THE ARCHIVE OF MECHANICAL ENGINEERING

VOL. LX 2013 Number 2

10.2478/meceng-2013-0012

Key words: Shell Eco-Marathon, fuel economy

PAWEŁ SULIKOWSKI [*], *RYSZARD MAROŃSKI* [**]

QUASI-PERIODIC CONTROL TECHNIQUE FOR MINIMIZING FUEL CONSUMPTION DURING RECORD VEHICLE COMPETITION

The problem of the optimal driving technique during the fuel economy competition is reconsidered. The vehicle is regarded as a particle moving on a trace with a variable slope angle. The fuel consumption is minimized as the vehicle covers the given distance in a given time. It is assumed that the run consists of two recurrent phases: acceleration with a full available engine power and coasting down with the engine turned off. The most fuel-efficient technique for shifting gears during acceleration is found. The decision variables are: the vehicle velocities at which the gears should be shifted, on the one hand, and the vehicle velocities when the engine should be turned on and off, on the other hand. For the data of students' vehicle representing the Faculty of Power and Aeronautical Engineering it has been found that such driving strategy is more effective in comparison with a constant speed strategy with the engine partly throttled, as well as a strategy resulting from optimal control theory when the engine is still active.

1. Introduction

Modern motor companies design new cars mostly taking into account relatively small operation costs. Such tendencies are enforced by competition on the market. Also the fuel companies, including Royal Dutch Shell, propagate technologies reducing the fuel consumption by cars. The new ecological technologies are tested during different events, including the annual competition called the Shell Eco-Marathon. Its aim is to encourage young enthusiasts to design vehicles minimizing the fuel consumption. The competition is divided into two categories: the prototype vehicles and the urban cars. Each category is divided into classes with respect to the engine used

---

[*] *Student of Faculty of Power and Aeronautical Engineering, Warsaw University of Technology; E-mail: paw.sulikowski@gmail.com*

[**] *Institute of Aeronautics and Applied Mechanics, Warsaw University of Technology; E-mail: maron@meil.pw.edu.pl*

(piston engine or electrical one) and the fuel (gasoline, ethanol, gas, oil). The Students' Vehicle Aerodynamics Association affiliated at the Faculty of Power and Aeronautical Engineering, Warsaw University of Technology, participates in the category of prototype vehicles powered by gasoline piston engines.

The success in this competition depends on two elements. The first one is the optimal construction of the minimal mass vehicle, generating minimal aerodynamic and rolling losses, using a high efficient engine. The second element is the strategy employed during the competition. This also may be optimized. The following paper is devoted to that second element.

In the paper by Rogowski and Maroński (2009) the problem of the optimal driving strategy during the fuel economy competition is considered. The vehicle is regarded as a particle moving on the trace with a variable slope angle. This version of the vehicle is not equipped with a gear-box (the transmission ratio is constant). The velocity is controlled by the power setting. The problem is formulated in the optimal control approach and solved using a direct pseudospectral Chebyshev's method (Fahroo and Ross, 2002). This approach, despite being correct from the methodological point of view, gives the result worse than that obtained during the real event in 2006, Nogaro, France. Therefore, the minimum-fuel problem is reconsidered using the assumptions better adjusted to the real competition, the data for the current version of the students' vehicle and the data for the new trace in Lausitz, Germany.

2. Problem formulation

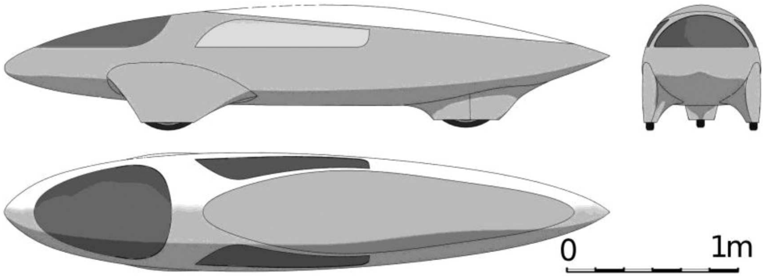
Fundamental assumptions of the mathematical model are as follows (cf. Rogowski and Maroński, 2009):

