

# The effect of anisotropy in the modified Jiles-Atherton model of static hysteresis<sup>\*</sup>

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**Abstract:** An extension of the modified Jiles-Atherton description to include the effect of anisotropy is presented. Anisotropy is related to the value of the angular momentum quantum number  $J$ , which affects the form of the Brillouin function used to describe the anhysteretic magnetization. Moreover the shape of magnetization dependent  $R(m)$  function is influenced by the choice of the  $J$  value.

**Key words:** soft magnetic materials, hysteresis, anisotropy, modelling

## 1. Introduction

Modelling of hysteresis loops in soft magnetic materials is important for optimal design of magnetic circuits in electric devices. The shape of magnetization curves is affected by number of physical phenomena, e.g. eddy currents, anisotropy, applied stress etc., which should be taken into account when developing the appropriate descriptions of magnetization processes.

The macroscopic Jiles-Atherton hysteresis model [11] is one of the most popular models of the phenomenon due to its physical background and relatively easy form of model equations. The original model formulation has been developed in order to describe hysteresis loops in isotropic soft magnetic materials. Its essential features are:

- the averaged interactions between magnetic domains within the material are considered using the concept of “effective field”,
- the major loop branches are obtained by the introduction of an offset from the “anhysteretic” curve, which describes a hypothetical medium devoid of imperfections inherent in the magnetic material (inclusions, voids, dislocations of crystalline lattice etc.). For the description of the anhysteretic curve the model developers have chosen the modified Langevin function.

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The description has been later extended to include the effect of anisotropy by the introduction of an additional energy term in the argument of the modified Langevin function [17]. This concept has been developed in a number of subsequent papers, including the recent one [16].

In the present paper another possible approach is considered. The Langevin function is replaced with a more general Brillouin function. Moreover a magnetization dependent  $R(m)$  function is introduced into the model equation. The form of the  $R(m)$  function depends on the value of the angular momentum quantum number  $J$ , which affects the form of the Brillouin function. Modelling of major hysteresis loops is carried out for different classes of contemporary soft magnetic materials, i.e. Fe-based amorphous alloys, NiFe alloys and Fe-based nanocrystalline materials.

## 2. Model description

The modified version of the Jiles-Atherton model equations, outlined in recent papers [2-4, 6, 7] is considered. In the modified description the equation for total differential susceptibility is written in the form similar to that envisaged by Gy. Kádár in his product Preisach model [13-15]:

$$\frac{dM}{dH} = R(m) \left[ \beta + \frac{dM_{irr}}{dH} \right], \quad (1)$$

where  $\beta$  [-] is a model parameter, interpreted as initial susceptibility, whereas  $dM_{irr}/dH$  may be obtained from the fundamental Jiles-Atherton relationship

$$\frac{dM_{irr}}{dH_e} = \frac{\delta_M (M_{an} - M_{irr})}{k \delta}, \quad (2)$$

where  $H_e = H + \alpha M$  is the so-called effective field,  $\delta$  is the sign of time derivative of flux density, which is the input variable in the normalized measurement conditions (IEC60404),

$$\delta_M = 0.5 \left[ 1 + \text{sign} \left[ (M_{an} - M_{irr}) \cdot \frac{dB}{dt} \right] \right]. \quad (3)$$

$\alpha$  [-],  $a$  [A/m] and  $k$  [A/m] are the model parameters. The values of the latter two parameters may be expressed as power laws with respect to magnetization in order to obtain an accurate representation of minor loops [4, 6-7].  $m \in \langle 0, 1 \rangle$  denotes reduced magnetization, which is actual magnetization referred to saturation magnetization.

The function  $R(m)$ , interpreted as a measure of active domain wall surface [1] in the first approximation is given as

$$R(m) = 1 - m^2. \quad (4)$$

$M_{an}$  in Eq. (2) is the anhysteretic magnetization, which is envisaged to be the modified Brillouin function

$$M_{an} = M_s B_J(x) = M_s \left[ \frac{2J+1}{2J} \coth \frac{2J+1}{2J} x - \frac{1}{2J} \coth \frac{x}{2J} \right], \quad (5)$$

where  $J$  is the angular momentum quantum number in the considered magnetic material, whereas  $x$  denotes the reduced effective field  $x = H_e/a$  for brevity.  $M_s$  [A/m] is a model parameter (saturation magnetization).

