

Anti-synchronization in different new chaotic systems via active nonlinear control

AYUB KHAN and RAM PRAVESH PRASAD

In this paper, we discuss anti-synchronization between two identical new chaotic systems and anti-synchronization between another two identical new chaotic systems by active nonlinear control. The sufficient conditions for achieving the anti-synchronization of two new chaotic systems are derived based on Lyapunov stability theory. Numerical simulations are provided for illustration and verification of the proposed method.

Key words: anti-synchronization, new chaotic system, active nonlinear control

A. Introduction

In 1990, Pecora and Carroll [1] induced a method to synchronize two identical chaotic systems with different initial conditions which have wide applications in several field such as biological science, physical system [2], chemical system [3], ecological system [4], secure communication [5] etc.

Chaos synchronization is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. In most of the chaos anti-synchronization approaches, the master -slave or drive -response is used. A particular chaotic system is called the master or drive system and another chaotic system is called slave or response system. Then idea of chaos anti-synchronization is used for the output of the master system to control the slave system so that output of the slave system follow the output of the master system asymptotically. There are different types of method for chaos anti-synchronization of the chaotic non-linear systems, such as OGY [6], Active control method [7], Adaptive control method [8], sample-data feedback method [9], time delay feedback method [10], backstepping method [11], sliding mode control method [12], etc.

So far, many types of chaos synchronization method have been presented such as complete synchronization [1], phase synchronization [13], generalized synchronization

A. Khan is with Department of Mathematics, Zakir Husain Delhi College, University of Delhi, New Delhi-01. R.P. Prasad is with Department of Mathematics, University of Delhi, New Delhi-07, e-mail: ram.mbhu@gmail.com

Received 8.02.2013.

[14], anti-synchronization [15,16], projective synchronization [17], generalized projective synchronization [18,19], etc.

Complete synchronization is characterized by the convergence of the two chaotic trajectories and has been observed in mutually coupled, unidirectionally coupled and even noise induced chaotic oscillator. Projective synchronization is characterized by the fact that the master and slave systems synchronize up to a scaling factor, where as in generalized projective synchronization the responses of the synchronized dynamical states synchronize up to constant scaling matrix α . It is shown that the complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix are $\alpha = I$ and $\alpha = -I$, respectively.

This paper has organized in five sections. Section 2 is related to the materials and methodology, In section 3, we discuss the system description, section 4 and section 5 are discuss about the anti-synchronization of two new chaotic systems. section 6 contains the conclusion.

B. Materials and methodology

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \quad (1)$$

where $x \in R^n$ are the states of system, A is the $n \times n$ matrix of the system parameters and $f : R^n \rightarrow R^n$ is the nonlinear part of the system. We consider the system (1) as the master or drive system

As the slave or response system, we consider the following system.

$$\dot{y} = By + g(y) + u \quad (2)$$

where $y \in R^n$ are the states of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \rightarrow R^n$ is the nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two identical chaotic systems. If $A \neq B$ or $f \neq g$, then x and y are the states of two different chaotic systems. For anti-synchronization design a controller u , which anti-synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), y(0) \in R^n$.

We define the anti-synchronization error as

$$e = y + x \quad (3)$$

The synchronization error dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \quad (4)$$

Thus, the anti-synchronization problem is to find a controller u to stabilize the error dynamics (4) for all initial conditions $e(0) \in R^n$. Hence, we find a controller u so that

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0 \quad \text{for all } e(0) \in R^n. \quad (5)$$

Consider a Lyapunov function $V(e) = e^T P e$, where P is a positive definite matrix, then $V : R^n \rightarrow R$ is a negative definite function. Thus, by Lyapunov stability theory [20], the error dynamics (4) is globally exponentially stable and hence the states of the master system (1) and slave system (2) will be globally and exponentially anti-synchronized.

