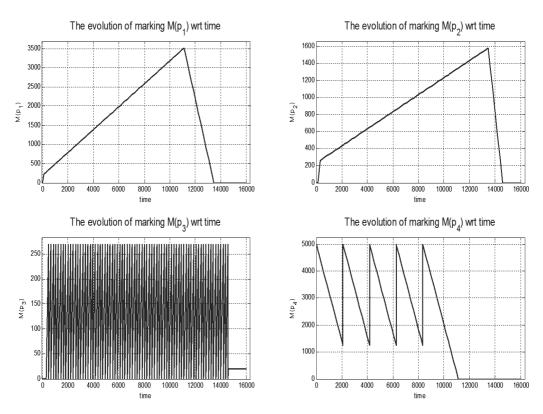


Figure 5. The dynamics behavior of the material flows of the production line in their individual scales - i.e. the marking evolution of the continuous places $P_1 - P_4$ in time.

2.3. Description of the simulation results

The results of simulation of the production line are displayed in Fig. 4 and Fig. 5. They illustrate the dynamic development of marking $M(P_i)$ of the continuous places P_i , i = 1..., 5. In other words, they represent the courses of the material flows throughout these places. $M(P_5)$ models a feedback flow. A batch of granules is poured into the Holder in order to avoid its emptying. Namely, when the feed of the Exhauster by granules is aborted, the Exhauster cannot work and restoring its function can take a time. The temperature of the hot mass of plastics inside the Exhauster has to be kept in order to avoid stiffening the mass. The power costs for this escalate the worse and lower the effectiveness of the production. Of course, the drawing of the foil will be aborted too and the bale of the foil will have insufficient mass. $M(P_4)$ represents the flow of granules throughout the Holder. $M(P_1)$ is the flow of the exhausted bubble from the Exhauster, $M(P_2)$ expresses the flow of the drawing foil and $M(P_3)$ represents spooling the particular bales of the foil. The finished bale is immediately removed in order to spool the next



bale. When the flow of granules is purposefully stopped, the other flows are gradually finished too.

3. Cooperation of hybrid agents

In the previous section the hybrid agent and its function were described in details. Having a group of hybrid agents the cooperation has to be synthesized. Let us examine how to do. In substance, there are to principle possibilities based on P/T PN: (i) the agent cooperation by means of DES supervision methods; (ii) the agent negotiation by means of DES supervision methods.

As to the agent supervision, there are several approaches. One of them was presented in [3]- [6], namely, how to realize the agent cooperation based on P/T PN P-invariants. In general, consider N lines for the foil production and $M \le N$ lines for producing the bags rolls. These lines are hybrid and they can be understood to be hybrid autonomous agents. The lines producing the foil can work independently, even simultaneously in time. The same is valid for the lines producing the rolls of bags. However, the problem is that N lines producing the foil have to share only M rolling lines. To solve this problem a strategy has to be defined for the sharing. When such a strategy is defined the conditions for the supervisor synthesis can be formulated and the corresponding supervisor can be synthesized. The simplest form of the agents cooperation is in case when there exists a buffer at any line producing the foil. The lines can produce foil and store the bales into their buffers. The idle rolling lines can take the bales of foil from the buffers as they want and need. A form of scheduling seems to be a more sophisticated strategy which makes an optimizing possible - e.g. minimizing the idle time. However, it depends also on actual needs of foil parameters (especially on its width and thickness).

However, the negotiation based on the offers and demands seems to be the most sophisticated strategy. The P/T PN-based modeling can be usable also on that way - see [7]. Let us introduce the example of the negotiation process of two agents.

3.1. The negotiation of two agents

The example of negotiation of a couple of agents A_1 , A_2 can be described as follows. The structure of the P/T PN model of the Agent 1 displayed in Fig. 6 (on its left side) is created by the places $p_1 - p_3$ and transitions $t_1 - t_4$, while the structure of the P/T PN model of the Agent 2 (on the right side) is created by the places $p_4 - p_6$ and transitions $t_5 - t_8$. The P/T PN model of the communication channel for realizing the negotiation represents the Interface being located between the agents. It consists of $p_7 - p_8$ and $t_9 - t_{12}$. The interpretation of P/T PN places is the following: $p_1 - A_1$ does not want to communicate; $p_2 - A_1$ is available; $p_3 - A_1$ wants to communicate; $p_7 - communication;$ and p_8 - availability of the communication by means of the interface (a communication channel). It is necessary to emphasize the interpretation of the transitions being the part



F. ČAPKOVIČ

of the Interface, namely: t_9 - fires the communication when A_1 is available and A_2 wants to communicate with A_1 ; t_{10} - fires the communication when A_2 is available and A_1 wants to communicate with A_2 , and t_{12} - fires the communication when both A1 and A2 wants to communicate each other.