1. The record vehicle is regarded as a particle.
2. The motion takes place in the vertical plane. The shape of the trace is described by a given smooth function h(x) of the horizontal coordinate x.
3. The vehicle is equipped with a gear-box, therefore the optimal shifting strategy should be computed. This is the first fundamental difference in comparison with the paper by Rogowski and Maroński (2009).
4. The brakes are not used.
5. The velocity of the vehicle is quasi-periodically controlled via turning on and off the engine. When the engine is running it generates its maximal power (no engine throttling). The vehicle velocities when the engine should be turned on and off are not known. This is the second fundamen-

tal difference in comparison with the paper by Rogowski and Maroński (2009).

6. The wind and any other random factors, like the existence of another competitors, are not considered.
7. The mass of the vehicle is constant. The amount of the fuel consumed is insignificant in comparison with the total mass of the vehicle. This assumption is not valid in the aircraft dynamics (Panasz and Maroński, 2005).
8. The local slope of the trace β is small, therefore $\cos\beta \approx 1$.
9. The indicators on the dashboard show: the vehicle velocity, the rotational speed of the engine, the covered distance and the time from the start. Basing on these parameters, the optimal strategy should be realized by the driver during the event.

Assumption 5 needs an explanation. In the paper by Rogowski and Maroński (2009), the optimal result is about 37% worse in comparison with the result obtained during real competition in Nogaro, France, 2006. It is due to the assumption that the engine is constantly active during the run and the velocity is controlled via throttling the engine power. The diagram of the specific fuel consumption versus rotational speed and power setting shows that the specific fuel consumption takes minimal value close to the maximal power setting and it is relatively high for minimal power setting (cf. Fig. 3 in Rogowski and Maroński, 2009). It gave us the impulse for further analysis of the problem.



	symbol	value	unit
vehicle's mass	m_0	46	kg
driver's mass	m_k	64	kg
frontal area	A	0,3743	m^2
wheel diameter	-	0,479	m

Fig. 1. Students' record vehicle "Droplet" ("Kropelka" in Polish) and its geometrical and mass data

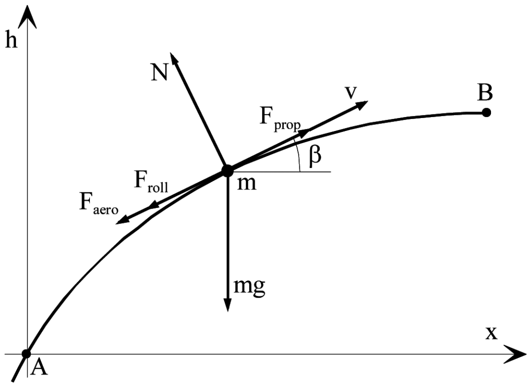


Fig. 2. Particle model of the record vehicle. The forces exerted on the vehicle are: F_{aero} – aerodynamic drag, F_{roll} – rolling resistance, F_{prop} – forward propulsive force, N - normal reaction of the ground, mg - weight. A and B are initial and final points respectively, h(x) is the shape function of the trace, x is the horizontal coordinate (approximately covered distance), m is the mass of the vehicle with the driver, v is the velocity of the vehicle, β is the local slope of the trace

The equations of vehicle motion resulting from the second Newton law are as follows (cf. Arczyński, 1994; Maroński, 1999; Maroński, 2002; Rogowski and Maroński, 2009)

$$m\,\frac{dv}{dt} = F_{prop} - F_{aero} - F_{roll} - F_g\,, \tag{1}$$

$$\frac{dx}{dt} = v\cos\beta \approx v, \tag{2}$$

where:

$$F_g = m\,g\sin\beta, \tag{3}$$

$$F_{roll} = N\;C_{roll} = m\,g\cos\beta\,C_{roll} \approx m\,g\,C_{roll} = \text{const}\,., \tag{4}$$

$$F_{aero} = 0.5\,\rho\,A\,C_D\,v^2, \tag{5}$$

$$F_{prop} = \frac{M\,[\omega\,(v,u)]\;\;\omega\,(v,u)\;\;\eta\,(u)}{v}. \tag{6}$$

The symbols used are: t – time, F_g – gravitational force component onto direction of motion, g – gravitational acceleration, C_{roll} – rolling resistance coefficient, ρ - air density, A – frontal projection area, C_D – drag coefficient, M – torque, ω – engine rotational speed, u – transmission ratio, η – efficiency.