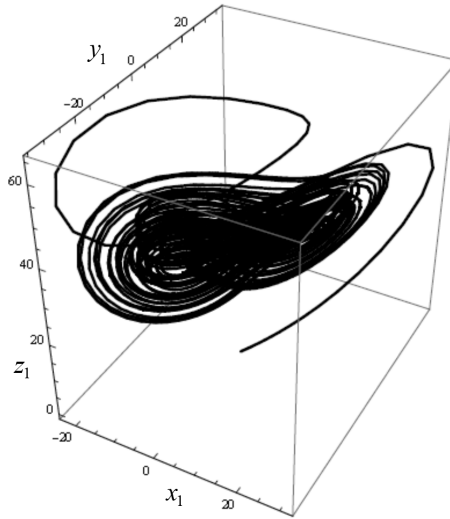


Figure 1. Chaotic behavior of system (6).

C. System description

The new chaotic system is defined by

$$\begin{aligned} \dot{x} &= a_1(y - x) + w \\ \dot{y} &= c_1y - xz \\ \dot{z} &= xy - b_1z + y \\ \dot{w} &= xz + r_1w + xz \end{aligned} \quad (6)$$

where x, y, z , and w are state vectors and a_1, b_1, c_1 and r_1 , are positive constant parameters. The system is chaotic when $a_1 = 39, b_1 = 3, c_1 = 25$ and $r_1 = 1.3$, which is shown in Fig. 1.

Another new chaotic system is given by

$$\begin{aligned}
 \dot{x} &= a(x - y) + w + y \\
 \dot{y} &= dx - xz + cy \\
 \dot{z} &= xy - bz + wz \\
 \dot{w} &= yz + rw + xy
 \end{aligned} \tag{7}$$

where x , y , z , and w are state vectors and a , b , c , d and r , are positive constant parameters. The system is chaotic when $a = 38$, $b = 3$, $c = 14$, $d = 7$ and $r = 0.5$, which is shown in Fig. 2.

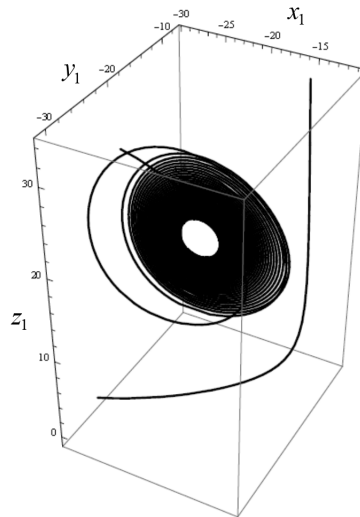


Figure 2. Chaotic behavior of system (7).

D. Anti-synchronization between two identical new chaotic systems

In this section, based on the active control theory, anti-synchronization between two identical new chaotic systems is achieved. The drive and response systems are given as follows

$$\begin{aligned}
 \dot{x}_1 &= a_1(y_1 - x_1) + w_1 \\
 \dot{y}_1 &= c_1 y_1 - x_1 z_1 \\
 \dot{z}_1 &= x_1 y_1 - b_1 z_1 + y_1 \\
 \dot{w}_1 &= x_1 z_1 + r_1 w_1 + x_1 z_1
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 \dot{x}_2 &= a_1(y_2 - x_2) + w_2 + u_1 \\
 \dot{y}_2 &= c_1 y_2 - x_2 z_2 + u_2 \\
 \dot{z}_2 &= x_2 y_2 - b_1 z_2 + y_2 + u_3 \\
 \dot{w}_2 &= x_2 z_2 + r_1 w_2 + x_2 z_2 + u_4
 \end{aligned} \tag{9}$$

where u_1, u_2, u_3 and u_4 are control functions and the system parameters are chosen such that the both systems (8) and (9) are in chaotic states when the control function $u_i = 0, (i = 1, 2, 3, 4)$. Our goal is to determine the control function from active control method. In order to observe the anti-synchronization between (8) and (9), we get

$$\begin{aligned}
 \dot{e}_1 &= a_1(e_2 - e_1) + e_4 + u_1 \\
 \dot{e}_2 &= c_1 e_2 - x_1 z_1 - x_2 z_2 + u_2 \\
 \dot{e}_3 &= x_1 y_1 - b_1 e_3 + e_2 + x_2 y_2 + u_3 \\
 \dot{e}_4 &= 2x_1 z_1 + r_1 e_4 + 2x_2 z_2 + u_4
 \end{aligned} \tag{10}$$

where $e_1 = x_2 + x_1, e_2 = y_2 + y_1, e_3 = z_2 + z_1, e_4 = w_2 + w_1$. According to the of active control method, the control functions $u_i, (i = 1, 2, 3, 4)$ can be designed by

$$\begin{aligned}
 u_1 &= v_1 \\
 u_2 &= x_1 z_1 + x_2 z_2 + v_2 \\
 u_3 &= -x_1 y_1 - x_2 y_2 + v_3 \\
 u_4 &= -2x_1 z_1 - 2x_2 z_2 + v_4.
 \end{aligned} \tag{11}$$

