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Loopshaping of motor torque controller

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The controller synthesis problem of the motor torque is presented. The tuning of the II^2 controller parameters of the electromagnetic motor torque is introduced. The results are obtained by applying the weighted sensitivity method (nominal performance) which is the optimization in $\mathcal{H}_\infty$ space. The waterbed effect for some weighting functions is presented. The results, which are obtained by a parametric optimization (nonlinear programming), are analysed by the calculations of the stability margins.

Key words: electric drive, II^2 controller, robust control, stability region, parametric optimization, nonlinear programming, waterbed effect, weighted sensitivity, modulus criterion

1. Introduction

Parametric optimization of a motor torque controller is discussed in the paper. The obtained results are presented for a generalized mathematical model of the torque production circuit in electric machines [20]. Thus, this transfer-function (the plant model) is the second-order with derivative element and integral-plus-double integral (II^2) controller is selected for good steady-state performance, at low frequencies (offset-free reference tracking).

General parametric optimization of motor torque or current controllers are included in several books e.g. [9, 11, 14]. Taking into account the motor torque constraints is shown in [7, 15, 18, 22] and the weighted sensitivity for constrained transient of torque is considered in the paper.

Loopshaping of the closed-loop system is called nominal performance or weighted sensitivity, too. This controller synthesis problem is presented in many publications such as [1, 8, 24] and [2] where the controller has fixed structure.

Robust controller synthesis for electrical drives can be found in [3, 11, 12, 13, 21].

The optimization method in the space $\mathcal{H}_\infty$ as minimizing norm $||w_P S||_\infty$ is considered and weight w_P designing is introduced. Thus, the $\mathcal{H}_\infty$ problem as a static optimization of a constrained nonlinear multivariable function (constrained nonlinear programming) is

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realized in this paper. The stability region of a controller parameters is constraints of the optimization problem.

The results which are obtained in optimization process are checked by the simulation (frequency and step responses) and the stability margins (robustness) calculation. Furthermore, a waterbed effect of a sensitivity function is discussed for parameters changing of some weighting functions.

The parametric optimization is realized by the function *fmincon* from Optimization Toolbox (MATLAB). This method is more flexible than functions from Robust Toolbox (e.g. *hinfstruct*). The optimization algorithm is shown in appendix B.

$\mathcal{H}_\infty$ control presented in this paper is alternative to another synthesis methods [9, 10, 19, 18, 17, 21]. The closed-loop shaping can be used both for double-inertia and oscillatory plant model.

Robust analysis of the closed-loop torque control system is realized by classical stability margins (gain margin (GM), phase margin (PM) and stability margin $S_m = ||S||_\infty^{-1}$).

2. Weighted sensitivity

$\mathcal{H}_\infty$ norm for a SISO system is induced by $\mathcal{L}_2$ norm for signals. The physical interpretation of $\mathcal{H}_\infty$ norm corresponds to the maximum energy amplification over the input signal. It can be shown that $||G(s)||_\infty$ equals the supremum in the Bode plot of the transfer-function.

Typically (nominal performance) in the loopshaping magnitude of S, which should be small, is only considered. The sensitivity function is compared with upper bound $\frac{1}{w_P(s)}$ where w_P is weight (fixed stable transfer-function). The loopshaping performance requirement is satisfied by the condition

$$|S(j\omega)| < \frac{1}{|w_P(j\omega)|} \tag{1}$$

Optimization in $\mathcal{H}_\infty$ space is the minimization of $||w_P(s)S(s)||_\infty$ and this problem usually has solution by MATLAB function *hinfstruct*. *Hinfstruct* is included in the Robust Control Toolbox and based on the paper [2]. These tools can tune arbitrary control architectures consisting of feedback loops and fixed-order, fixed-structure controllers. But the assumption of this function is, there are no common roots of a numerator and a denominator of the product $w_P(s)S(s)$. If this assumption is not satisfied so designer has to take advantage of the classical optimization methods (e.g. Optimization Toolbox).

Usually the function w_P is shaped (type of transfer-function and parameters) by the designer and this weight is stable but not necessarily asymptotic stable. The robust control literature [1, 8, 24, 25] includes many formulas of w_P and two of them are applied in this paper:

$$w_P(s) = \frac{1}{M} + \frac{\omega_B}{s}, \quad 1 \leqslant M \leqslant 2 \tag{2}$$

or

$$w_P(s) = \frac{s/M + \omega_B}{s + \omega_B A_m}, \quad 0 < A_m \ll 1, \;\; 1 \leqslant M \leqslant 2 \tag{3}$$

where A_m, M, ω_B are infimum (at low frequency), supremum (at high frequency) and bandwidth frequency of $\frac{1}{w_P(s)}$, respectively. So discussed weighting function determines the **nominal performance**. Moreover, the frequency responses of the upper bound of S which based on previously presented weighting functions, and it can be considered as