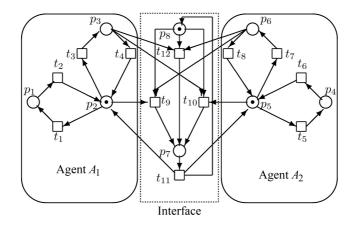


Figure 6. The P/T PN model of the negotiation process.

As it is clear from Fig. 6, both agents and the interface are modules of P/T PN (PN subnets). Thus, the agent negotiation can be modelled in analytical term (2). The P/T PN based model of the agents communication has the following parameters



$\mathbf{G}^{T} = \begin{pmatrix} \mathbf{G}_{A_{1}}^{T} & 0 & \mathbf{G}_{A_{1}-A_{2}}^{T} \\ 0 & \mathbf{G}_{A_{2}}^{T} & \mathbf{G}_{A_{2}-A_{1}}^{T} \\ 0 & 0 & \mathbf{G}_{neg}^{T} \end{pmatrix} =$	$\begin{pmatrix} 1 \end{pmatrix}$	0	0	0	0	0	0	0	0	0	0	0 \	١
	0	1	0	1	0	0	0	0	0	0	1	0	
	0	0	1	0	0	0	0	0	0	0	0	0	
$\left(\mathbf{C}^{T} 0 \mathbf{C}^{T} \right)$	-	_	_	_	-	_	_	—	-	—	_	_	
$\mathbf{C}^{T} = \left(\begin{array}{ccc} \mathbf{G}_{A_{1}} & 0 & \mathbf{G}_{A_{1}-A_{2}} \\ 0 & \mathbf{C}^{T} & \mathbf{C}^{T} \end{array} \right)$	_ 0	0	0	0	1	0	0	0	0	0	0	0	
$\mathbf{G} = \begin{bmatrix} \mathbf{G} & \mathbf{G}_{A_2} & \mathbf{G}_{A_2-A_1} \\ \mathbf{G} & \mathbf{G} & \mathbf{G}^T \end{bmatrix}$	0	0	0	0	0	1	0	1	0	0	1	0	
$\langle 0 0 \mathbf{G}_{neg}^{r} \rangle$	0	0	0	0	0	0	1	0	0	0	0	0	
	-	_	—	_	-	_	_	—	-	—	_	_	
	0	0	0	0	0	0	0	0	1 0	1	0	1	
	(0	0	0	0	0	0	0	0	0	0	1	0 /	J

To apply the model with these parameters it is necessary to choose an initial state. Of course, modeling of more cooperating agents in such a way is possible too.

However, it is necessary to say that there is no time for a circuitous negotiation process in real industrial processes, especially on the level of production lines. The negotiation is more suitable on a management or business level of an enterprise or factory.

3.2. Cooperation of hybrid production lines by supervision

The supervision seems to be a suitable tool for cooperation of the production lines. Consider the six production lines in a factory recycling the collected waste plastic mentioned above in the section 3. They are schematically displayed in Fig. 7. Four upper lines produce the plastic double foil from the granulate prepared from the waste plastic. The FOHPN model of such a production line was presented in section 2. Here, only the cooperation of the lines will be discussed. Two lower lines produce rolls of plastic bags from the double foil. In Fig. 7 only a rough schema of the cooperation of two groups of lines is displayed. The interpretation of the PN place is as follows:

- $\{p_1, p_4, p_7, p_{10}\}$ represent the continuous production of the foil
- $\{p_2, p_5, p_8, p_{11}\}$ represent the cutting a bales of the foil with a determined weight and delivering the bale to the buffers $\{p_3, p_6, p_9, p_{12}\}$
- ${p_{13}, p_{16}}$ represent the continuous rolling lines processing the double foil into the form of the belt of bags
- $\{p_{14}, p_{17}\}$ represent the rolling the belt into rolls of a prescribed length (pres-

cribed number of bags) and

 $\{p_{15}, p_{18}\}$ represent buffers of the rolls.

However, such a structure of the production lines is not sufficient because it is not clear how the lines will cooperate each other. Especially, the sharing four rolling lines by the six drawing lines has to be specified. Methods of supervision have to be used. The supervisor synthesized by means of such a method and/or by their combination will help



on this way. It will be synthesized in the following section and displayed in the following figure.