The Brillouin function is a more general function than the Langevin function  $M_{an} = M_s [\coth x - 1/x]$  used in the original description.  $J$  takes discrete values, the lowest possible one is 0.5 – this corresponds to a two level, uniaxial anisotropy case. Typical values of this parameter, appropriate for description of magnetization phenomena in modern oriented and non-oriented steels are 0.5 and 1, respectively [4, 6-7]. The Brillouin function for  $J = 0.5$  becomes then the hyperbolic tangent. For  $J \rightarrow \infty$  (isotropic case) the Brillouin function becomes the Langevin function. These two extreme cases are depicted in Fig. 1.

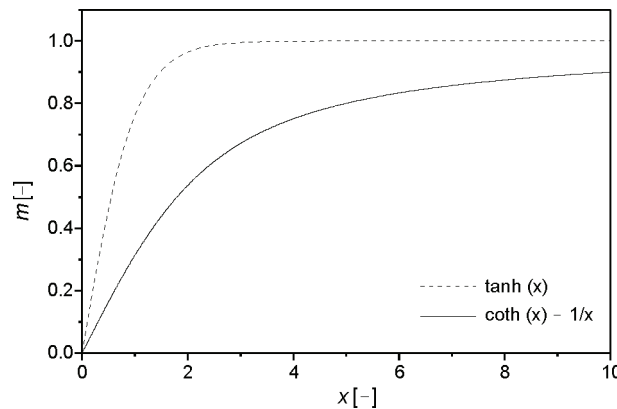


Fig. 1. Two limiting cases of the Brillouin function

It can be easily understood, that the  $J$  value is a measure of anisotropy of the magnetic material. It affects the  $M_{an} = f(x)$  dependence, but moreover it also affects the form of the  $R(m)$  function. In Refs. [14-15] a concept has been expressed, that  $R(m)$  could be the derivative of the Brillouin function with respect to  $x$ , expressed in terms of magnetization itself, cf. Fig. 2

$$R(m) = \frac{dB_J(x)}{dx} = f(m). \quad (6)$$

Analytical calculation is possible only for  $J = 0.5$ ,  $d \tanh(x)/dx = 1/\cosh^2(x) = 1 - \tanh^2(x) = 1 - m^2$ . For other  $J$  values one has to resort to numerical methods to evaluate the value of  $R(m)$  function for a given value of  $x$ .

From Figures 1 and 2 it follows that anisotropy, whose measure could be the  $J$  value, affects the shape of modelled hysteresis loop in a complex way: it changes the shape of the anhysteretic curve, but it also modifies the total susceptibility by a change of the shape of the  $R(m)$  function. The latter impact should be pronounced for higher excitation levels.

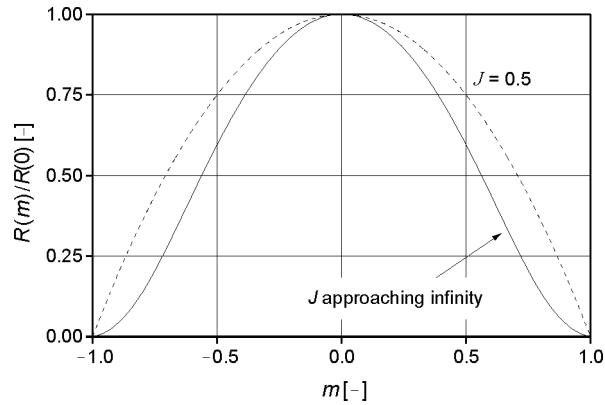


Fig. 2. The effect of changing the  $J$  value on the shape of  $R(m)$  function

The modified Jiles-Atherton description has been developed to describe magnetization curves of electrical steels [2, 4, 6-7]. For these soft magnetic materials the assumed values of  $J$  parameter could be equal to 0.5 (for grain oriented steels) or to 1 (for non-oriented steels). Recently, the model has been applied to evaluate the effects of anisotropy and ambient temperature variations in MnZn ferrites [3].

### 3. Modelling

In the present paper the proposed description is applied to model magnetization curves of chosen soft magnetic materials used in electrical engineering and power electronics, cf. Fig. 3. Model equations (1-3, 5, 6) are transformed to yield the relationship  $dM/dB$ , necessary for the so-called inverse model.