$$\left|\frac{1}{w_P(\omega)}\right| = \frac{M\omega}{\sqrt{\omega^2 + M^2\omega_B^2}} \tag{4}$$

for weight (2) the magnitude at bandwidth frequency ω_B equals

$$\left|\frac{1}{w_P(\omega_B)}\right| = \frac{M}{\sqrt{1+M^2}} \tag{5}$$

The frequency responses of the upper bound of S and the magnitude at bandwidth frequency ω_B for second weight (3) is in the following form

$$\left|\frac{1}{w_P(\omega)}\right| = M\sqrt{\frac{\omega^2 + A_m^2\omega_B^2}{\omega^2 + M^2\omega_B^2}}, \qquad \left|\frac{1}{w_P(\omega_B)}\right| = M\sqrt{\frac{1+A_m^2}{1+M^2}} \tag{6}$$

The values of the parameters A_m, M, ω_B have a significant influence on **waterbed effect**, which results from the Bode sensitivity integral for stable open-loop transfer-function $L(s)$ [1, 8, 24, 25]

$$\int_0^\infty \ln|S(j\omega)|d\omega = 0 \tag{7}$$

So formula (7) implies that areas of sensitivity reduction ($\ln|S(j\omega)|$ negative) and sensitivity bandwidth ($\ln|S(j\omega)|$ positive) are equal.

Thus, decreasing the value of the parameter A_m (3) at low frequency and over a larger range (ω_B increase) results in a larger peak of $|S|$. Then the inequality $||w_P(s)S(s)||_\infty < 1$ will not be possible to satisfy if the parameter M remains unchanged (parameter value should increase). The illustrative examples are included in sec. 4 and 5.

Example 1 (Selection of weighting functions) *Proper operation of many electric drives in dynamic states is guaranteed for the following limitations of the state variables:*

$$\left\{\begin{array}{ll} |M_e(t)| \leqslant M_{max} = \lambda_N M_N & \text{– motor torque limitation,} \\ \left|\dfrac{dM_e(t)}{dt}\right| \leqslant pM_N & \text{– limitation of torque derivative} \end{array}\right\} \tag{8}$$

where λ_N, p are positive constants.

The simplest (1st order) transfer-function which satisfies the constraints (8) *is the complementary sensitivity function* $T = \frac{1}{\frac{\lambda_N}{p}s+1}$. *Thus, from* $S+T=1$ *and* (1) *the weight* w_P *is determined in the following form*

$$\left|\frac{s}{s+\frac{p}{\lambda_N}}\right| < \frac{1}{|w_P(s)|} \tag{9}$$

Hence (9) *is the critical weight and can be rewrite as* (2) *with* $M=1, \omega_B = \frac{p}{\lambda_N}$

$$w_P(s) = \frac{s+M\omega_B}{Ms} = \frac{s+\frac{p}{\lambda_N}}{s} \tag{10}$$

$\mathcal{H}_\infty$ *optimization for weight* (10) *can lead to the waterbed effect. Moreover, the weight* (11) *is easier to shape*

$$w_P(s) = \frac{s/M+\omega_B}{s+\omega_B A_m} \tag{11}$$

with $M>1,\ 0<A_m<1$. *Thus, function* (11) *is more popular.*

The upper bound of S for weights (10) *and* (11) *is shown in Fig. 1 where* $p=50, \lambda_N = 2, \omega_B = 8\frac{rad}{s}, A_m = 0.1, M = 1.6$. *These functions are used in further researches.*

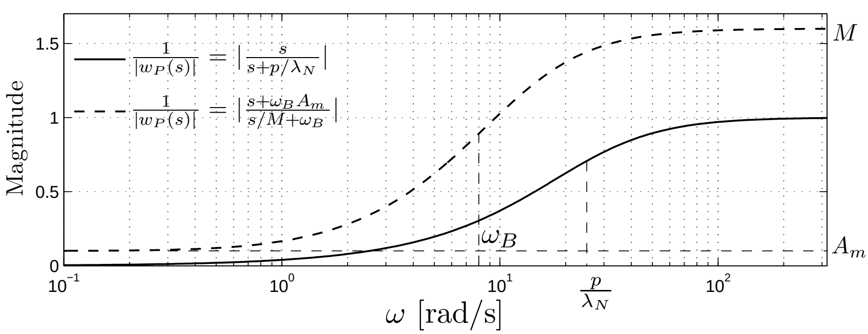

Figure 1. Comparison of frequency responses of weighting functions (10) and (11) with magnitude at bandwidth frequency

3. Electric drive

The generalizations of the mathematical models of various motors and the reference frames were introduced in the papers [20, 23]. **Motor torque** model is without approximation (usually to first order lag element) and without decoupling:

$$G(s)\Big|_{M_m=0} = \frac{M_e(s)}{U(s)} = \frac{A_1 s}{BTs^2+Bs+1} \tag{12}$$

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where M_m is load torque, M_e is motor torque, U is voltage in torque production axis, B,T are time constants and A_1 is parameter.