The propulsive force F_{prop} is computed from equation (6) if both of the following conditions are satisfied:

- the engine is turned on,
- the rotational speed of the engine is greater than its minimal value ω_{MIN} =209.3 rad/s (2000 rev/min).

If the first condition is satisfied but the second is not, the clutch is activated. It happens especially at the start to the event. Further details one can find in Sulikowski (2011).

The boundary conditions represent the vehicle velocities at the beginning and at the end of the race:

$$v(t_A) = v_A = 0\,, \qquad v(t_B) = v_B = v_{FINISH}\ . \tag{7}$$

Minimized is the total amount of fuel G_p used to cover the given distance from A to B

$$G_p = \int_A^B sfc(v,u)\ M(v,u)\ \omega\,(v,u)\ \eta\,(u)\ dt\ , \tag{8}$$

where sfc is the specific fuel consumption. The integrand in (8) attains zero as the engine is not active.

From the regulations of the competition it follows that the average velocity v_{AV} should be no less than 30 km/h, $v_{AV} \geq 30$ km/h. According to equation (5), the aerodynamic drag rapidly increases with a velocity, therefore in the presented reasoning the inequality constraint is replaced by the equality constraint

$$v_{AV} = \frac{L}{t_B - t_A} = 30\,km/h = 8.33\,m/s, \tag{9}$$

where L is the total length of the trace [m].

The goal of the analysis is minimization of the overall fuel consumption during Shell-Eco Marathon represented by the integral (8). The conditions (1)-(7) and (9) should be satisfied. Due to assumption 5, the vehicle control function is quasi-periodic. It means that the distance is divided into sections. Each of them contains two phases: the active phase (acceleration), where the engine propels the vehicle with a maximal power setting, and the passive phase (deceleration) when the vehicle coasts down. The switching velocities, the maximal v_{MAX} and the minimal one v_{MIN}, during the phases are the same in the whole race and they should be computed. During the first phase (acceleration), the shifting strategy minimizing the fuel consumption should be found. The switching points also depend on the vehicle velocity and this can be easily observed and remembered by the driver. The shifting strategy remains the same in further part of the trace, regardless of the local slope of terrain.

3. Data of the record vehicle

The basic geometrical data of students' vehicle "Droplet" are taken from an existing construction – the current version of the vehicle. The mass data are derived from the protocol after the technical inspection during the event, in May 2010. The frontal projection area is obtained by integrating the area of the vehicle shadow during projection on the plane perpendicular to the direction of motion. Fig. 1 contains these geometrical and mass data.

The estimation of the rolling resistance F_{roll} and the dependence of the velocity on aerodynamic drag (eq. 5) is of primary importance. The share of the aerodynamic and rolling losses is different for a record vehicle in comparison with the classical car (Piechna, 2000), therefore the rolling and aerodynamic resistances have been measured during the series of experiments conducted by the Students' Vehicle Aerodynamics Association. For this study it is assumed that the data from a free coasting down give the better results in comparison with towing. Finally, the rolling resistance is F_{roll} =4.22 N and the drag coefficient is C_D =0.1373. Further details referring to the experiments and the method of estimation one can find in Sulikowski (2011).

The students' record vehicle is powered by the petrol piston engine Honda GX25, having a capacity of 25 cm^3. For the standard regulations of this engine and the maximal power setting, the specific fuel consumption versus rotational speed is given in Fig. 3a. Fig. 3b illustrates the maximal power and the maximal torque versus rotational speed (Honda engines, 2010).