Hence the error dynamics system becomes

$$\begin{aligned}
 \dot{e}_1 &= a_1(e_2 - e_1) + e_4 + v_1 \\
 \dot{e}_2 &= c_1 e_2 + v_2 \\
 \dot{e}_3 &= -b_1 e_3 + e_2 + v_3 \\
 \dot{e}_4 &= r_1 e_4 + v_4
 \end{aligned} \tag{12}$$

The error system (10) to be controlled is a linear system with control input v_1, v_2, v_3 and v_4 as the function of error states e_1, e_2, e_3 and e_4 . As long as these feedbacks stabilize the system, e_1, e_2, e_3 and e_4 converge to zero as time $t \rightarrow \infty$. This implies that the two identical new chaotic systems are anti-synchronize with feedback control. There are many possible choice for the control inputs v_1, v_2, v_3 and v_4 . We choose

$$\begin{aligned}
 v_1 &= -a_1 e_2 - e_4 \\
 v_2 &= (-c_1 - 1)e_2 \\
 v_3 &= -e_2 \\
 v_4 &= -(r_1 + 1)e_4.
 \end{aligned} \tag{13}$$

Then the error dynamics is

$$\begin{aligned}
 \dot{e}_1 &= -a_1 e_1 \\
 \dot{e}_2 &= -e_2 \\
 \dot{e}_3 &= -b_1 e_3 \\
 \dot{e}_4 &= -e_4.
 \end{aligned} \tag{14}$$

Let choose the Lyapunov function as follows

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2). \tag{15}$$

The time derivative of the Lyapunov function along the trajectory is

$$\dot{V} = e_1 \dot{e}_1 + e_2 \dot{e}_2 + e_3 \dot{e}_3 + e_4 \dot{e}_4 = -[a_1 e_1^2 + e_2^2 + b_1 e_3^2 + e_4^2]. \tag{16}$$

Since the Lyapunov function V is positive definite and its derivative \dot{V} is negative definite in the neighborhood of zero solution for the system (12), by Lyapunov stability theory [20], the error dynamics can converge to the origin asymptotically. This implies that two identical new chaotic systems are anti-synchronized.

Numerical result

In numerical simulation, to verify and demonstrate the effectiveness of the proposed method to solve the system and we take the initial values of drive and response systems are taken as $x_1(0) = 0.5$, $y_1(0) = 0.8$, $z_1(0) = -0.1$, $w_1(0) = -0.3$ and $x_2(0) = -0.4$, $y_2(0) = -0.2$, $z_2(0) = 0.6$, $w_2(0) = 0.1$ respectively. Hence the initial values of error system are $e_1(0) = 0.1$, $e_2(0) = 0.6$, $e_3(0) = 0.5$, $e_4(0) = -0.2$. The simulation results are illustrated in Fig. 3. Fig. 3 displays the time evolutions of the drive and response systems.

E. Anti-synchronization between another two identical chaotic systems

The drive and response systems are given by

$$\begin{aligned}
 \dot{x}_1 &= a(y_1 - x_1) + w_1 + y_1 \\
 \dot{y}_1 &= dx_1 - x_1 z_1 + cy_1 \\
 \dot{z}_1 &= x_1 y_1 - bz_1 + w_1 z_1 \\
 \dot{w}_1 &= y_1 z_1 + rw_1 + x_1 y_1,
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 \dot{x}_2 &= a(y_2 - x_2) + w_2 + y_2 + u_1 \\
 \dot{y}_2 &= dx_2 - x_2 z_2 + cy_2 + u_2 \\
 \dot{z}_2 &= x_2 y_2 - bz_2 + w_2 z_2 + u_3 \\
 \dot{w}_2 &= y_2 z_2 + rw_2 + x_2 y_2 + u_4.
 \end{aligned} \tag{18}$$