In spite of the construction differences of electric motors (separately excited DC, BLDC, PMSM or induction motors) , the mathematical model of the voltage-electromagnetic torque relationship is the second order transfer-function with electromagnetic T and electromechanical B time constants. On the basis of relationship between these constants, one can specify if the motor is:

1. $B \geqslant 4T$ second-order lag with derivative element,
2. $B < 4T$ oscillatory with derivative element.

The first case is simple and the modulus criterion [7, 9, 10] can be applied to the torque controller parametric optimization. The second case is more difficult because no simplification of transfer-function can be used. State-space methods of PI controller tuning are presented in [18, 19].

The mathematical models of the **power converters** can be considered in the form of the transfer-function [4, 5, 6, 9, 11, 14, 21]

$$G_p(s) = \frac{K_p}{\tau_0 s + 1} \tag{13}$$

But the fact that τ_0 is much smaller than the time constants (B,T) of electric motor can lead to simplification of the model (13) to the following form [7, 16, 18, 22]

$$G_p(s) = K_p \tag{14}$$

Such models of the power converter are used in the parametric optimization of the motor torque controllers [7, 9, 11, 17].

4. II^2 controller of electromagnetic torque – general case

The relation between constants B and T cannot be checked, thus it does not matter whether the electric motor is oscillatory or lag element. The transfer-function (12) is with derivative, obviously.

From the fact that the operator s occurs in numerator of the motor transfer-function (12) the II^2 controller is selected in the following form

$$G_R(s) = \frac{K_1 s + K_2}{s^2} \tag{15}$$

If Y, K_p, A_1 are gain of measurement system, average gain of power converter and nominator parameter of motor torque transfer-function (12) then one $A = K_p A_1 Y$ parameter can be used.

In this section two models (13), (14) of the power converter are used separately:

1^o $G(s) = K_p$ the transfer-functions are in the forms

$$L(s) = \frac{A(K_1 s + K_2)}{BTs^3 + Bs^2 + s} \tag{16}$$

$$S(s) = \frac{s(BTs^2 + Bs + 1)}{BTs^3 + Bs^2 + (1 + AK_1)s + AK_2} \tag{17}$$

and from Liénard-Chipart criterion [1] the stability region in controller parameters space is calculated as

$$K_1 > -\frac{1}{A} \tag{18a}$$

$$K_2 > 0 \tag{18b}$$

$$K_2 < \frac{1}{T}K_1 + \frac{1}{AT} \tag{18c}$$

2^o $G(s) = \dfrac{K_p}{\tau_0 s + 1}$ the transfer-functions are in the forms

$$L(s) = \frac{A(K_1 s + K_2)}{BT\tau_0 s^4 + B(T + \tau_0)s^3 + (B + \tau_0)s^2 + s} \tag{19}$$

$$S(s) = \frac{s(BT\tau_0 s^3 + B(T + \tau_0)s^2 + (B + \tau_0)s + 1)}{BT\tau_0 s^4 + B(T + \tau_0)s^3 + (B + \tau_0)s^2 + (1 + AK_1)s + AK_2} \tag{20}$$

and the stability region is in the following form of three inequalities

$$K_1 > -\frac{1}{A} \tag{21a}$$

$$K_2 > 0 \tag{21b}$$

$$K_2 < -\frac{T\tau_0(1 + AK_1)^2}{AB(T + \tau_0)^2} + \frac{(B + \tau_0)(1 + AK_1)}{AB(T + \tau_0)} \tag{21c}$$

Example 2 *Stability regions of parameters (K_1, K_2) space for the presented control system with the power converter models are shown in the Fig. 2. The calculations based on inequalities* (18) *and* (21) *for electric drive parameters (appendix A).*

$\mathcal{H}_\infty$ control problem, as the constrained nonlinear optimization (constrained nonlinear programming) in $\boldsymbol{R}^2$, is formulated for:

objective function: $||w_P S||_\infty$ *where w_P is in the form* (2) *or* (3) *and S can be considered as transfer-functions* (17) *or* (20).

constraints: *stability region of the controller parameters which is defined by relations* (18) *or* (21).