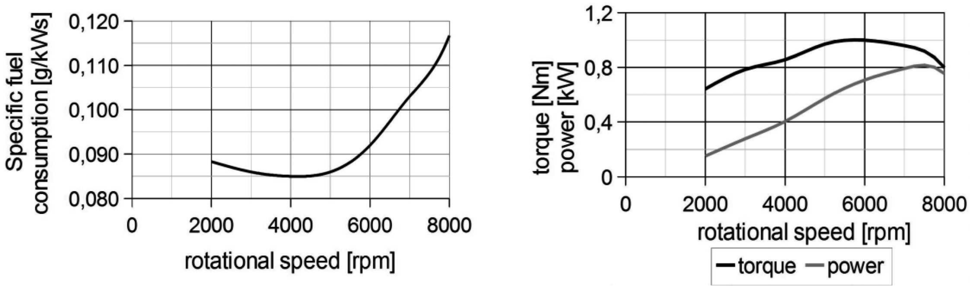


Fig. 3. Engine specific fuel consumption versus its rotational speed, and maximum available engine power and torque versus its rotational speed

The shape of the trace in Lausitz, Germany (elevation of the terrain above horizontal versus distance) is depicted in Fig. 4 (grey line). This figure shows a profile of one lap. During the competition the vehicles cover 8 laps.

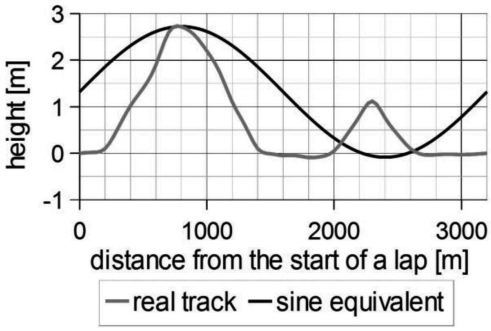


Fig. 4. Shape of the trace (one lap)

4. Optimal shifting strategy

The record vehicle is equipped with two reducers and a gear-box. The first reducer decreases the rotational speed of the engine. Its ratio is constant and equal to u=1.8. Further, the torque is transmitted to the rear wheel hub via the chain transmission, whose ratio is u=7.0. The gear-box within the hub has 14 gears. The ratios and efficiencies for different gears are depicted in Fig. 5 and computed basing on producer's data (Rohloff, 2010).

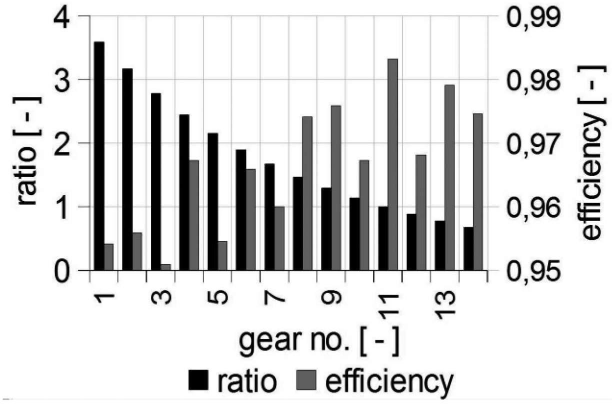


Fig. 5. Ratios and efficiency at different gears

Summing up, the driver may select the actual ratio from a range of 8.316 to 45.108.

The optimization is put into practice under the following assumptions:

- Only one cycle is considered when the vehicle accelerates with maximal power of the engine up to the maximal velocity equal to 15 m/s (this limitation follows from the safety conditions). The vehicle moves on the flat sector of the trace.

- From the construction of the gear-box it follows that shifting the gears is done one by one (any gear cannot be omitted).
- The amount of fuel consumed during acceleration is minimized. Optimized is the number of the gear (one from fourteen) and the vehicle velocity when this gear should be shifted up.

The genetic algorithm, called the simulated annealing, is used for the solution of this problem. It is employed along with the AB-mutation of the variables. The details one can find in Sulikowski (2011), Kirkpatrick et al. (1983), Grygiel (2000).