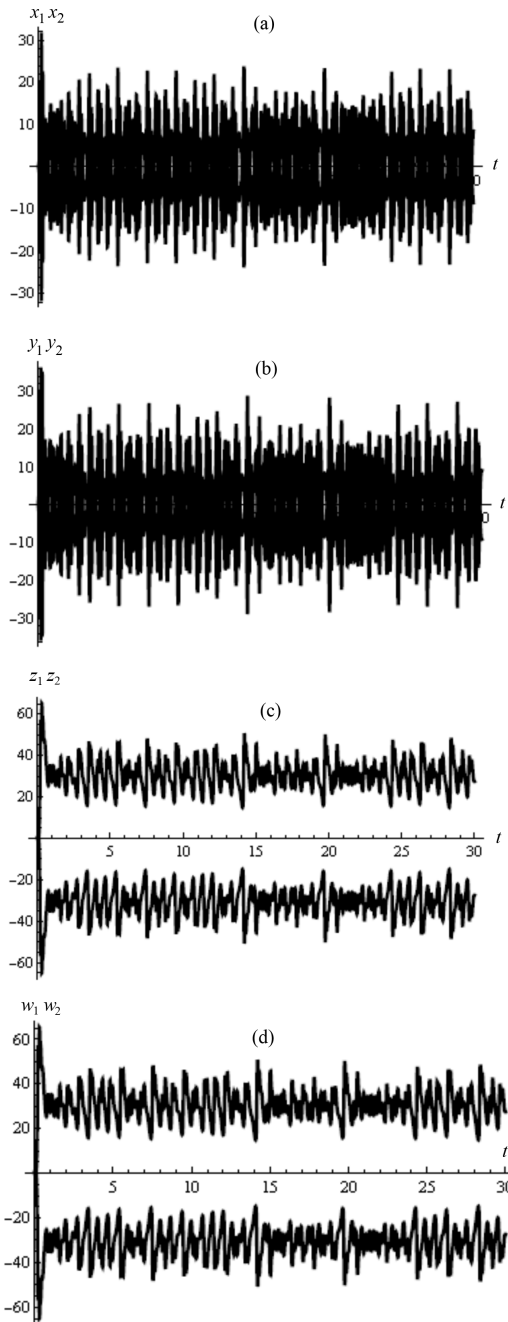


Figure 3. (a), (b), (c) and (d) are shows the state trajectories of drive and response systems.

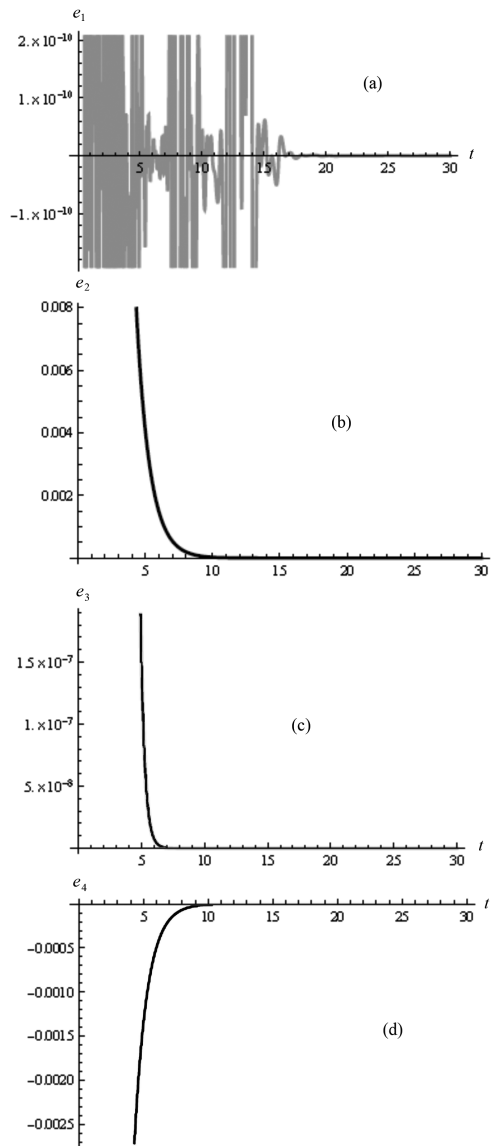


Figure 4. (a) Dynamics of the anti-synchronization errors e_1 , (b) Dynamics of the anti-synchronization errors e_2 , (c) Dynamics of the anti-synchronization errors e_3 , and (d) Dynamics of the anti-synchronization errors e_4 via active control.

