

# Use of the finite element method for parameter estimation of the circuit model of a high power synchronous generator

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**Abstract.** The paper presents the two-dimensional, field-circuit model of a high power synchronous generator verified by measurements. The model enables determining the waveforms of electromagnetic quantities in steady and transient states. Verification of the model was based on comparison of the measured and calculated waveforms after a disturbance in the voltage regulation system of a TWW-200-2 generator operating in Połaniec Power Plant. There are also presented the field methods for determining electromagnetic parameters (synchronous reactances and time constants) when using the distributions of static and quasi-static, magnetic and electromagnetic fields calculated by the finite element method (FEM). The set of these parameters was used as the starting parameters of the optimization algorithm for estimation of electromagnetic parameters of the synchronous generator circuit model. The dynamic waveforms under the generator load conditions calculated by the finite element method are the basis of parameter estimation. The parameter estimation of the generator model was performed with the use of the least squares method.

**Key words:** synchronous generator models, finite element method, field-circuit model measurement verification, circuit model parameter estimation.

## 1. Introduction

In simulation investigations of the Polish Power System (PPS) there is a database of mathematical model parameters of generating units most often used. The values of these parameters are determined based on the catalog and design data delivered by manufacturers of component elements of generating units or are estimated on the basis of typical data published in scientific-technical elaborations. The parameter values of the generating unit mathematical models determined in such a way are approximate, loaded with a large error in many cases and they do not represent the real values of the parameters of operating units. The parameter values given by manufacturers do not take into account the actual operating conditions of generating units, including changes of their properties caused by long exploitation, repairs and modernization. As a consequence, the results of PPS simulation investigations which are the basis for planning the system development are uncertain.

Nowadays it is thought that the field-circuit modeling of synchronous generators is one of the most accurate calculation methods, since it makes it possible to take into account essential electromagnetic and electromechanical phenomena deciding on the machine properties, such as: nonlinearity of magnetizing characteristics of magnetic cores, influence of eddy currents in the conductive elements of the rotor, movement of the rotor [1, 2]. The main factors limiting the use of field-circuit models for simulations of power systems are a long computation time and the need to use computers of high computing power. However, such models are more and more often used for determining the parameters of synchronous machine circuit models. The field-circuit calculations can be carried out at the stage of machine design, they can also

simulate measurement tests which are sometimes very difficult to be realized under the generator operating conditions. In this paper, there is presented a two-dimensional, field-circuit model of a high power synchronous generator enabling determining the waveforms of electromagnetic quantities in steady and transient states. Verification of the model was based on comparison of the measured and calculated waveforms after a disturbance in the voltage regulation system of a TWW-200-2 generator operating in Połaniec Power Plant. The verified field-circuit model was used, first of all, for determining a set of starting parameters of the optimization algorithm, and next for calculating the dynamic waveforms being the basis of estimation of electromagnetic parameters of the generator circuit model.

## 2. The field-circuit model of a synchronous generator

In calculations of the transient state there was used the two-dimensional, field-circuit model of a synchronous generator in which the Kirchhoff equations describing the electromagnetic state of individual windings are related with the equations describing the space-time distribution of the electromagnetic field. The latter equations result from the Maxwell equations.

The field-circuit calculations were made for a cylindrical high-power synchronous generator with the following ratings:  $S_n = 235.5$  MV·A,  $U_n = 15.75$  kV,  $I_n = 8625$  A,  $I_{fn} = 2680$  A,  $\cos \phi_n = 0.85$ .

In the calculation model there was assumed [3]:

- two-dimensional distribution of the electromagnetic field in the generator cross-section,

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- nonlinear magnetizing characteristics of the stator and rotor cores,
- constant rotational speed of the rotor,
- eddy currents induced in the solid block and slot wedges of the rotor,

while there was neglected:

- skin effect in the stator and rotor windings,
- eddy currents in the stator lamination.

Due to the same magnetic field distribution under each pair of poles, the machine cross-section including one pole pitch (Fig. 1) was assumed for calculations. There was assumed the zero Dirichlet boundary condition for the magnetic vector potential on the outer surface of the stator ( $\Gamma_1$ ). On the two other edges  $\Gamma_2$  and  $\Gamma_3$  separated by a pitch pole, there were assumed the conditions of potential aperiodicity. The considered cross-section of the generator computational model was discretized by means of a mesh consisting of 40684 triangular elements. In the calculations of the transient state there was assumed a constant step for integrating equations in the time domain equal to  $\Delta t = 0.0002$  s.