Up to the year 2009, the "Droplet" was equipped with a transmission with the constant ratio u=14.5. Application of the gear-box lets the engine operate with the rotational speed close to the minimum of the specific fuel consumption (cf. Fig. 3a). Fig. 6 shows the optimal velocities where the gears should be shifted (black squares). For example, for the velocity 2.13 m/s the gear should be shifted from 1 via 2 and 3 to 4. Omitting the gears number 2 and 3 is not possible for this type of gear-box. The solid black line (near horizontal) shows the specific fuel consumption for the vehicle equipped with gear-box. The solid grey line shows the specific fuel consumption for the previous version of the vehicle with constant transmission ratio u=14.5. Acceleration from 0 to 15 m/s using optimal shifting the gears gives 43% saving in comparison with acceleration using a constant transmission ratio (previous version of the "Droplet").

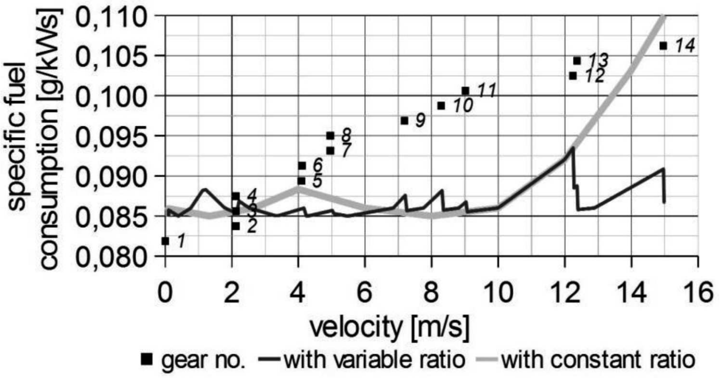


Fig. 6. Optimal shifting the gears from 0 to 15 m/s (black squares). Specific fuel consumption for optimal shifting (black line) and for the constant transmission ratio (grey line)

5. Optimal engine control strategy

Due to assumption 5, the vehicle is powered by the engine that is quasi-periodically turned on and off during the run. The distance may be divided into sections (not necessary equidistant) where the vehicle accelerates with the maximal engine power setting, and where the vehicle coasts down with the engine turned off. Typically, the engine works with its maximal efficiency for the power setting referring to 80÷90% of maximal power for a given rotational speed. The savings in the specific fuel consumption is about 1÷5% in comparison with a non throttled regime (Edgar, 2008). For the sake of simplicity of the driving technique used during the race, it is much more comfortable to use maximal power than a sophisticated throttle control (Rogowski and Maroński, 2009).

It is known from the literature that the vehicle's velocity at the end of the run should be relatively low. Such an element of the strategy is called "the negative kick" in competitive running (Maroński, 1996). It makes it possible to consume the kinetic energy of the vehicle at the finish. The finish velocity $\mathrm{v}_{\mathrm{FINISH}}$ is not known, and it should be computed. The other decision variable is the minimal velocity $\mathrm{v}_{\mathrm{MIN}}$ when the engine is turned on after each phase of free coasting down (the passive, unpowered section). The unknown is also the maximal velocity $\mathrm{v}_{\mathrm{MAX}}$ where the engine is turned off after each acceleration phase. However, this variable may be computed from the equality constraint v_{AV} =30 km/h.

The problem of the optimal engine control strategy is formulated as follows. Find the vehicle velocities ($\mathrm{v}_{\mathrm{MIN}}$, $\mathrm{v}_{\mathrm{MAX}}$, $\mathrm{v}_{\mathrm{FINISH}}$) minimizing the fuel consumed during the event. The distance to be covered is 25 485 m. The strategy assumes the cycles consisting of accelerations with the maximal engine power and decelerations when the engine is not active, therefore the differential equation of motion (1) should be held in each point of the trace. The optimal shifting strategy described in section 4 is employed during acceleration phases. The vehicle starts with zero velocity and the average velocity during the whole race should be equal to 30 km/h. The formulated problem has been solved using an exhaustive search algorithm.