Our goal is to determine the control functions u_i , ($i = 1, 2, 3, 4$) to make the anti-synchronization in two identical new chaotic systems via active nonlinear control. From equation (17) and (18) we get

$$\begin{aligned}
 \dot{e}_1 &= a(e_2 - e_1) + e_4 + e_2 + u_1 \\
 \dot{e}_2 &= de_1 - x_1z_1 + ce_2 - x_2z_2 + u_2 \\
 \dot{e}_3 &= x_1y_1 - be_3 + w_1z_1 + x_2y_2 + w_2z_2 + u_3 \\
 \dot{e}_4 &= y_1z_1 + re_4 + x_1y_1 + y_2z_2 + x_2y_2 + u_4
 \end{aligned} \tag{19}$$

where $e_1 = x_2 + x_1$, $e_2 = y_2 + y_1$, $e_3 = z_2 + z_1$, $e_4 = w_2 + w_1$. To achieve the asymptotic stability of the zero solution of the error system (19), we take the active control function as follows

$$\begin{aligned}
 u_1 &= v_1 \\
 u_2 &= x_1z_1 + x_2z_2 + v_2 \\
 u_3 &= -x_1y_1 - w_1z_1 - x_2y_2 - w_2z_2 + v_3 \\
 u_4 &= -y_1z_1 - x_1y_1 - y_2z_2 - x_2y_2 + v_4,
 \end{aligned} \tag{20}$$

hence the error system becomes

$$\begin{aligned}
 \dot{e}_1 &= a(e_2 - e_1) + e_4 + e_2 + v_1 \\
 \dot{e}_2 &= de_1 + ce_2 + v_2 \\
 \dot{e}_3 &= -be_3 + v_3 \\
 \dot{e}_4 &= re_4 + v_4
 \end{aligned} \tag{21}$$

According to method of active control, v_i , ($i = 1, 2, 3, 4$) are chosen as

$$\begin{aligned}
 v_1 &= -ae_2 - e_4 - e_2 \\
 v_2 &= -de_1 - (c + 1)e_2 \\
 v_3 &= 0 \\
 v_4 &= -(r + 1)e_4.
 \end{aligned} \tag{22}$$

Then the error dynamics is

$$\begin{aligned}
 \dot{e}_1 &= -ae_1 \\
 \dot{e}_2 &= -e_2 \\
 \dot{e}_3 &= -be_3 \\
 \dot{e}_4 &= -e_4.
 \end{aligned} \tag{23}$$

Therefore the closed loop system (19) has eigenvalues $-a, -1, -b$ and -1 . By the Lyapunov stability theory [20], this leads to the error states e_1, e_2, e_3 and e_4 converging to zero as $t \rightarrow \infty$. That is to say that the systems (17) and (18) can achieve global

asymptotical anti-synchronization.

Numerical result

In the numerical simulation to solve the drive and response systems, and we take the initial values of the drive and response systems $x_1(0) = -0.1$, $y_1(0) = 0.2$, $z_1(0) = -0.6$, $w_1(0) = 0.4$ and $x_2(0) = -1$, $y_2(0) = 0.4$, $z_2(0) = -0.2$, $w_2(0) = 1$ respectively. The initial values of error dynamics is $e_1(0) = -1.1$, $e_2(0) = 0.6$, $e_3(0) = -0.8$, $e_4(0) = 1.4$. The simulation results are illustrated in Fig. 5. Fig. 5 displays the time evolutions of the drive and response systems.

F. Conclusion

In this paper, we have studied the anti-synchronization between two identical new chaotic systems and another two identical new chaotic systems via active nonlinear control. The anti-synchronization results derived in this paper have been established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is very convenient and efficient for the anti-synchronization of two identical new chaotic systems. Numerical simulations have been performed to illustrate the anti-synchronization results derived in this paper.