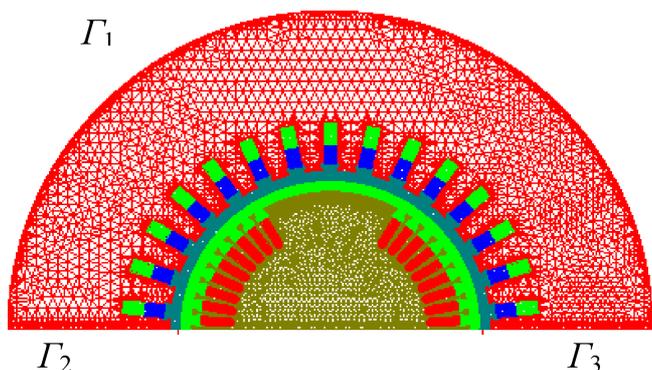


Fig. 1. Cross-section and finite element mesh of the TWW-200-2 generator

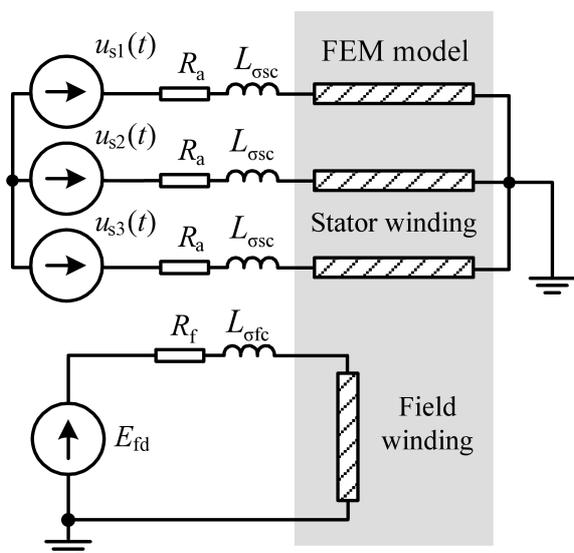


Fig. 2. External circuits connected to the turbogenerator field model

Figure 2 shows the circuit part of the generator computational model. It consists of external electric circuits connected to the windings modeled in the field part. These circuits contain supply sources as well as resistances and leakage inductances of the winding ends [4] which are not included in the two-dimensional field model.

### 3. Measurement verification of the synchronous generator field-circuit model

The investigated TWW-200-2 generator installed in Połaniec Power Plant is equipped with a static excitation system. The test was made under the generator no-load conditions at the constant rotational speed equal to  $n = 3000$  rot/min. A disturbance, in the form of a step change in the voltage regulator reference voltage (by  $\pm 10\%$ ), initiated the transient state during which the waveforms of the field and stator voltage of the generator were recorded. The field voltage  $E_{fd}$  (Fig. 3a) has non-negative values, which results from the operation of an excitation rectifier included in the generator voltage regulation system. Figure 3b shows the comparison of the calculated and measured waveforms of the generator stator voltage. From the comparison of these waveforms it follows that the developed field-circuit model of the TWW-200-2 synchronous generator represents the phenomena occurring in the machine at no-load in steady and transient state accurately enough.

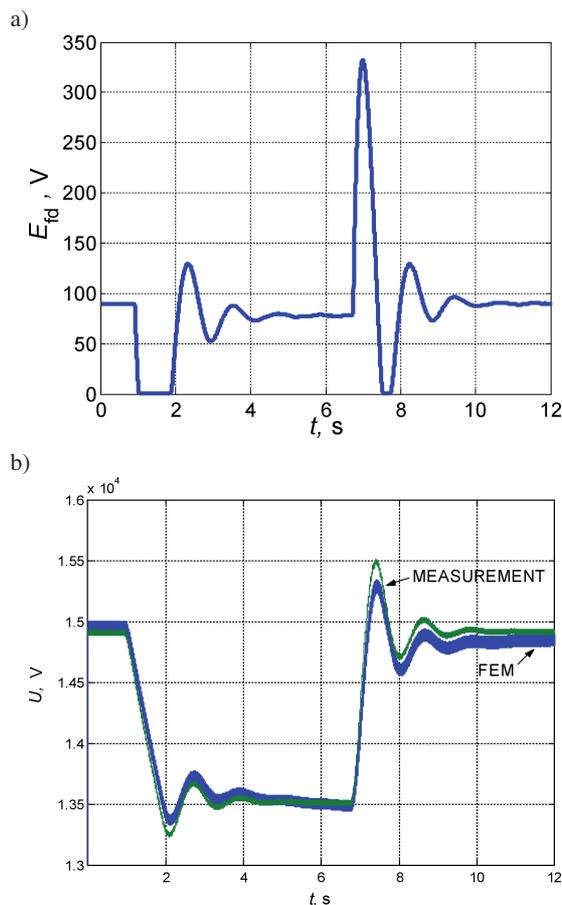


Fig. 3. The measured waveform of the field voltage (a) and the measured and calculated waveform of the generator stator voltage (b)

Certain discrepancies between the calculation results and the measured waveforms of the stator voltage can result from different values of the electrical conductivity of the solid block and slot wedges of the rotor in the real machine and the computational model.