At the beginning, the assumptions and the method have been verified on an idealized sinusoidal trace similar to Lausitz event track (cf. black line in Fig. 4). Two strategies are compared: a simple constant velocity strategy (v=8.33 m/s), and an optimal quasi-periodic strategy. In both cases, the same distance (25 485 m) has been covered in the same time. Computations indicate that for such simplified trace the optimal fuel consumption is 15.81 g and that is equivalent to the distance to be covered 1134 km using 1 liter of fuel. The constant velocity strategy gives the fuel consumption equal to 28.45 g. Such poor result follows from the relatively high specific fuel consumption for partly throttled regimes which are necessary for keeping the vehicle's velocity constant.

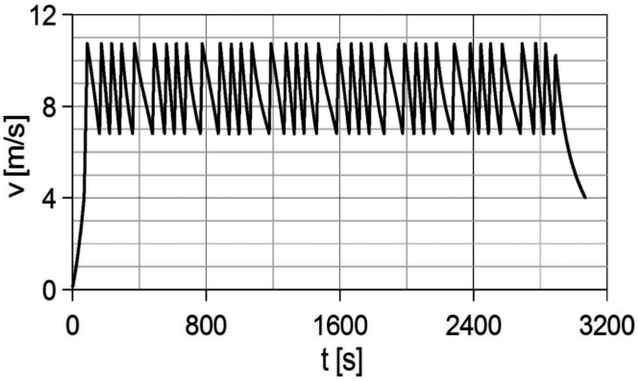


Fig. 7. Optimal vehicle velocity versus time for simplified sinusoidal track

The optimal results for Lausitz event track (grey line in Fig. 4) are depicted in Fig. 8. Here the vehicle accelerates just after the start, then the quasi-periodic strategy is used. The vehicle coasts down at the finish to the velocity v_{FINISH} that is less than v_{MIN}. The disturbances of the periods in the velocity diagram follows from the local slope of terrain that varies with the distance. The engine is turned on 39 times during the record run (excluding the start of the engine at the beginning of the race).

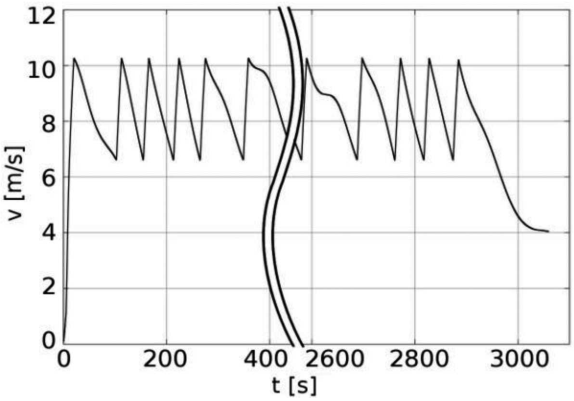


Fig. 8. Optimal vehicle velocity versus time for Lausitz event track

Fig. 9 shows the distance to be covered using 1 liter of the fuel p_2 versus v_{MIN} and v_{FINISH} (v_{MAX} results from the constraint that v_{AV} =30 km/h). It follows from the picture that the optimal result is relatively insensitive to the velocity at the finish v_{FINISH}. It is due to the fact that kinetic energy accumulated in the vehicle's body is relatively small in comparison with the energy necessary for covering the whole distance (more than 25 km).

The quasi-periodic strategy has been compared with the constant velocity strategy. The gear is selected in such a manner that the rotational speed of