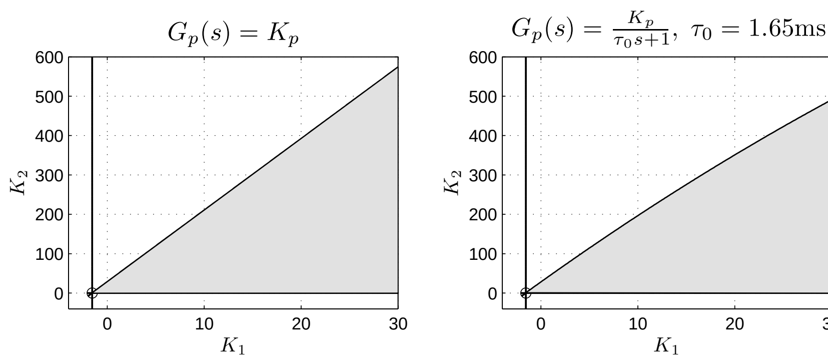

Figure 2. Stability regions for various model of power converter

starting point: $x_0 = [K_{10} \;\; K_{20}]$ *which has to be chosen within the stability region* (18) *or* (21).

The MATLAB function with the interior-point algorithm can be used directly for all previously presented transfer-functions but zero-pole cancellation should be realized before writing objective function.

The interior-point constrained optimization gives the best result (convergence, speed), comparing with remained options of function *fmincon*, for considered problem. The *hinfstruct* function can be used only for the problems where zero-pole cancellation does not exist. So *fmincon* is more reliable than *hinfstruct*, but the code program is larger.

Example 3 (Parametric optimization of II^2 controller for DC drive with weighting function (2)) *The motor and the power converter parameters are included in appendix A and the sensitivity function is described by* (17). *In DC motor $M_e = \psi_e I$ (ψ_e is flux linkage, I is armature current) so the torque control is equal to the current control. Thus, the mathematical model of the motor is in the following form [7, 9, 11, 17, 18, 20]*

$$G(s) = \frac{I(s)}{U(s)} = \frac{\frac{B}{R}s}{BTs^2+Bs+1}, \quad B = J\frac{R}{\psi_e^2}, \quad T = \frac{L}{R} \tag{22}$$

where R, L are armature generalized resistance and armature total inductance, respectively.

a: *Firstly, the weight* (2) *in critical form* (10) *is considered. The standard* MATLAB *function* hinfstruct *cannot be used in the $\mathcal{H}_\infty$ optimization of*

$$||w_P S||_\infty = \left|\left|\frac{(s+\frac{p}{\lambda_N})(BTs^2+Bs+1)}{BTs^3+Bs^2+(1+AK_1)s+AK_2}\right|\right|_\infty \tag{23}$$

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because of simplification Laplace operator s from numerator of S and denominator of w_P*. This fact is the reason for the application of the interior-point method (function* fmincon *in* MATLAB*). Presented result with* 10^{-6} *absolute tolerance on the function value will be obtained. Algorithm* fmincon *is a static optimization of constrained nonlinear multivariable function, so the inequalities* (18) *are used as constraints and* $||w_P S||_\infty$ *is the performance index.*

The optimization results are presented in Fig. 3, where frequency responses (emphasis magnitude at high frequency), Bode diagram (emphasis magnitude at low frequency) and step responses of closed-loop system are shown. The step responses may be interpreted as p.u. transient.

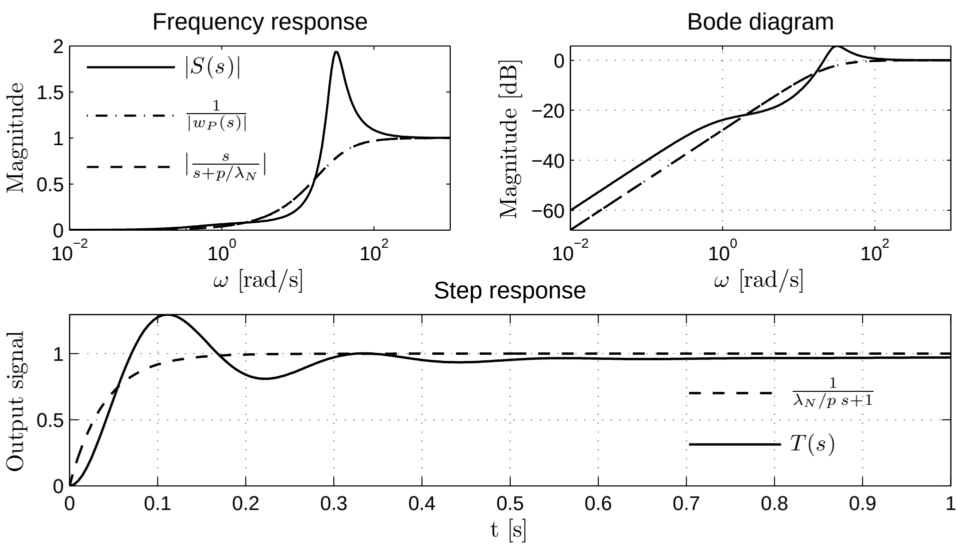

Figure 3. Optimization results (II^2 controller) for weight (2) – $\omega_B = \frac{p}{\lambda_N}$ rad/s, $M = 1$ and power converter model (14)