#### 4. Machine parameters determined based on the distributions of magnetostatic fields

Magnetic fluxes linked with the stator and field winding are determined based on the calculated distribution of the magnetic potential. Figure 4 shows the exemplary distributions of the magnetic field lines in the d and q axis of the machine.

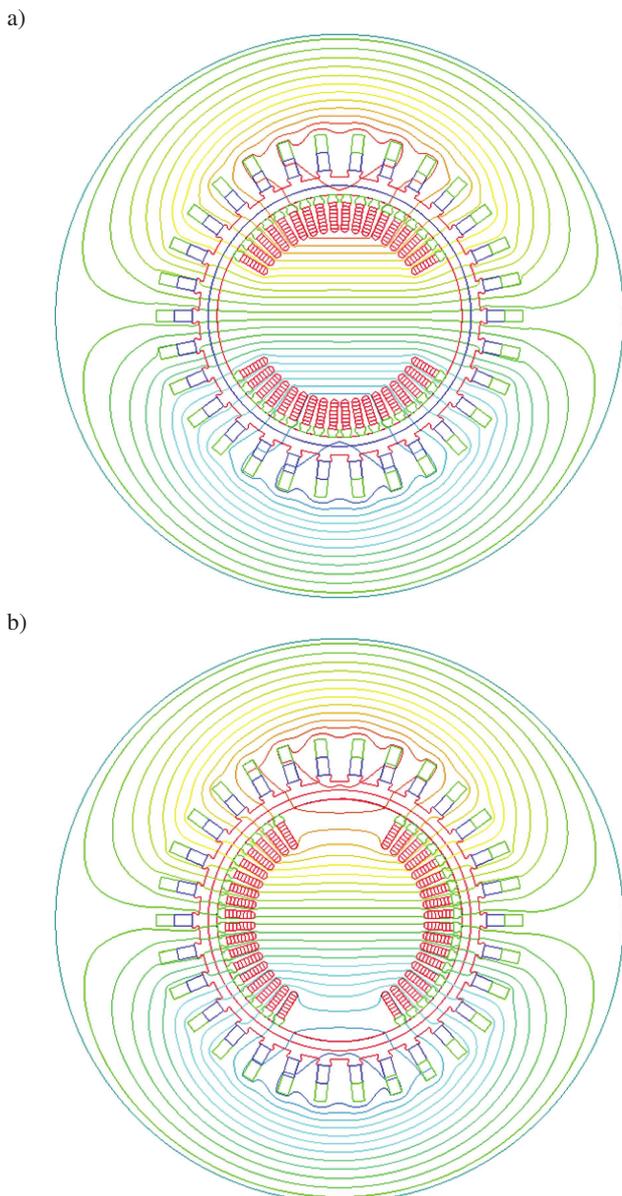


Fig. 4. Exemplary distributions of the magnetic field lines in d (a) and q (b) axis of the machine

In order to determine the synchronous reactance in the d axis ( $X_d$ ), one should force the current in the phase circuits

of the stator winding so that the magnetomotive force along the d axis is obtained. Based on the calculated current ( $I_d$ ) and flux linkage ( $\Psi_d$ ) in the d axis, there was determined the synchronous reactance:

$$X_d = \frac{\Psi_d \omega_r}{I_d}, \quad (1)$$

where  $\omega_r$  – relative pulsation of the generator stator voltage.

The synchronous reactance in the q axis was determined in an analogous way. Then the leakage reactances of the stator winding ends, which were not included in the two-dimensional model of the machine, were added to the calculated synchronous reactances in both axes.

In Fig. 5 there are depicted the characteristics of the synchronous reactances as a function of the currents in the d and q axis of the stator. The changes in the reactance values are caused by saturation of the machine magnetic cores.