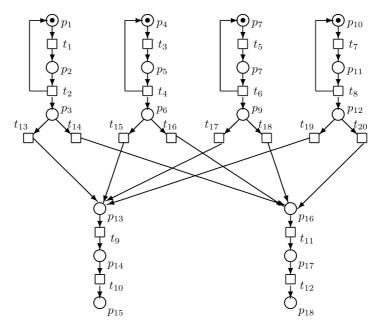


Figure 7. The Petri net-based model of the rough conception of the supposed cooperation of the production lines.

4. DES supervision and the agent cooperation

Supervision is very important in DES and hybrid systems, because the classical control methods utilized in continuous systems and discrete-time systems cannot be used. A supervisor for DES subsystems can be suitable in general for avoiding the egoistic effort of autonomous agents, especially in case of limited sources (working space, raw materials or semiproducts, energy, etc.). Namely, by means of prohibition some states of the global system describing the multi-agent system (MAS), e.g. like the so called mutex (<u>mutual exclusion</u>), a useless 'haggle' of agents each other for a priority can be removed on behalf of the global system purposes. However, on the contrary, the supervision process can be understood to be a carrier of the cooperation in the sense of the global system politics. In such a way the conditions for the supervisor synthesis represent the desired cooperation of agents in a group of agents or in MAS. In both cases some constraints has to be satisfied in order to achieve the desired behavior (i.e. to synthesize the supervisor). Here, two kinds of constrains known from supervising methodology in DES control theory will be considered. There are several approaches to the DES supervisor synthesis. However, we will used here two of them, namely





- 1. based on P/T PN place invariants (called as P-invariants),
- 2. more general one based on usage of state vector, control vector and Parikh's vector.

4.1. Supervision based on P-invariants

The principle of the method is based on the PN P-invariants [14, 11, 12] being the vectors, **v**, with the property that multiplication of these vectors with any state vector $\mathbf{x}_k \in X_{reach}$ (i.e. reachable from a given initial state vector $\mathbf{x}_0 \in X_{reach}$) yields the same result (the relation of the state conservation):

$$\mathbf{v}^T \cdot \mathbf{x}_k = \mathbf{v}^T \cdot \mathbf{x}_0 \tag{4}$$

Because of (2)

$$\mathbf{v}^T \cdot \mathbf{x}_k = \mathbf{v}^T \cdot \mathbf{x}_0 + \mathbf{v}^T \cdot \mathbf{B} \cdot \mathbf{u}_{k-1}$$
(5)

Hence, to satisfy the previous definition of P-invariants, the condition

$$\mathbf{v}^T \cdot \mathbf{B} = \mathbf{0} \tag{6}$$

has to be met because the occurrence of discrete events (represented by the vectors \mathbf{u}_k) cannot be affected. P-invariants are useful in checking the property of mutual exclusion. To eliminate a selfish behaviour of autonomous agents at exploitation of limited joint resources it is necessary to allocate the sources to individual agents rightly, with respect to the global goal of MAS. Such a constraint of the agents behaviour and violation of their autonomy is rather in favour of MAS than in disfavour. In case of the existence of several (e.g. n_x) invariants in a PN, the set of the P-invariants is created by the columns of the $(n \times n_x)$ -dimensional matrix V being the solution of the homogeneous system of equations

$$\mathbf{V}^T \cdot \mathbf{B} = \boldsymbol{\emptyset} \tag{7}$$

This equation represents the base for the supervisor synthesis method. Some additional PN places (slacks) can be added to the PN-model in question. The slacks create the places of the supervisor. Therefore, the relation (7) can be rewritten into the form

$$[\mathbf{L},\mathbf{I}_{s}].\begin{bmatrix}\mathbf{B}\\\mathbf{B}_{s}\end{bmatrix} = \emptyset$$
(8)

Hence,

$$\mathbf{L}.\mathbf{B} + \mathbf{B}_s = \mathbf{\emptyset}; \quad \mathbf{B}_s = -\mathbf{L}.\mathbf{B} \tag{9}$$

where:

- \mathbf{I}_s is $(n_s \times n_s)$ -dimensional identity matrix with $n_s \leq n_x$ being the number of slacks
- L is $(n_s \times n)$ -dimensional matrix of integers which represents (in a suitable form) the cooperation conditions L.x $\leq b$

 $L.x \leq b$ is the form of the cooperation conditions with b being the vector of integers



F. ČAPKOVIČ

representing the restrictions imposed on marking of the original PN $\mathbf{B}_s = \mathbf{G}_s^T - \mathbf{F}_s$ is $(n_s \times m)$ -dimensional matrix which yields (after its finding by computing) the structure of the PN-based model of the supervisor.