MATLAB *script which solves the II*^{2} *controller tuning problem (loopshaping) is presented in appendix B.*

This result with $||w_P S||_\infty = 2.46$ *(controller parameters* $K_1 = 18, K_2 = 15,8$*) is obtained, thus frequency responses of* $\frac{1}{w_P(s)}$ *and* $S(s)$ *are not fitted and, moreover, magnitude of* $|S(j\omega)|$ *has large peak and additionally, stability margin is small* $S_m = 0.51$*. The step response overshoot of the closed-loop transfer-function* $T(s)$ *is about* 25%*, so it is not proper optimization result. Moreover, the robustness factors are* $GM = +\infty$*,* $PM = 34,9^o$ $(\omega_c = 27 rad/s)$.

Concluding ***the frequency*** ω_B ***is too high and the magnitude M is too small. This means the waterbed effect.***

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b: *In order to improve previous optimization result the parameters* $M = 1.6$ *and* $\omega_B = 0.32\frac{p}{\lambda_N}$ *are assumed. In Fig. 4 the optimization results* ($||w_P S||_\infty = 1.045$) *are shown. The controller parameters* $K_1 = 4.9, K_2 = 11.6$ *are obtained The transfer-*

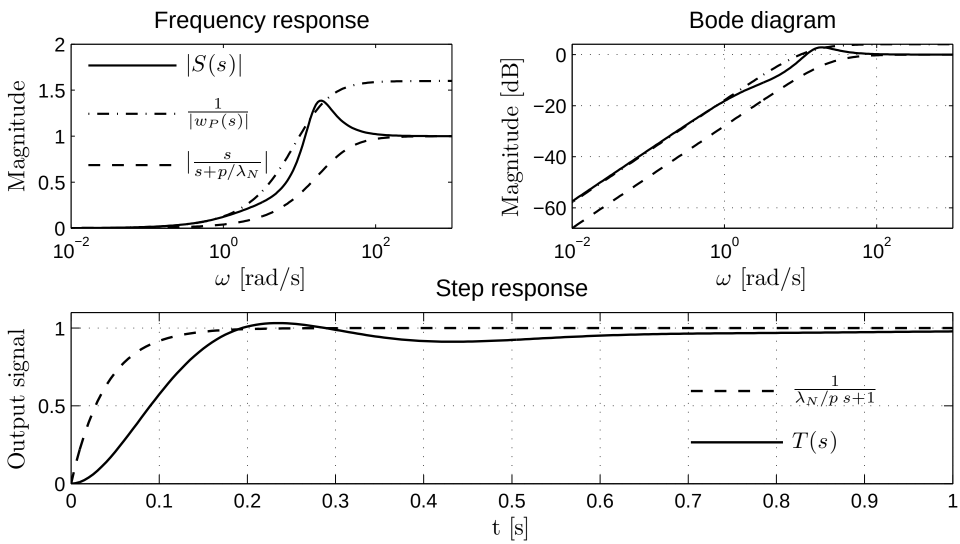

Figure 4. Optimization results (II^2 controller) for weight (2) – $\omega_B = 0.32\frac{p}{\lambda_N}$ rad/s, $M = 1.6$

function $S(s)$ *is good shaped and overshoot is very small (step response of* $T(s)$*). The robustness factors are* $GM = +\infty$, $PM = 60.7^o$ ($\omega_c = 11.9 rad/s$), $S_m = 0.72$.

Example 4 (Parametric optimization of II^2 controller for DC drive with weight (3)**)** *The performance index is in the following form*

$$||w_P S||_\infty = \left|\left|\frac{s(s/M+\omega_B)(BTs^2+Bs+1)}{(s+\omega_B A_m)(BTs^3+Bs^2+(1+AK_1)s+AK_2)}\right|\right|_\infty \tag{24}$$

with parameters $M = 1.6$, $A_m = 0.01$ *and* $\omega_B = 0.32\frac{p}{\lambda_N}$.

After optimization process the controller parameters are $K_1 = 5.2, K_2 = 11.3$ *and the performance index is* $||w_P S||_\infty = 1.061$. *Thus the frequency response of S is similar to the result from Fig. 4 and the robustness factors are* $GM = +\infty$, $PM = 59.9^o$ ($\omega_c = 12.4 rad/s$), $S_m = 0.71$.

Example 5 (Parametric optimization of II^2 controller with first-order lag model of power converter) *The sensitivity function is in the form* (20) *(where* $\tau_0 = 1.37ms$*) and weight is the same as in the example 4.*

Thus, the controller parameters are $K_1 = 4.8, K_2 = 11.1$ *and the performance index is* $||w_P S||_\infty = 1.061$. *Hence, the frequency response of S is similar to result from Fig. 4 and the robustness factors are* $GM = 44.4$, $PM = 60.3^o$ $(\omega_c = 11.8 rad/s)$, $S_m = 0.7$.