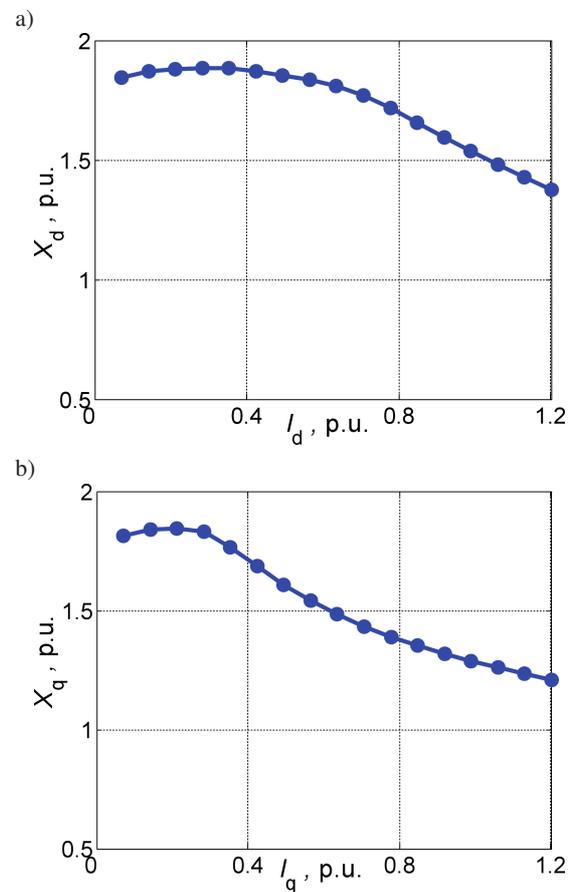


Fig. 5. Characteristics of the synchronous reactances in d (a) and q (b) axis of the machine

To determine the field winding reactance characteristic and no-load characteristic of the machine, one should calculate the flux linked with the field winding  $\Psi_f$  and the stator winding  $\Psi_{md}$  for selected values of the exciting current at the open stator winding.

Figure 6 shows the characteristic of the excitation reactance ( $X_f$ ) as a function of the exciting current when taking into account the reactance of the field winding ends as well

as the no-load characteristic. Based on these characteristics there was determined the magnetizing reactance in the d axis for the non-saturated circuit.

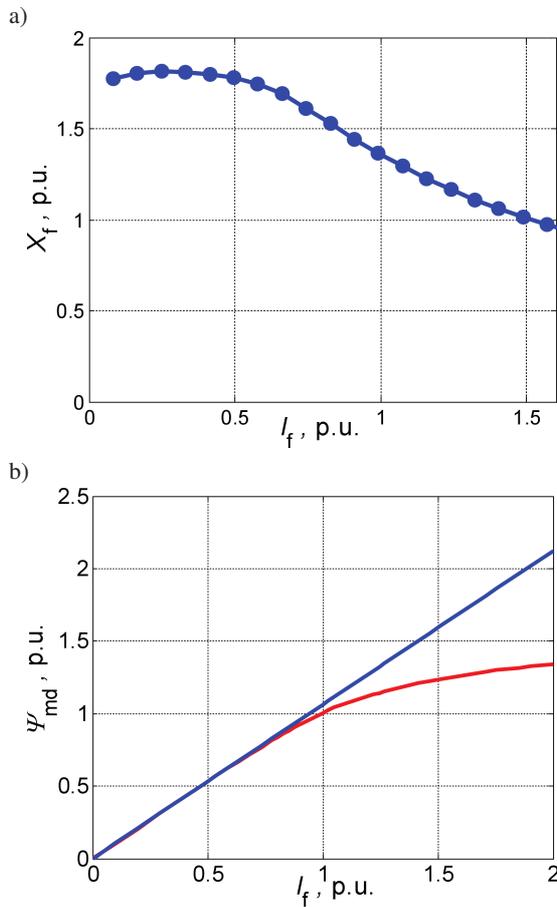


Fig. 6. Excitation reactance (a) and no-load (b) characteristics

In the electrical machine theory, there is applied the principle of division of the resultant magnetic field into the main field and leakage fields [5]. This principle, resulting from engineering intuition and practice, was used for determination of the leakage fluxes of the stator and rotor windings for the non-saturated magnetic circuit of the machine. Knowing the excitation reactance value – measured or calculated on the basis of generally known design dependencies [4, 6] – as well as the values of the calculated reactances, one can determine the transient parameters of the machine in the d axis.

## 5. Machine parameters determined based on the distributions of sinusoidally variable fields

On the basis of the distributions of the fields varying sinusoidally in time, there were determined the parameters of the damping circuits in the rotor as well as the subtransient reactances and time constants dependent on them. These reactances can be determined when supplying the stator winding with a sinusoidal current of frequency higher than the rated

one at the motionless rotor placed longitudinally or transversely in relation to the stator flow. The subtransient reactances were determined at the open excitation winding, based on the distribution of the electromagnetic field varying sinusoidally, when supplying the stator winding with a current of frequency equal to 500 Hz.