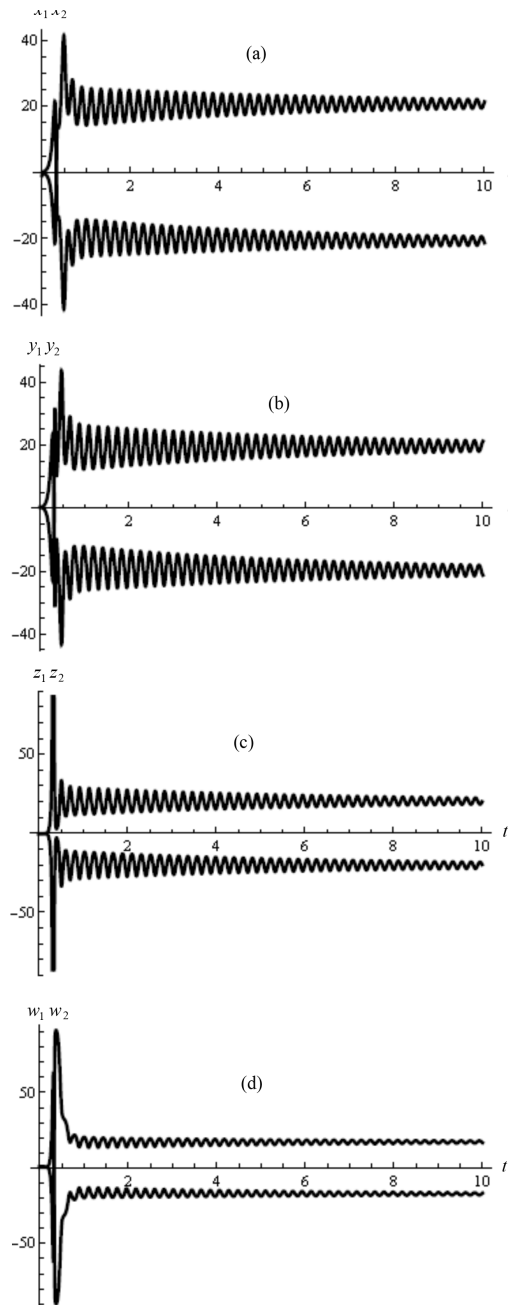


Figure 5. ((a), (b), (c) and (d) are shows the state trajectories of drive and response systems.

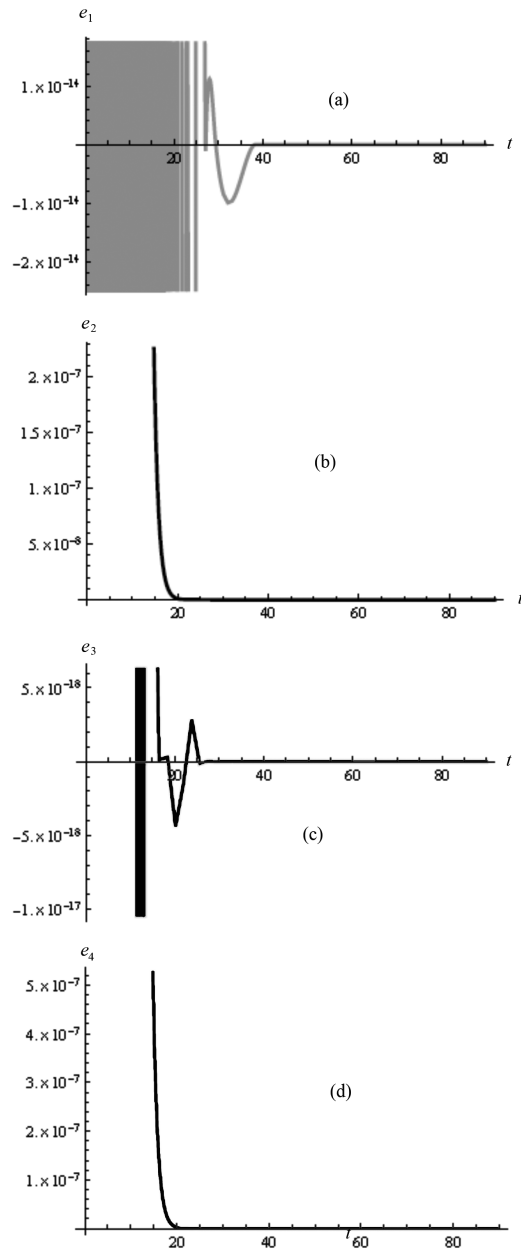


Figure 6. (a) Dynamics of the anti-synchronization errors e_1 , (b) Dynamics of the anti-synchronization errors e_2 , (c) Dynamics of the anti-synchronization errors e_3 , and (d) Dynamics of the anti-synchronization errors e_4 via active control.