5. II^2 controller of electromagnetic torque – simplified case

If transfer function (12) satisfies the condition $B \geqslant 4T$ then mathematical model can be rewritten as

$$G(s) = \frac{M_e(s)}{U(s)} = \frac{As}{(B_1 s + 1)(T_1 s + 1)}, \quad B_1 \geqslant T_1 \tag{25}$$

and for simple calculations the controller is considered in the following form

$$G_R(s) = K_2 \frac{K_1' s + 1}{s^2}, \quad K_1 = K_1' K_2 \tag{26}$$

After plant pole $-\frac{1}{B_1}$ compensation by controller zero $-\frac{1}{K_1'}$ the first controller parameter is

$$K_1' = B_1 \tag{27}$$

and the transfer-functions are obtained as

$$L(s) = \frac{K_2 A}{s(T_1 s + 1)}, \qquad S(s) = \frac{T_1 s^2 + s}{T_1 s^2 + s + K_2 A} \tag{28}$$

The performance index with weight (3) is in the form

$$||w_P S||_\infty = \left|\left| \frac{(s + M\omega_B)(T_1 s + 1)}{M(T_1 s^2 + s + AK_2)} \right|\right|_\infty \tag{29}$$

The stability region of the transfer function S is $K_2 > 0$. In similarity to general case (sec. 4) the constrained nonlinear optimization is formulated for: objective function (29), constraints $K_2 > 0$ and starting point $K_{20} > 0$.

Example 6 (Parametric optimization of II^2 controller with performance index (29)**)**
The similar results as in Fig. 3 are obtained for $M = 1, \omega_B = \frac{p}{\lambda_N}$. *So, it means the waterbed effect and parameters of weighting function have to be changed e.g. as in example 3. Then the result is better and* $||w_P S||_\infty = 1.24$ *for controller parameters* $K_1 = 3.45, K_2 = 19.4$. *Moreover, robustness factors are* $GM = +\infty$, $PM = 51.9^o$ $(\omega_c = 9.8 rad/s)$, $S_m = 0.68$. *In the Fig. 5 the optimization results are shown.*

If the frequency ω_B *will be small then the condition* $\left|\left| w_P(s)S(s) \right|\right|_\infty < 1$ *can be satisfied (Fig. 6).*

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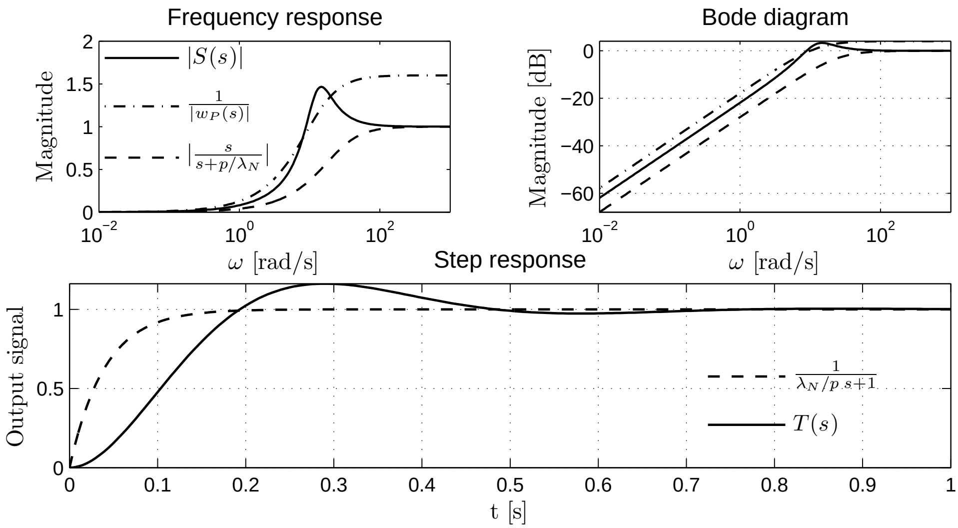

Figure 5. Optimization results (II^2 controller) for $K_1^{'} = B_1$ and norm (29) – $\omega_B = 0.32\frac{p}{\lambda_N}$ rad/s, $M = 1.6$