Based on the space-time distribution of the magnetic potential, there were calculated the magnetic fluxes linked with the stator phase windings at the open excitation winding. Figure 7 shows the exemplary distributions of the magnetic field lines in the d and q axis of the machine produced by the stator current for the frequency equal to 500 Hz. It can be seen that the magnetic flux is forced to pass through the air-gap by eddy currents induced in the rotor electric circuits. The characteristics of the subtransient reactances in the d and q axis ( $X''_{d0}$ ,  $X''_q$ ) at the open excitation winding are presented in Fig. 8.

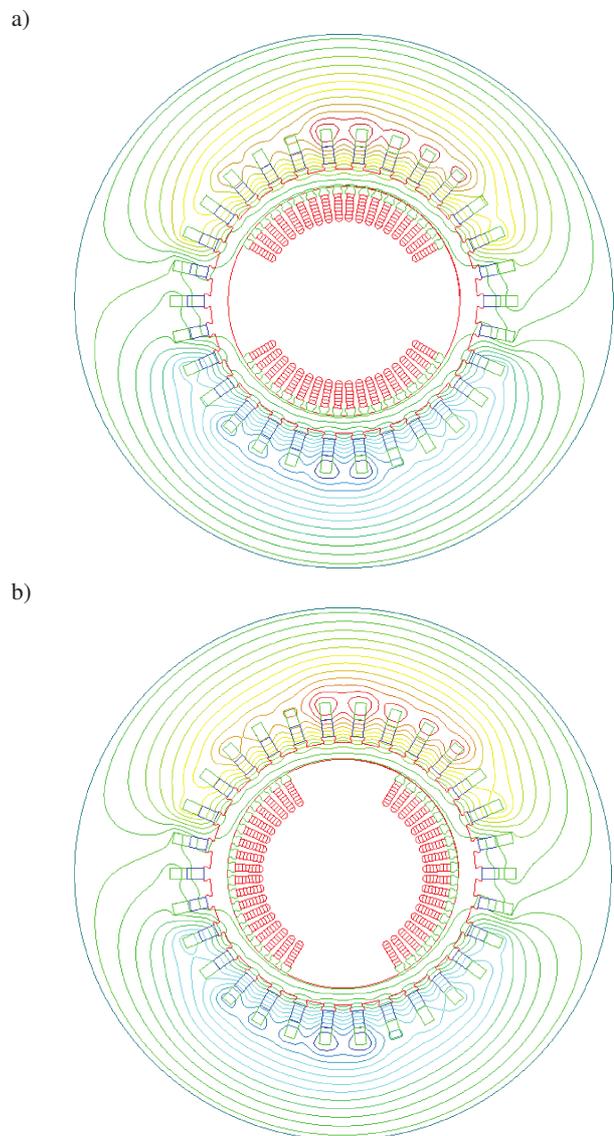


Fig. 7. Distribution of the magnetic field lines in d (a) and q (b) axis of the machine when supplying the stator with a current of frequency equal to 500 Hz

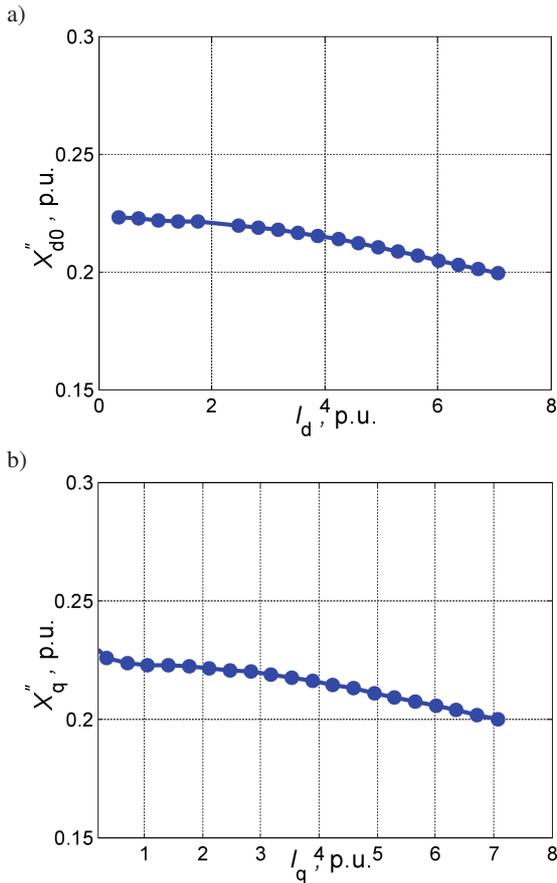


Fig. 8. Characteristic of the subtransient reactance in d axis at the open excitation winding (a) and the subtransient reactance in q axis (b)

Based on the calculated (by the finite element method) subtransient reactance  $X''_{d0}$  at the open excitation winding, there was determined the subtransient reactance  $X''_d$ .