References

- [1] L.M. PECORA and T.L. CARROLL: Synchronization in chaotic systems. *Physical Review Letters*, **64** (1990), 821-824.
- [2] M. LAKSHMANAN and K. MURALI: Chaos in nonlinear oscillators: controlling and synchronization, World Scientific, Singapore, 1996.
- [3] S.K. HAN, C. KERRER and Y. KURAMOTO: D-phasing and bursting in coupled neural oscillators. *Physical Review Letters*, **75** (1995), 3190-3193.
- [4] B. BLASIUS, A. HUPPERT and L. STONE: Complex dynamics and phase synchronization in spatially extended ecological system. *Nature*, **399** (1999), 354-359.
- [5] M. FEKI: An adaptive chaos synchronization scheme applied to secure communication. *Chaos Solitons and Fractals*, **18** (2003), 141-148.
- [6] E. OTT, C. GREBOGI and J.A. YORKE: Controlling chaos. *Physical Review Letters*, **64** (1990), 1196-1199.
- [7] L. TIAN, J. XU and M. SUN: Chaos synchronization of the energy resource chaotic system with active control. *Int. J. of Nonlinear Science*, **3** (2007), 228-234.
- [8] J.H. PARK, S.M. LEE and O.M. KWON: Adaptive synchronization of Genesio-Tesi chaotic system via a novel feedback control. *Physical Letters A*, **371** (2007), 263-270.
- [9] J. ZHAO and J. LU: Using sampled data feedback control and linear feedback synchronization in a new hyper-chaotic system. *Chaos Solitons and Fractals*, **35** (2008), 376-382.
- [10] J.H. PARK and O.M. KWON: A novel criterion for delayed feedback control of time delay chaotic systems. *Chaos Solitons and Fractals*, **17** (2003), 709-716.
- [11] X. WU and J. LU: Parameter identification and backstepping control of uncertain Lu system. *Chaos Solitons and Fractals*, **18** (2003), 721-729.
- [12] H.T. YAU: Design of adaptive sliding mode controller for chaos synchronization with uncertainties. *Chaos Solitons and Fractals*, **22** (2004), 341-347.
- [13] Z.M. GE and C.C. CHEN: Phase synchronization of coupled chaotic multiple time scales systems. *Chaos Solitons and Fractals*, **20** (2004), 639-647.
- [14] Y.W. WANG and Z.H. GUAN: Generalized synchronization of continuous chaotic systems. *Chaos Solitons and Fractals*, **27** (2006), 97-101.

- [15] X. ZHANG and H. ZHU: Anti-synchronization of two different hyper-chaotic systems via active and adaptive control. *Int. J. of Nonlinear Science*, **6** (2008), 216-223.
- [16] T. CHIANG, J. LIN, T. LIAO and J. YAN: Anti-synchronization of uncertain unified chaotic systems with dead-zone nonlinearity. *Nonlinear Analysis*, **68** (2008), 2629-2637.
- [17] J. QIANG: Projective synchronization of a new hyper-chaotic Lorenz system. *Physical Letters A*, **370** (2007), 40-45.
- [18] Y. JIAN-PING and L. CHANG-PIN: Generalized projective synchronization for the chaotic Lorenz system and the chaotic Chen system. *J. Shanghai Univ.*, **10** (2006), 299-304.
- [19] R.H. LI, W. XU and S. LI: Adaptive generalized projective synchronization in different chaotic systems based on parameter identification. *Physical Letters A*, **367** (2007), 199-206.
- [20] W. HAHN: *The Stability of Motion*, Springer-Verlag, New York, (1967).