Thus, the supervisor structure respects the actual structure of the matrix \mathbf{L} respecting the restrictions \mathbf{b} . The supervised system (the group of autonomous agents augmented for the supervisor) is characterized by the augmented state vector and the augmented structural matrices as follows

$$\mathbf{x}_{a} = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_{s} \end{bmatrix}; \ \mathbf{F}_{a} = \begin{pmatrix} \mathbf{F} \\ \mathbf{F}_{s} \end{pmatrix}; \ \mathbf{G}_{a}^{T} = \begin{pmatrix} \mathbf{G}^{T} \\ \mathbf{G}_{s}^{T} \end{pmatrix}$$
(10)

where \mathbf{F}_s and \mathbf{G}_s^T represent the supervisor structure. They correspond to the interconnections of the supervisor and autonomous agents.

Analogically, the initial state of the supervisor ${}^{s}\mathbf{x}_{0}$ can be computed as

$$[\mathbf{L} | \mathbf{I}_s] \cdot \begin{bmatrix} \mathbf{x}_0 \\ {}^s \mathbf{x}_0 \end{bmatrix} = \mathbf{b} ; \quad {}^s \mathbf{x}_0 = \mathbf{b} - \mathbf{L} \cdot \mathbf{x}_0$$
(11)

where **b** is the column vector of the corresponding dimensionality (i.e. $(n_s \times 1)$) with integer entries representing the limits for number of common tokens - i.e. the maximum numbers of tokens that the corresponding places can possess altogether (i.e. to share them).

4.2. More general supervision

Of course, the previous approach to supervision based on P-invariants does not cover all cases occurring in practice. To widen a class of cooperation fashions the more general approach has to be used. One of possibilities how to do this is to impose the conditions also on PN transitions. On this way also the Parikh's vector is very important and useful.

First of all the Parikh's vector has to be defined. Namely, developing (2)

$$\mathbf{x}_1 = \mathbf{x}_0 + \mathbf{B} \cdot \mathbf{u}_0 \tag{12}$$

$$\mathbf{x}_2 = \mathbf{x}_1 + \mathbf{B} \cdot \mathbf{u}_1 = \mathbf{x}_0 + \mathbf{B} \cdot (\mathbf{u}_0 + \mathbf{u}_1)$$
 (13)

$$\mathbf{x}_{k} = \mathbf{x}_{0} + \mathbf{B}.(\mathbf{u}_{0} + \mathbf{u}_{1} + \dots + \mathbf{u}_{k-1}) = \mathbf{x}_{0} + \mathbf{B}.\mathbf{v}$$
(14)

where $\mathbf{v} = \sum_{i=0}^{k-1} \mathbf{u}_i$ is named to be the Parikh's vector. It gives us information about how many times the particular transitions are fired during the development of the system dynamics from the initial state \mathbf{x}_0 to the state \mathbf{x}_k .

Because not only the behaviour of agents but also the cooperation of agents exhibit attributes of DES we can utilize PN model for cooperating modules too.



The general linear constraints for supervisor synthesis were simply described in [13] as follows

$$\mathbf{L}_{p}.\mathbf{x} + \mathbf{L}_{t}.\mathbf{u} + \mathbf{L}_{v}.\mathbf{v} \leqslant \mathbf{b} \tag{15}$$

where \mathbf{L}_p , \mathbf{L}_t , \mathbf{L}_v are, respectively, $(n_s \times n) -, (n_s \times m) -, (n_s \times m) - \text{dimensional matrices}$. When $\mathbf{b} - \mathbf{L}_p \cdot \mathbf{x} \ge \mathbf{0}$ is valid - see e.g. [13] - the supervisor with the following structure and initial state

$$\mathbf{F}_{s} = \max(\mathbf{0}, \mathbf{L}_{p}, \mathbf{B} + \mathbf{L}_{v}, \mathbf{L}_{t}); \ \mathbf{L}_{pv} = \mathbf{L}_{p}, \mathbf{B} + \mathbf{L}_{v}$$
(16)

$$\mathbf{G}_{s}^{T} = \max(\mathbf{0}, \mathbf{L}_{t} - \max(\mathbf{0}, \mathbf{L}_{pv})) - \min(\mathbf{0}, \mathbf{L}_{pv})$$
(17)

$${}^{s}\mathbf{x}_{0} = \mathbf{b} - \mathbf{L}_{p} \cdot \mathbf{x}_{0} - \mathbf{L}_{v} \cdot \mathbf{v}_{0}$$
⁽¹⁸⁾

guarantees that constraints are verified for the states resulting from the initial state. Here, the max(.) is the maximum operator for matrices. However, the maximum is taken element by element. Namely, in general, for the matrices **X**, **Y**, **Z** of the same dimensionality $(n \times m)$, the relation $\mathbf{Z} = \max(\mathbf{X}, \mathbf{Y})$ means that $z_{ij} = \max(x_{ij}, y_{ij})$, i = 1, ..., n, j = 1, ..., m. The same holds for min(.).