The equivalent time constants in the d and q axis of the rotor damping circuits, which in the investigated generator are the solid block and slot wedges, were determined based on the distributions of the harmonic magnetic field at the open excitation winding (based on the active  $P_r$  and reactive  $Q_r$  power in the conducting parts of the rotor) from the relationship:

$$T_{D,Q} = \frac{X_{D,Q}^{\bullet}}{R_{D,Q}^{\bullet} \omega_r} = \frac{Q_{rD,Q}}{\omega P_{rD,Q}}. \quad (2)$$

The powers  $P_r$  and  $Q_r$  determine the complex power expressed by the complex Poynting vector in the harmonic field:

$$-\oint_s \underline{S} \cdot ds = P_r + jQ_r. \quad (3)$$

The active  $P_r$  and reactive  $Q_r$  powers were determined based on the distributions of the electromagnetic field in the d and q axis from the formulas:

$$P_r = \frac{1}{2} \int_V \frac{J_m^2}{\gamma} dV, \quad (4)$$

$$Q_r = 2j\omega(W_m - W_e) \approx 2j\omega W_m, \quad (5)$$

where  $J_m$  – current density amplitude,  $\gamma$  – electrical conductivity,  $W_m$ ,  $W_e$  – average values of the energy stored in the magnetic and electric field. In the case considered there was neglected the average value of the electric field energy  $W_e$ .

For a two-dimensional electromagnetic field, integrating the volumes and surfaces of the area is reduced to integrating the appropriate surfaces and contours.

Based on the time constants (2), one can determine the resistances of the equivalent damping circuits in the d and q axis referred to the stator side as well as the subtransient time constants:

$$R_{D,Q}^{\bullet} = \frac{X_{D,Q}^{\bullet}}{T_{D,Q} \omega_r}, \quad T'_{d0} = T_D \left( 1 - \frac{X_{md}^2}{X_f^{\bullet} X_D^{\bullet}} \right), \quad (6)$$

$$T''_{q0} = T_Q. \quad (7)$$

The stator current frequency, which in the machine field calculation model equals the frequency of the currents induced in the rotor and decides on the intensity of the skin effect, influences significantly the results of calculations of the subtransient time constants. The calculations were carried out for the frequency equal to 3 Hz for which the calculation results were close to the measurement results [7, 8].

Table 1 presents the calculation results of a TWW-200-2 synchronous generator parameters based on the spatial distributions of the magnetic and electromagnetic field determined by the finite element method for the non-saturated magnetic circuit of the machine. These parameters are given in the generator relative values.

Table 1  
The calculation results of the standard parameters of the TWW-200-2 synchronous generator

Parameter	$X_d$	$X_q$	$X'_d$	$X''_d$	$X''_q$	$X_\sigma$	$T''_{d0}, s$	$T'_{d0}, s$	$T''_{q0}, s$
value	1.88	1.8	0.293	0.214	0.224	0.19	0.033	5.67	0.013

Many works show that the determination of induction parameters with the use of the finite element method is loaded with a small error compared to numerical values measured on real objects. Another problem is to determine the time constants of the solid block and conducting slot wedges of the rotor which strongly depend on the phenomenon of the skin effect in the rotor, and therefore on the selection of a frequency in the calculation model. This problem is discussed in the work [7] in which there are presented the results of parameter calculations by the finite element method, verified by measurements for a synchronous generator. The parameters calculated by FEM can be applied as the starting parameters of optimization algorithms, basing on which one can determine the parameters of the generator model when using dynamic waveforms or static characteristics measured on real objects.

## 6. The methodology of electromagnetic parameter estimation of the synchronous generator circuit model

In the parameter estimation process there was used the circuit mathematical model of the generator GENROU expressed in Park coordinates (d, q), containing one equivalent damping

circuit in the rotor in the d axis and two equivalent circuits in the q axis. In this model the phenomenon of magnetic core saturation was taken into account in an approximate way, while the stator transformation voltages were neglected [9, 10].

The parameters of the synchronous generator mathematical model can be determined (in both axes) based on the analysis of dynamic waveforms caused by a disturbance of the machine steady operation [11–14]. The dynamic waveforms under generator load conditions calculated by the finite element method are the basis of parameter estimation. The calculations were carried out for the synchronous generator operating in a single-machine power system, in which the transient state was caused by a step change in the excitation voltage equal to +5%  $E_{fdn}$ . In the field-circuit model, there was assumed that in the steady state the generator was loaded with the rated active  $P_n$  and reactive  $Q_n$  power, and the absolute value of the space vector of the stator voltage was equal to  $U_n$ . The calculation time of transient states of a generator with the use of the finite element method is very long. That is why the considerations were limited to one type of a disturbance of the machine steady operation.