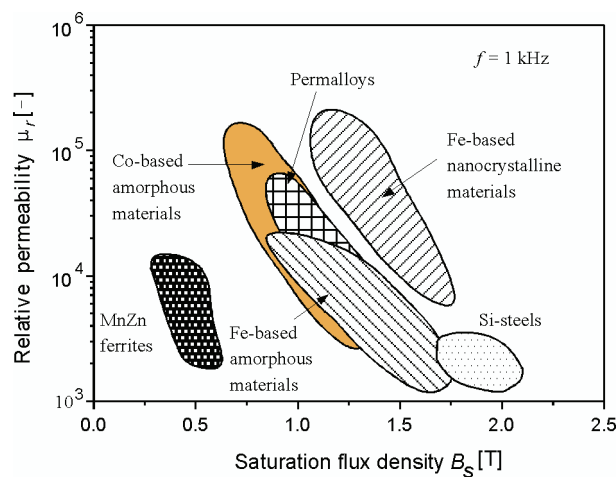


Fig. 3. Properties of chosen soft magnetic materials

Measurements of hysteresis loops are carried out on toroid samples of permalloys, Fe-based amorphous and nanocrystalline materials using a laboratory setup, which fulfills all the requirements of IEC60404 standard. The excitation frequency is kept as low as possible, in order to avoid the disturbing effect of eddy currents on hysteresis loop [18]. In modelling this effect is therefore neglected.

In order to recover the optimal set of model parameters, the Matlab implementation of the global optimization algorithm DIRECT is used [9, 12]. This robust procedure has already been successfully applied for this purpose in the context of the “classical” Jiles-Atherton model in Ref. [5]. The fitness value is the squared sum of errors between the measured and the modelled values of magnetization in a number of points (given in the first column of Table 1) on hysteresis loops. The number of the considered data points is chosen larger than the problem dimension i.e. redundant, so that the effects from inevitable measurement errors could be avoided.

Table 1.

	$J$ [-]	$\alpha x 10^{-6}$ [-]	$a$ [A/m]	$\beta$ [-]	$k$ [A/m]	$M_s x$ $10^6$ [A/m]	fitness [(A/m) <sup>2</sup> ]	$t$ [s]	evaluations [-]
Metglas (31 points)	0.5	3.926	8.12	5185	3.45	1.138	$1.178 \cdot 10^{11}$	209	523
	1.0	3.952	5.97	12687	3.38	1.170	$1.078 \cdot 10^{11}$	249	551
	2.5	3.938	4.12	15313	3.38	1.221	$8.768 \cdot 10^{10}$	401	667
	5.0	3.945	3.57	12017	3.38	1.259	$7.387 \cdot 10^{10}$	534	681
	10.0	3.945	3.29	8319	3.38	1.295	$6.377 \cdot 10^{10}$	681	785
	<b>25.0</b>	<b>3.952</b>	<b>3.11</b>	<b>3657</b>	<b>3.38</b>	<b>1.322</b>	<b><math>6.325 \cdot 10^{10}</math></b>	<b>897</b>	<b>737</b>
Vitroperm 800 F (40 points)	0.5	0.443	1.796	40349	1.308	1.098	$8.937 \cdot 10^{11}$	182	443
	1.0	0.442	1.798	52268	0.876	1.141	$5.311 \cdot 10^{11}$	265	539
	2.5	0.444	1.798	29721	0.807	1.243	$2.280 \cdot 10^{11}$	350	539
	5.0	0.442	1.798	36098	0.716	1.308	$1.387 \cdot 10^{11}$	753	685
	<b>10.0</b>	<b>0.443</b>	<b>1.798</b>	<b>43386</b>	<b>0.691</b>	<b>1.328</b>	<b><math>1.309 \cdot 10^{11}</math></b>	<b>965</b>	<b>623</b>
	25.0	0.443	1.789	60922	0.661	1.328	$1.802 \cdot 10^{11}$	1184	515
Permalloy 79 NiFe (51 points)	0.5	8.677	6.71	30394	4.69	0.695	$1.943 \cdot 10^{10}$	228	629
	2.0	9.594	5.69	48464	4.56	0.726	$2.147 \cdot 10^{10}$	207	551
	2.5	8.669	3.12	63551	4.69	0.743	$1.284 \cdot 10^{10}$	411	817
	5.0	8.027	2.46	60393	4.82	0.765	$1.132 \cdot 10^{10}$	437	639
	<b>10.0</b>	<b>9.542</b>	<b>2.33</b>	<b>12938</b>	<b>4.71</b>	<b>0.775</b>	<b><math>1.087 \cdot 10^{10}</math></b>	<b>501</b>	<b>673</b>
	25.0	9.612	2.15	40657	4.74	0.774	$8.866 \cdot 10^{10}$	642	587