4.3. Solving the problem of the hybrid agents

At forming the rules defining the mutual cooperation of the production lines mentioned above we have to respect the facts as follows:

- (i) any bale of the foil from output buffers of the four foil production lines can enter only one of the two rolling machines;
- (ii) only one bale can enter any rolling machine;
- (iii) next bale can enter the rolling machines after finishing the rolling process

While (i), (ii) mean that the transition functions of the PN transitions $t_{13} - t_{20}$ (being the components of the control vector \mathbf{u}_k in any step k - see the description of (1)) have to satisfy the following restrictions

$$\gamma_{t_{13}} + \gamma_{t_{15}} + \gamma_{t_{17}} + \gamma_{t_{19}} \leqslant 1 \tag{19}$$

$$\gamma_{t_{14}} + \gamma_{t_{16}} + \gamma_{t_{18}} + \gamma_{t_{20}} \leqslant 1 \tag{20}$$

It means that in (15) only the matrix \mathbf{L}_t and \mathbf{b} will occur in the following forms



(iii) means that the places $p_{13} - p_{16}$ has to meet the restrictions as follows

$$\sigma_{p_{13}} + \sigma_{p_{14}} \leqslant 1 \tag{21}$$

$$\sigma_{p_{16}} + \sigma_{p_{17}} \leqslant 1 \tag{22}$$

Hence, in (15) only the matrix \mathbf{L}_p and \mathbf{b} will occur in the following forms

The conditions can be satisfied by means of the supervision theory for DES. Hence, the supervisor with the structure

and the initial state

$${}^{s}\mathbf{x}_{0} = \begin{pmatrix} 1\\ 1 \end{pmatrix}$$

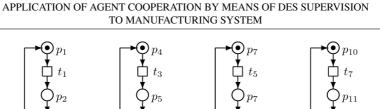
can be found. Then, the PN model of cooperating lines is given in Fig. 8, where the PN places p_{19} , p_{20} together with the corresponding oriented arcs (i.e. structural interconnections represented by the incidence matrices \mathbf{F}_s , \mathbf{G}_s^T) realize the supervisor.

5. Conclusion

The paper points out the possibility how the cooperation of agents can be synthesized by means of methods of the DES supervision. P/T PN were utilized here in order to model agents (DES subsystems) while the individual hybrid agents (containing a mix of continuous material flows and discrete operations) were modeled by means of the FOHPN. The cooperation of the agents was modeled by means of P/T PN. The instance of the particular practical application was introduced. The lines producing the plastic double foil from the granules prepared from waste plastics and the rolling lines producing roles of plastic bags were taken into account. First of all the dynamic behavior of the production line was tested by means of simulation in Matlab. Thereby, the satisfying applicable results were obtained. Finally, the hybrid agents cooperation by means of DES supervision is introduced in order to underline soundness of the approach. w czasonisma nan n







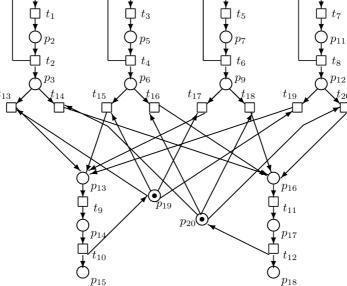


Figure 8. The Petri net-based model of the supervised cooperation of the production lines.

In spite of the fact that the specific kind of manufacturing systems was investigated in this paper, the P/T PN modeling DES and FOHPN modeling hybrid systems can be applied on arbitrary complex system having such a character. The area of applying P/T PN modeling and supervisory control covers especially flexible manufacturing system (e.g. including robots, numerically controlled machine tools, etc.), some kinds of transport systems (including a nets of automatically guided vehicles inside of manufacturing systems) and even some kinds of communication systems (applicable also inside of manufacturing systems). The P/T PN-based negotiation can be applied on the management or business level of an enterprise or factory, in the e-business, e-commerce and in enegotiation in general. The area of applying FOHPN modeling covers complex systems having character of hybrid system where some devices cannot be sufficiently identified in order to be described by a mathematical model - e.g. filling and packing production lines, etc. Of course, the supervision of such systems can also be performed by means of the methods described in this paper.

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