Description of matrix converter in frequency domain[*]

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Abstract: The paper presents abilities and advantages following from the use of the harmonic balance method for analysis of steady state of a multiphase system with switching devices on example of a matrix converter. Switching elements are modelled as resistances with step-wise variable parameters, what allows to describe the converter by a linear infinite set of equations. The analysis in frequency domain is presented on example of the one-periodic control strategy. External systems were also added using the Thevenin method approach. The numerical calculation results of a linear equations set were verified by the variable structure method in a time domain and the numerical convergence was confirmed. Furthermore, the exemplary complex system was analysed using the cascade method and current waveforms were obtained.

Key words: harmonic balance method, matrix converter, frequency domain

1. Introduction

Matrix converter (MC) on account of its universality has been investigated by research institutes in the whole world [13]. It is an array of bidirectional switches which are able to connect directly any of input and output phases. To ensure the proper operation conditions some basic switching rules have to be fulfilled by a control strategy [5]. The strategy is crucial and decides about the destination, functionality and operation features of the converter [1].

Quality and effectiveness of the control strategy are assessed by its detailed qualitative and quantitative analysis. The result of this analysis determines usability of the strategy [3]. Kind of the analysis, its application and proper interpretation of its results are of the fundamental importance for assessment of the control strategy. Because of the frequency analysis of that kind of units [4], description of the controller in a frequency domain is desirable [14]. However, the most popular description is in time domain [6]. The reason is that this description is easy to create by using popular programs for modelling. The further analysis, which based on

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time domain waveforms, can be used to obtain frequency relations, but it requires a large number of simulations and these relations aren't precise. In the paper the analysis in a frequency domain is presented on example of the scalar technique, called the "one-periodic control strategy" [8-10], which can be used to convert *N*-phase system into *M*-phase system with different frequency and phase then the input one. It has some important advantages such as: high dynamic and control precision, limited amount of harmonics in input and output signals, the low sampling frequency [7, 11].

These features give possibilities to use the MC with the one-periodic control strategy as a component of a FACTS unit, which allows to control power flow in a transmission line. One of its most interesting applications, is to use it as a power network interconnection device to avoid power flow oscillations resulting from disturbances in a power system. It is of the fundamental importance from a dynamic stability point of view. This application requires a dedicated control strategy, which ensures suitable conditions. They seem to be fulfilled by the one-periodic control strategy, which can control the power flow by shifting the phase angle of a transferred voltage [12].

This paper presents decomposition of the MC time domain equations for individual symmetrical components. Furthermore, it shows the Thevenin method for describing complex systems with the MC and example calculations of a power transmission line.

2. Description of MC in frequency domain

In the analysis bidirectional switching elements are modelled as ideal (switching on and off in an arbitrary time instant) and represented by a set of conductances. Under such assumptions an electric circuit with the topology in the Figure 1 represents the MC, in which pairs of input and output voltages and currents: $(u_{i,n}, i_{i,n})$ and $(u_{o,n}, i_{o,n})$ for $n \in \{1, 2, ..., N\}$, represent external systems. To simplify the analysis the number of input phases is the same as output ones $(N = M)$. This circuit can be rather easily described by the node potential method what leads to the Equations (1).
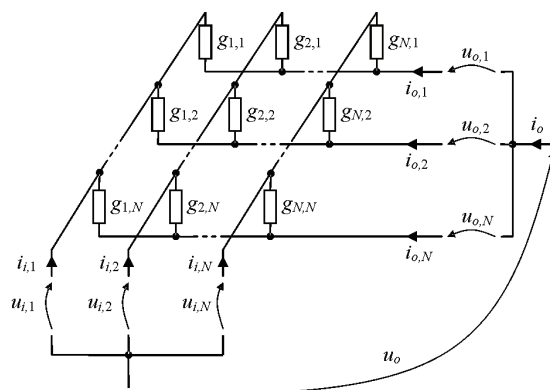