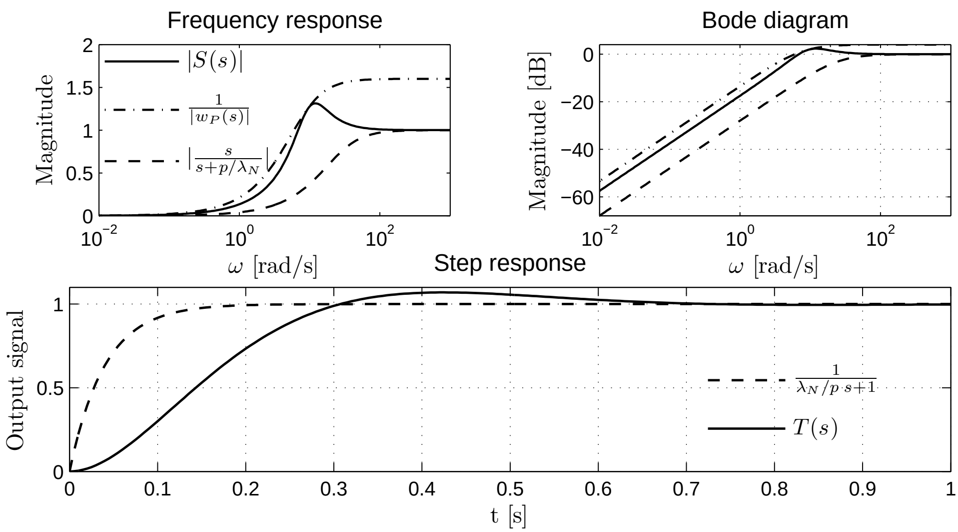

Figure 6. Optimization results (II^2 controller) for $K_1^{'} = B_1$ and norm (29) – $\omega_B = 0.2\frac{p}{\lambda_N}$ rad/s, $M = 1.6$

6. Comparison with modulus criterion

The DC drive system is considered in this section and the mathematical model of the motor is in the following form (for constant flux linkage ψ_e) [4, 7, 9, 14, 20]:

$$\begin{cases} J\frac{d\omega(t)}{dt} & = M_e(t) - M_m(t), \;\; M_e(t) = \psi_e I(t) \\ U(t) & = RI(t) + \underbrace{\psi_e \omega(t)}_{E(t)} + L\frac{dI(t)}{dt} \end{cases} \tag{30}$$

and the thyristor power converter is assumed as (13) with $\tau_0 = 1.67$ms.

The *modulus criterion* [9, 17], is the classical approach in electrical drives, with PI controller

$$G_R(s) = K_R \frac{T_R s + 1}{T_R s}, \quad K_R = \frac{TR}{2K_p Y \tau_0}, \quad T_R = T$$

The modulus criterion based on simplification of the mathematical model (e.m.f. is neglected, so $E(t) = 0$). Moreover pole (plant) by zero (controller) compensation and optimal tracking postulate $\left|\frac{G_R G}{1+G_R G}\right| \to 1$ are applied.

Thus, the closed-loop transfer-function is theoretically ($E(t) = 0$) in the following form

$$G_c(s) = \frac{I(s)}{I_{ref}(s)} = \frac{1}{2\tau_0^2 s^2 + 2\tau_0 s + 1} \tag{31}$$

but without the motor model simplification ($E(t) \neq 0$) the transfer-function is

$$G_c(s) = \frac{I(s)}{I_{ref}(s)} = \frac{K_R A s + \frac{K_R}{T} A}{BT\tau_0 s^3 + B(T + \tau_0)s^2 + (B + \tau_0 + K_R A)s + 1 + \frac{K_R}{T} A} \tag{32}$$

The the closed-loop frequency response loopshaping is limited, but the advantages are the basic structure and ready to use formulas for calculations. The modulus criterion gives very fast step response, so limitation of the motor torque derivative can be realized by setpoint value prefilter which, additionally, should compensate steady-state error [18].

Moreover, the PI controller is not robust on the load torque input, but this disadvantageous effect can be compensated [22].

7. Conclusions

In this approach the sensitivity upper bound has been considered. The results of research in frequency- and time-domain are satisfied.

The waterbed effect of the sensitivity function for various weights are shown in Fig. 7. To sum up: if ω_B will be smaller than $\frac{p}{\lambda_N}$, then the area below and above 0 is less than in the critical case. Moreover, the step response overshoot of $T(s)$ is the smallest for Fig. 7.d. but the speed of time response is characterized by low dynamics.

Presented method can be used for typical stator or rotor flux linkage reference frame in induction or permanent magnet synchronous motors, because the mathematical models of torque production are very similar [20]. Hence, $\mathcal{H}_\infty$ optimization is useful for motor torque or quadrature current controller synthesis.