Modelling strategy consists in sweeping the  $J$  value over an interval, what implies an appropriate adjustment of the shape of  $R(m)$  function, followed by the search for optimal values of other parameters. Briefly speaking, the DIRECT algorithm allows us to set two stopping conditions: exceeding a specified number of iterations or fitness function evaluations, if the global minimum is unknown. The results are given in Table 1. In all cases the algorithm has

stopped due to exceeding the prescribed number of iterations. The last column provides the information about the final number of fitness function evaluations. The fitness is defined here as the squared sum of errors for a number data points belonging to the experimental and the modelled hysteresis loops.

The bounds, which define the search space for the optimization routine, are preset in a manner similar to that explained in the earlier paper [5]. Parameters  $a$  and  $k$  should take values close to coercivity, thus  $a, k \in \langle 0.5H_c; 3H_c \rangle$ . The value of saturation magnetization  $M_s$  is expected to be larger than magnetization at loop tip  $M_{TIP}$ . The theoretical value at the zero K temperature decreases for most soft magnetic materials according to the well known Bloch's  $3/2$  law [8, 10], thus magnetization values at room temperature are significantly lower. It is assumed, that  $M_s \in \langle M_{TIP}; 1.4M_{TIP} \rangle$ . The mean field parameter  $\alpha$  should be of the order  $H_c / M_s$ . Finally, the values of  $\beta$  parameter should be comparable to values presented in data sheets and reference textbooks. It is also useful to consider Rayleigh equation for a small amplitude loop and obtain a first approximation, if direct measurements of initial permeability at  $H = 0.4$  A/m cannot be carried out properly e.g. due to the drift problems. This approach is followed in the present paper, but the search range for this parameter is deliberately enlarged to  $\langle 0.5\beta_1; 5\beta_1 \rangle$ , where  $\beta_1$  is the aforementioned first guess value. This explains a quite large scatter of the final values for this parameter and different values of  $J$  quantum number in some materials, what is noticeable in Table 1. The obtained values of  $\beta$  parameter correspond to those published in the available literature.

The set of model parameters, which exhibits the lowest fitness value for the considered material, is assumed as the optimal one. Figures 4-6 depict the measured and modelled hysteresis loops for the optimal cases from Table 1. It can be stated, that a reasonable agreement between the measured and the modelled loops is obtained.

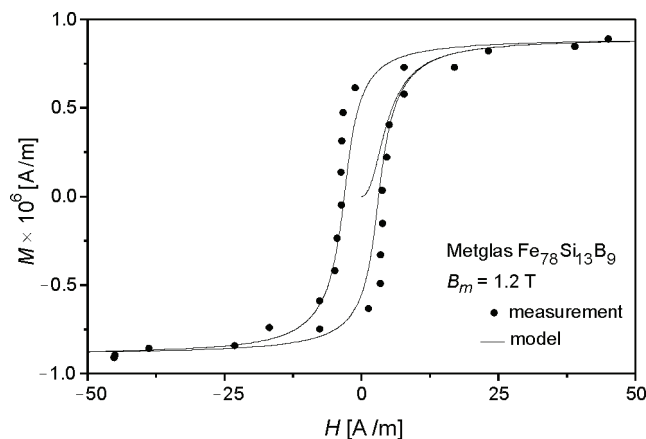


Fig. 4. The measured and the modelled hysteresis loops for the Metglas sample

The differences between the measured and the modelled hysteresis loops measured in the characteristic points of the magnetization curve (coercivity, remanence) did not exceed 15%.