Fig. 1. A circuit representation of the *MC*

$$\begin{bmatrix} \mathbf{i_o} \\ \mathbf{i_i} \\ \mathbf{i_z} \end{bmatrix} = \begin{bmatrix} \mathbf{g_{oo}} & \mathbf{g_{oi}} & \mathbf{g_{oz}} \\ \mathbf{g_{io}} & \mathbf{g_{ii}} & \mathbf{g_{iz}} \\ \mathbf{g_{zo}} & \mathbf{g_{zi}} & \mathbf{g_{zz}} \end{bmatrix} \begin{bmatrix} \mathbf{u_o} \\ \mathbf{u_i} \\ \mathbf{u_z} \end{bmatrix}, \tag{1}$$

where:

$$\mathbf{i_i^T} = [i_{i,1}, i_{i,2}, \ldots, i_{i,N}], \ \mathbf{i_o^T} = [i_{o,1}, i_{o,2}, \ldots, i_{o,N}], \ \mathbf{i_z} = [i_o],$$

$$\mathbf{u_i^T} = [u_{i,1}, u_{i,2}, \ldots, u_{i,N}], \ \mathbf{u_o^T} = [u_{o,1}, u_{o,2}, \ldots, u_{o,N}], \ \mathbf{u_z} = [u_o],$$

$$\mathbf{g_{oo}} = \sum_{n=1}^{N} \mathrm{diag}[g_{n,1}, g_{n,2}, \ldots, g_{n,N}], \ \mathbf{g_{ii}} = \sum_{n=1}^{N} \mathrm{diag}[g_{1,n}, g_{2,n}, \ldots, g_{N,n}],$$

$$\mathbf{g_{oi}} = -\begin{bmatrix} g_{1,1} & g_{2,1} & \cdots & g_{N,1} \\ g_{1,2} & g_{2,2} & \cdots & g_{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{1,N} & g_{2,N} & \cdots & g_{N,N} \end{bmatrix}, \ \mathbf{g_{io}} = \mathbf{g_{oi}^T}, \ \mathbf{g_{oz}} = \sum_{n=1}^{N}[g_{n,1}, g_{n,2}, \ldots, g_{n,N}]^T,$$

$$\mathbf{g_{oz}} = \mathbf{g_{zo}^T}, \ \mathbf{g_{iz}} = -\sum_{n=1}^{N}[g_{1,n}, g_{2,n}, \ldots, g_{N,n}]^T, \ \mathbf{g_{zi}} = \mathbf{g_{iz}^T}, \ \mathbf{g_{zz}} = \sum_{m=1}^{N}\sum_{n=1}^{N} g_{m,n},$$

    To carry out further analysis the conductances, representing switching elements, should be determined as time functions for a steady-state performance. These functions depend on the control strategy of the MC. For open control strategy the states of switching elements are a'priori done, but for close loop control strategy it can be found as a result of the control system operation at a steady-state condition. In [8] and [10] open control strategies are developed, which eliminate short circuits in the MC and which allow to keep symmetry both on the input as well as on the output of the MC. In this paper the control strategy patented in Poland [8] is considered for the MC control, which is named the one-periodic strategy, because all MC switching elements change the states periodically with the period T_s. According to the one-periodic control strategy switches are determined by the state functions in the form of conductance function of time $g(t)$ (Fig. 2). The switching element is in the conducting state, when the state function takes the value G_p and it is in the non-conducting state, when this function equals G_z.
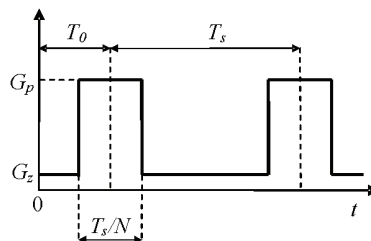

Fig. 2. A basic state function

When the state of the element $g_{1,1}$ is determined by the state function $g(t)$, states of other switching elements $g_{m,n}$ are determined by the state functions (2).

$$g_{m,n} = g\left(t - (m-1)\frac{T_s}{N} - (n-1)\frac{T_s}{N}\right). \tag{2}$$

According to this formula N switches are conducting all the time. Each of them is situated in a different row and column what assures lack of short circuit and provides the current path. The switching period is two times greater than the main frequency of external systems. In order to obtain the same output and input frequencies ($f_{out} = f_{in} = 50$ Hz), the sampling frequency $f_s = 100$ Hz should be chosen.

To change a phase of the output voltages it is enough to shift all state functions by a given time interval T_0, respectively, to the required phase angle β, called the control angle.