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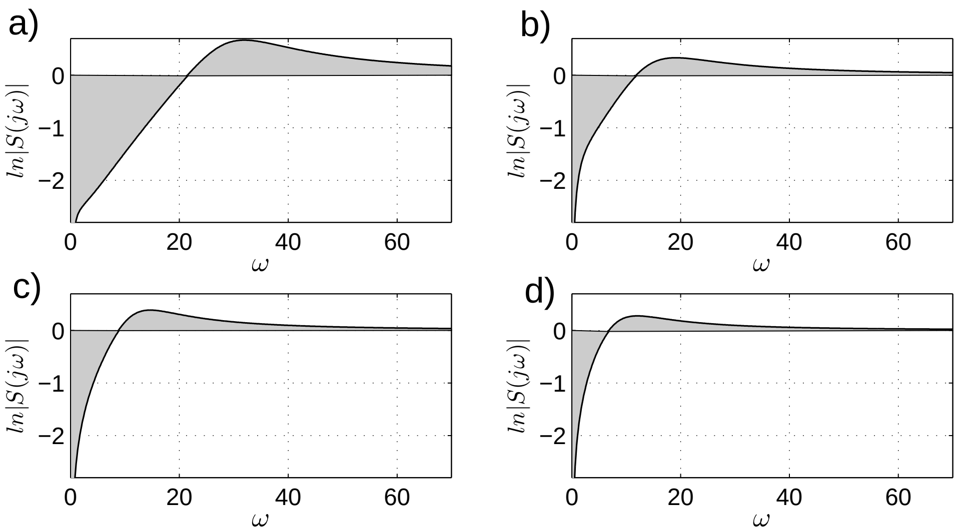

Figure 7. Waterbed effect for a) Fig. 3, b) Fig. 4, c) Fig. 5, d) Fig. 6

The main disadvantages of the loopshaping is the necessity to use nonlinear programming (numerical methods), but the program which is presented in appendix B is not very complicated.

This method can be enlarged on electric drive uncertainty which leads to the robust performance and the loopshaping performance can be satisfied for all possible plants. Thus, the optimization in $\mathcal{H}_\infty$ space [1, 8, 24] leads to the condition

$$\left|\left|\, |w_P(s)S(s)| + |w_I(s)T(s)| \,\right|\right|_\infty < 1 \qquad (33)$$

This problem for multiplicative uncertainty of a power converter was analysed in [21].

A. DC drive parameters

$$P_N = 18[kW], \quad U_N = 440[V], \quad I_N = 47[A],$$
$$n_N = 1800[rpm], \omega_N = 188[\tfrac{rad}{s}], \omega_0 = 200,3[\tfrac{rad}{s}],$$
$$R = 1,8[\Omega] \quad L = 99[mH], \quad T = L/R = 55[ms],$$
$$\psi_{eN} = 2,197[\tfrac{Vs}{rad}], \lambda_N = 2[\tfrac{I_{max}}{I_N}], \quad J = 0,69[kgm^2],$$
$$K_p = 69[\tfrac{V}{V}], \quad p = 50[\tfrac{A}{A}].$$

B. Matlab optimization algorithm

```
%  wS=(s+MwB)/(Ms)
function [K1,K2,S,fv]=me_ii2_ws_fmin_ogr()
global  alpha B T A wS M wB

Mn=In*Psi;   B=J*R/(Psi^2);   A=Kp*B/R*Y

s=tf('s');

alpha=p/lambda;
M=1.6; wB=0.32*alpha; wS = (s+M*wB)/(M*s)
K1=0.3;K2=0.6;      % initial values

WS=((s+M*wB)/M*(B*T*s^2+B*s+1))/(B*T*s^3+B*s^2+(1+K1*A)*s+K2*A);
x0=[K1 K2];
op=optimset('Display','iter','Algorithm','interior-point','LineSearchType',...
            'quadcubic','MaxIter',40,'TolFun',1e-6, 'TolX',1e-6);
[x,fv,exitflag,output]=fmincon(@Hnorm,x0,[],[],[],[],[],[],@confun,op)
 K1=x(1);  K2=x(2); %controller parameters
 K=x

 C=(K1*s+K2)/s;   S=feedback(1,C*G)
[GM,PM,Wcg,Wcp] =margin(C*G)% robustness analysis
SM=1/norm(feedback(1,C*G),Inf) % stability margin

%% performance index
function f=Hnorm(x)
global  alpha B T A wS  M wB
s=tf('s');
K1=x(1); K2=x(2);

C=(K1*s+K2)/s;   G=tf([A],[B*T B 1]); S=feedback(1,C*G);
WS=((s+M*wB)/M*(B*T*s^2+B*s+1))/(B*T*s^3+B*s^2+(1+K1*A)*s+K2*A);

figure(3)
bode(S,1/wS,WS);
grid; drawnow;
f=norm(WS,Inf);

%% Constraints
function [c, ceq] = confun(x)
global  alpha B T A
% Nonlinear inequality constraints
K1=x(1);   K2=x(2);
c = [K2-K1/T-1/(T*A); -K2;-K1-1/A];
% Nonlinear equality constraints
ceq = [];
```

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