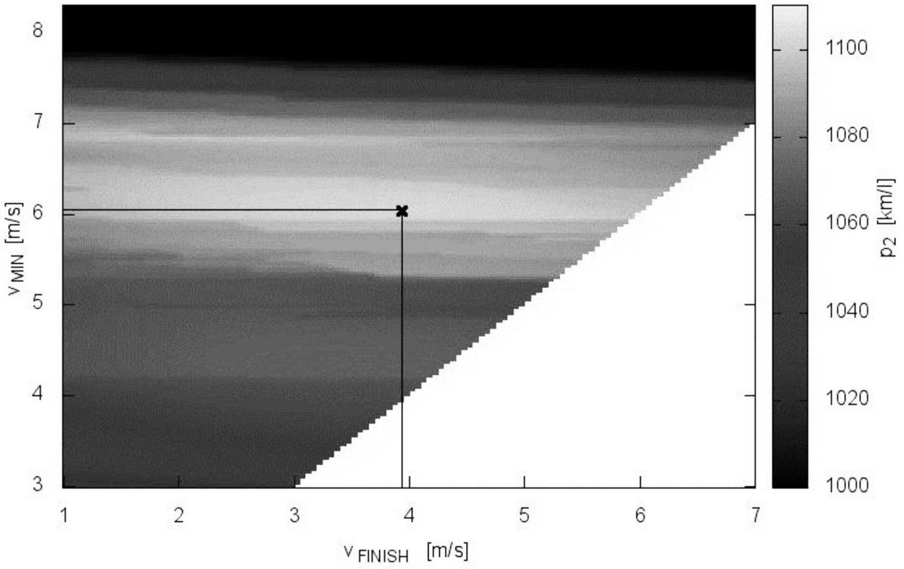


Fig. 9. The distance to be covered using 1 liter of fuel for different v_{MIN} and v_{FINISH}. The cross indicates the optimum

the engine is about 4 000 rev/min. That approximately refers to the minimal specific fuel consumption for the non-throttled regime. The power necessary for vehicle propulsion is about 15% of the maximal power, and it means that the specific fuel consumption is much greater (Edgar, 2008). Computations indicate that such a regime needs about 40% more fuel for covering the same distance with the same average velocity.

The question may arise on the superiority of the quasi-periodic strategy over that resulting from optimal control as far as the fuel consumption is concerned (Rogowski and Maroński, 2009). For comparing both strategies, the quasi-periodic control is computed using vehicle and track data published by Rogowski and Maroński (2009). The transmission ratio is assumed to be constant and the same as the previous one. For such a case, the optimum velocity and the throttle setting are given in Fig. 10. The engine is active only for 20.5% of the total time of the run. Computations indicate that the vehicle is able to cover the distance of 1033 km using 1 liter of fuel, in comparison with 307.4 km/l when using the previous strategy. This example confirms that the quasi-periodic strategy is much better.

6. Conclusions

In this study, the optimal driving strategy during the fuel economy competition is reconsidered. In contrary to the solution by Rogowski and Maroń-

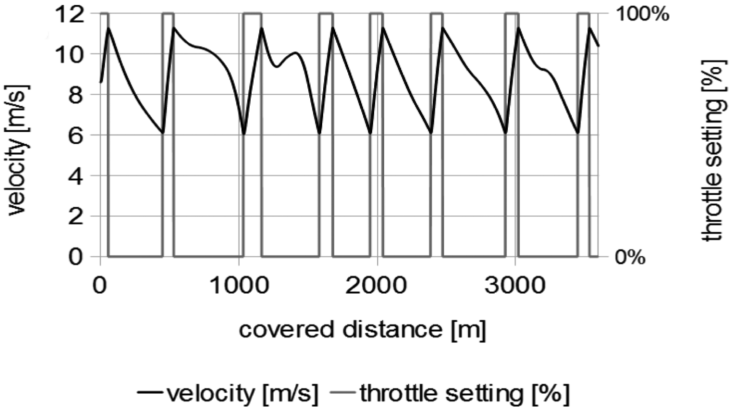


Fig. 10. Optimal velocity and throttle setting versus distance for the quasi-periodic control, and the trace and data published by Rogowski and Maroński (2009)

ski (2009), the quasi-periodic regime is employed. It contains sections where the vehicle accelerates to the maximal velocity v_{MAX} with maximal power setting and then it coasts down to the minimal velocity v_{MIN} where the engine is turned on again. The cycle is repeated as many times as necessary. Shifting the gears is optimized during the acceleration also with respect to the minimum fuel consumption. Comparison of the quasi-periodic strategy with the constant velocity strategy shows about 40% savings of fuel. The quasi-periodic strategy is also significantly fuel-saving in comparison with that one resulting from optimal control (Rogowski and Maroński, 2009). It may be put into practice easily. Here the velocity indications decide how to shift the gear and when the engine should be turned on/off. This is a practical advantage of the quasi-periodic strategy in comparison with a strategy following from an application of the optimal control theory, where the engine should be optimally throttled during the whole race.