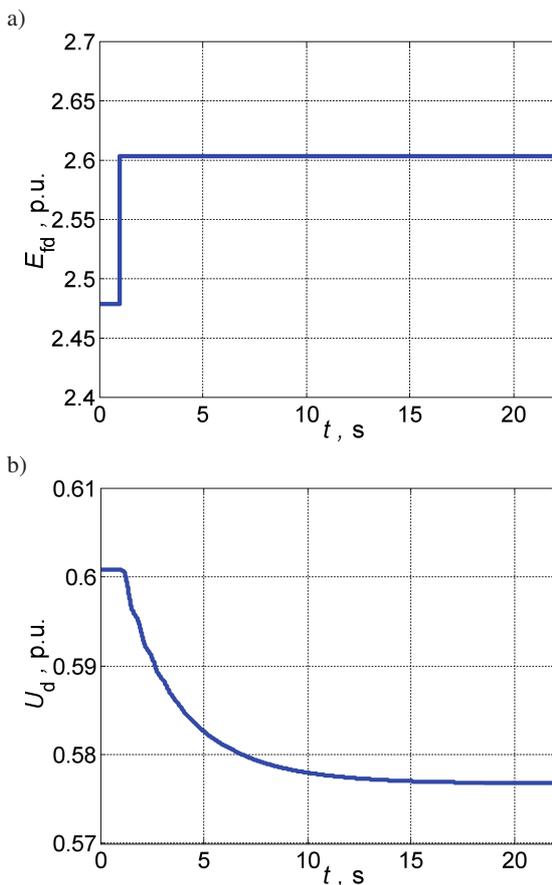


Fig. 9. Waveforms of selected input quantities of the generator circuit mathematical model at the step change in the excitation voltage equal to +5%  $E_{fdn}$

In the estimation process, the machine mathematical model parameters are determined in such a way as to minimize the objective function in the form of a mean-square error between the standard waveforms (calculated with FEM) and those calculated by means of the simulation model for the searched vector of parameters  $\mathbf{P}$ . For the case considered this vector can be expressed in the form

$$\mathbf{P} = [R_a \ X_\sigma \ X_{ad} \ X'_d \ X''_d \ T'_{d0} \ T''_{d0} \ X_{aq} \ X'_q \ X''_q \ T'_{q0} \ T''_{q0}].$$

The exemplary waveforms of the input quantities of the generator circuit mathematical model are shown in Fig. 9.

The least squares method was used for estimation. The mean-square error between the standard waveforms and those calculated by means of the simulation model for the searched vector of parameters  $\mathbf{P}$  was assumed to be determined by the following formula [10]:

$$\varepsilon(\mathbf{P}) = \sum_{i=1}^n \left( \left| \frac{I_{di}^m - I_{di}^s(\mathbf{P})}{I_{di}^m} \right|^2 + \left| \frac{I_{fi}^m - I_{fi}^s(\mathbf{P})}{I_{fi}^m} \right|^2 + \left| \frac{I_{qi}^m - I_{qi}^s(\mathbf{P})}{I_{qi}^m} \right|^2 \right), \quad (8)$$

where  $I_{di}^m$ ,  $I_{fi}^m$ ,  $I_{qi}^m$ ,  $I_{di}^s(\mathbf{P})$ ,  $I_{fi}^s(\mathbf{P})$ ,  $I_{qi}^s(\mathbf{P})$  – instantaneous values of the respective output signals calculated with FEM (m) and those of the circuit model calculated for the actual set of parameters  $\mathbf{P}$  (s).

## 7. Parameter estimation results of the synchronous generator circuit model

The minimization of the mean-square error defined by formula (8) was performed based on the Levenberg – Marquardt gradient algorithm with constraints from the Matlab Optimization Toolbox. The calculation results of the electromagnetic parameters of the generator are presented in Table 2. Figure 10 shows the comparison of the waveforms of the stator currents in the d and q axis and the excitation current calculated with the use of the field-circuit model (FEM) with those obtained from the circuit model (CM) for the calculated set of parameters.