Figure 7 depicts an exemplary action of the optimization procedure for the Vitroperm 800 F sample. It can be noticed, that in the presented case the fitness value achieves a steady state after some 20 iterations. Thus the arbitrarily chosen value 25 iterations is sufficient for obtaining acceptable modelling results, cf. Fig. 5.

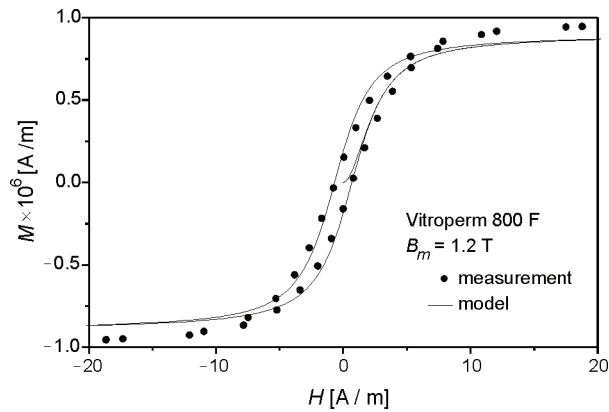


Fig. 5. The measured and the modelled hysteresis loops for the Vitroperm 800 F sample

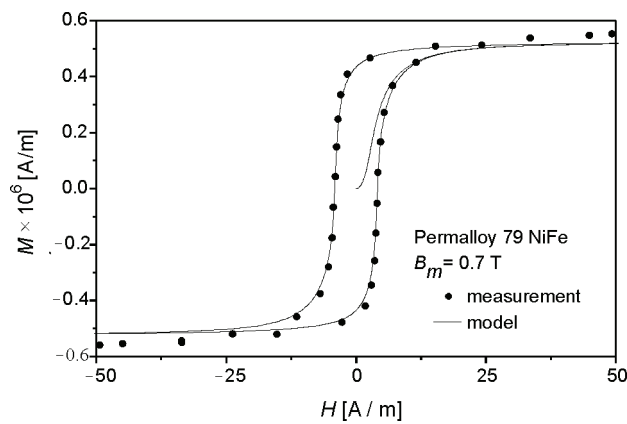


Fig. 6. The measured and the modelled hysteresis loops for the permalloy 79NiFe sample

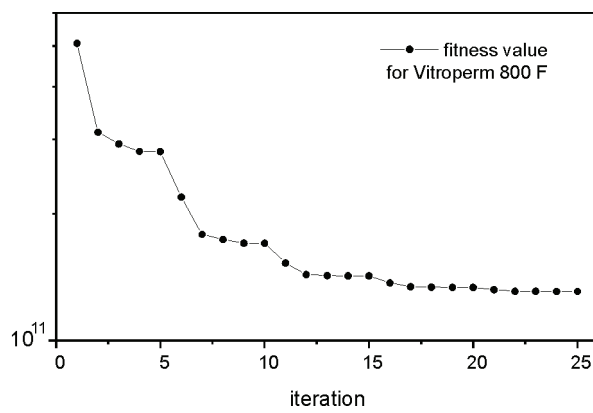


Fig. 7. Fitness variation during estimation of model parameters for the Vitroperm 800 F sample

## 4. Conclusions

The recently developed modified Jiles-Atherton description of magnetic properties was applied to model hysteresis loops of diverse magnetic materials: Fe-based amorphous alloys, NiFe alloys and Fe-based nanocrystalline materials. It was found that major hysteresis loops of the examined soft magnetic materials could be modelled using the proposed description with accuracy sufficient for engineering purposes. Different anisotropy levels of the considered materials could be modelled by an appropriate choice of the value of the angular momentum quantum number  $J$ . The form of the magnetization-dependent  $R(m)$  function was updated upon the change of  $J$  value. The form of model equations was thus related to the magnetic anisotropy (or its lack) of the examined material. For estimation of model parameters a robust “branch and bound” optimization algorithm was used. It should be stressed that the model identification issue was difficult, because the search domain for the  $J$  parameter was discrete, whereas for other parameters it was continuous. Another constraint for exhaustive search in the considered six dimensional space was the prohibitive estimation time, especially for higher values of  $J$  parameter. The use of “branch and bound” optimization routine as a black-box allowed us to obtain solve the estimation issue successfully within a reasonable time.

The forthcoming research shall be focused at description of minor loops in the considered soft magnetic materials with the presented approach.

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