Manuscript received by Editorial Board, February 23, 2012;
final version, February 14, 2013.

REFERENCES

[1] Arczyński S.: 1994, *Mechanika ruchu samochodu*, WNT, Warszawa (in Polish).
[2] Edgar J.: 2008, Brake specific fuel consumption, *AutoSpeed*, http://autospeed.com/cms/A_110216/article.html.
[3] Fahroo F., Ross M.: 2002, Direct Trajectory Optimization by a Chebyshev Pseudospectral Method, *Journal of Guidance, Control, and Dynamics*, **25**, 1, 160-166.
[4] Grygiel K.: Warszawa 2000, Algorytmy ewolucyjne z AB-mutacją, Materiały krajowej konferencji „Algorytmy ewolucyjne i optymalizacja globalna" (in Polish).
[5] Honda engines. Technical specifications, active in 2010, http://engines.honda.com/models/model-detail/gx25.

[6] Kirkpatrick S., Gelatt C.D., Vecchi M.P.: 1983, Optimization by simulated annealing, *Science. New Series 220*, nr 4598, 671-680.
[7] Maroński R.: 1999, *Metody rachunku wariacyjnego w biomechanice*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa (in Polish).
[8] Maroński R.: 2002, Optimization of cruising velocity in recreational cycling, *Acta of Bioengineering and Biomechanics*, **4**, Suppl. 1, 552-553
[9] Maroński R.: 1996, Minimum-time running and swimming: an optimal control approach, *Journal of Biomechanics*, **29**, 245-249
[10] Panasz P., Maroński R.: 2005, Commercial aircraft trajectory optimization by a Chebyshev's pseudospectral method, *The Archive of Mechanical Engineering*, **LII**, 1, 5-19.
[11] Piechna J.: 2000, *Podstawy aerodynamiki pojazdów*, WKŁ, Warsaw (in Polish).
[12] Rohloff: active in 2010, http://www.rohloff.de/en/technology/speedhub/efficiency_measurement/index.html.
[13] Rogowski K., Maroński R.: 2009, Driving techniques for minimizing fuel consumption during record vehicle competition, *The Archive of Mechanical Engineering*, **LVI**, 1, 26-35.
[14] Sulikowski P.: 2011, *Optimization of the engine regime for the record vehicle minimizing the fuel consumption during the Shell Eco-Marathon competition*, Praca Dyplomowa Inżynierska, Wydział Mechaniczny Energetyki i Lotnictwa Politechniki Warszawskiej (unpublished).

Quasi-okresowa strategia sterowania minimalizująca zużycie paliwa podczas zawodów pojazdów rekordowych

Streszczenie

Rozważono zagadnienie wyznaczenia optymalnej strategii sterowania pojazdem w czasie zawodów pojazdów rekordowych. Pojazd jest modelowany jak punkt materialny poruszający się po trasie o zmiennym kącie pochylenia. Minimalizowana jest ilość zużytego paliwa potrzebna do pokonania zadanego dystansu w zadanym czasie. Opracowano najbardziej efektywną technikę zmiany biegów w czasie przyspieszania. Założono, że pojazd jest napędzany w dwóch trybach: przyspieszania przy pełnej dostępnej mocy silnika i wybiegu, kiedy to silnik jest wyłączony. Zmiennymi decyzyjnymi są: numer biegu oraz prędkość pojazdu, przy której powinno nastąpić jego przełączenie z jednej strony, oraz prędkości pojazdu, przy których silnik powinien być włączony lub wyłączony, z drugiej. Wykorzystując dane pojazdu studentów reprezentujących Wydział Mechaniczny Energetyki i Lotnictwa Politechniki Warszawskiej wykazano, że taka strategia jest bardziej efektywna niż strategia jazdy ze stałą prędkością z częściowym dławieniem silnika oraz pewna strategia wynikająca z teorii sterowania optymalnego, gdzie silnik jest stale włączony.