Table 2  
 Calculation results of the synchronous generator parameters

Parameter	$X_{ad}$	$X'_d$	$X''_d$	$T'_{d0}, s$	$T''_{d0}, s$	$X_{aq}$
value	1.637	0.2509	0.2182	5.31	0.268	1.55
Parameter	$X'_q$	$X''_q$	$T'_{q0}, s$	$T''_{q0}, s$	$R_a$	$X_\sigma$
value	0.3547	0.235	2.202	0.0051	0.0014	0.192

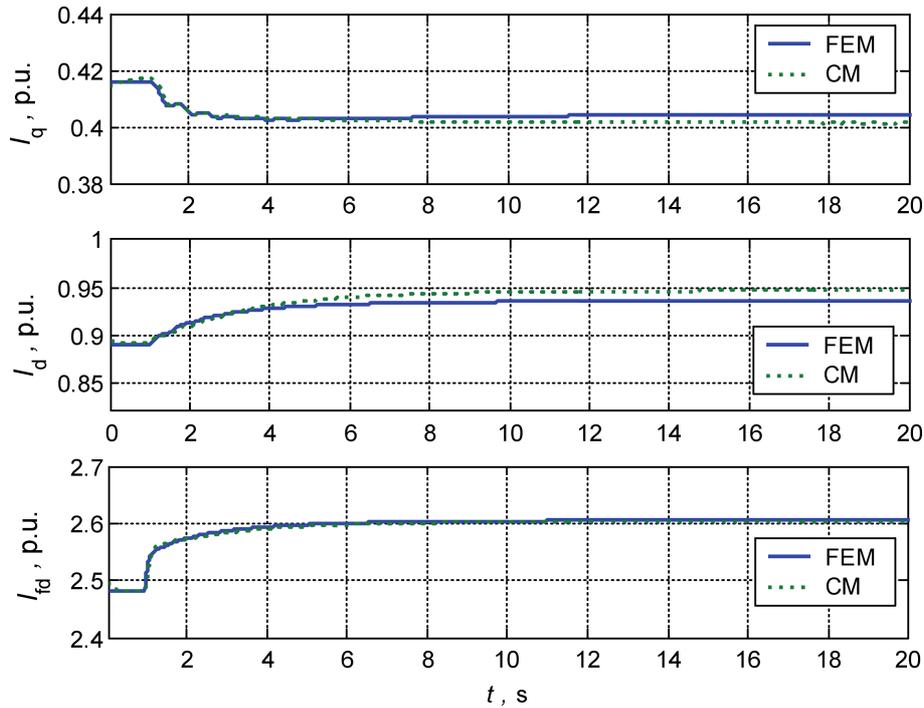


Fig. 10. Comparison of the waveforms of stator currents in d and q axis and the excitation current calculated based on the field-circuit model (FEM) with those calculated based on the circuit model (CM), caused by the step change in the excitation voltage equal to  $+5\%E_{fdn}$

## 8. Conclusions

In order to determine the set of electromagnetic parameters of the synchronous generator with the use of the method presented in the paper, it was necessary to have the complete set of the construction and material data based on which there were developed the field and field-circuit models of the generator. The field-circuit model was verified by the comparison of the measured and calculated waveforms of the stator voltage of the generator installed in Połaniec Power Plant at the step changes in the voltage regulator reference voltage of the generator under no-load conditions. From this comparison it follows that the developed field-circuit model represents the behavior of the machine in steady and transient states in a satisfactory way. This model was the basis of further investigations. The electromagnetic parameters of the generator circuit mathematical model in the d and q axis were calculated based on the distributions of the static and quasi-static, magnetic and electromagnetic fields in the machine cross-section. These distributions were calculated with FEM. Based on these calculations, there were determined the reactances of the steady and subtransient state. Since there is no clear boundary between the transient and subtransient state, the other parameters of the generator can be approximately determined for the mathematical model containing one equivalent damping circuit in the d axis and one damping circuit in the q axis. In the case of mathematical models containing a larger number of equivalent damping circuits, parameters of a synchronous generator can be determined based on transient waveforms of selected electrical and mechanical quantities of the machine. Due to the lack of possibility of taking measurements on the generating

unit in Połaniec Power Plant under the generator load conditions, these waveforms were calculated with the finite element method. The electromagnetic parameters of the synchronous generator model were determined in both axes based on the transient waveforms caused by the disturbance of the steady operation of a generator cooperating with the power system. A valuable advantage of this estimation method is no need of disconnecting the generator from the power system.

From the comparison of the waveforms of the stator current in the d and q axis and the excitation current calculated based on the field-circuit model with the waveforms calculated based on the circuit model for the determined set of parameters, it follows that the circuit model represents the investigated synchronous generator in a sufficiently accurate way. The visible discrepancies between the waveforms calculated on the basis of the field-circuit model and those calculated based on the circuit model can be the result of the simplified assumptions taken for the circuit model, e.g. neglecting the electromotive force of transformation.

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