It is very useful to introduce the symmetrical components for the further analysis (3) because it leads to simplification of the structure of matrices defined in the Equation (1).

$$\mathbf{u^o} = \mathbf{S_N u_o}, \ \mathbf{u^i} = \mathbf{S_N u_i}, \ \mathbf{i^o} = \mathbf{S_N i_o}, \ \mathbf{i^i} = \mathbf{S_N i_i}, \ \mathbf{i^z} = \mathbf{i_z}, \mathbf{u^z} = \mathbf{u_z}, \tag{3}$$

where:

$$\mathbf{S_N} = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & b & \cdots & b^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & b^{N-1} & \cdots & b^{(N-1)^2} \end{bmatrix}, b = \mathrm{e}^{j\frac{2\pi}{N}}.$$

Resulting equations take the form:

$$\begin{bmatrix} \mathbf{i^o} \\ \mathbf{i^i} \\ \mathbf{i^z} \end{bmatrix} = \begin{bmatrix} \mathbf{g^{oo}} & \mathbf{g^{oi}} & \mathbf{g^{oz}} \\ \mathbf{g^{io}} & \mathbf{g^{ii}} & \mathbf{g^{iz}} \\ \mathbf{g^{zo}} & \mathbf{g^{zi}} & \mathbf{g^{zz}} \end{bmatrix}\begin{bmatrix} \mathbf{u^o} \\ \mathbf{u^i} \\ \mathbf{u^z} \end{bmatrix}, \tag{4}$$

where:

$$\mathbf{i^o} = \left[i_o^0, i_o^1, \ldots, i_o^{N-1}\right]^T, \ \mathbf{u^o} = \left[u_o^0, u_o^1, \ldots, u_o^{N-1}\right]^T,$$

$$\mathbf{i^i} = \left[i_i^0, i_i^1, \ldots, i_i^{N-1}\right]^T, \ \mathbf{u^i} = \left[u_i^0, u_i^1, \ldots, u_i^{N-1}\right]^T,$$

$$\begin{aligned} \mathbf{g^{oo}} &= \mathbf{S_N g_{oo} S_N^{-1}}, & \mathbf{g^{oi}} &= \mathbf{S_N g_{oi} S_N^{-1}}, & \mathbf{g^{oz}} &= \mathbf{S_N g_{oz}}, \\ \mathbf{g^{io}} &= \mathbf{S_N g_{io} S_N^{-1}}, & \mathbf{g^{ii}} &= \mathbf{S_N g_{ii} S_N^{-1}}, & \mathbf{g^{iz}} &= \mathbf{S_N g_{iz}}, \\ \mathbf{g^{zo}} &= \mathbf{g_{zo} S_N^{-1}}, & \mathbf{g^{zi}} &= \mathbf{g_{zi} S_N^{-1}}, & \mathbf{g^{zz}} &= \mathbf{g_{zz}}. \end{aligned}$$

The one-periodic control strategy is symmetrical, what is visible in the formula of state functions creating (2), therefore the Eqation (4) divides into N possible equations sets. The first equations set relates zero order components of external currents and voltages with neutral signals (u_o, i_o).

$$\begin{bmatrix} i_o^0 \\ i_i^0 \\ i_o \end{bmatrix} = NG_0 \begin{bmatrix} 1 & -1 & -\sqrt{N} \\ -1 & 1 & -\sqrt{N} \\ -\sqrt{N} & -\sqrt{N} & N \end{bmatrix} \begin{bmatrix} u_o^0 \\ u_i^0 \\ u_o \end{bmatrix}. \tag{5}$$

Assuming symmetrical external systems this equation can be ignored. The other $N-1$ equations sets relate input signals of an order '*s*' with output signals of an order '*N-s*' for $s \in \{1,..., N-1\}$ (6).

$$\begin{bmatrix} i_o^{N-s} \\ i_i^s \end{bmatrix} = N \begin{bmatrix} G_0 & -g_{N-s}^{oi} \\ -g_{-(N-s)}^{oi} & G_0 \end{bmatrix} \begin{bmatrix} u_o^{N-s} \\ u_i^s \end{bmatrix}. \tag{6}$$

These signals are related by the DC component of the state function G_0. Assuming, that signals of one order (for instance $s = 1$) appear on the input and output side, it is enough to take into consideration only one equations set. The important advantage resulting from this description is that the *N*-phase MC is described in a time domain only by two Equations (6).

In order to proceed to the analysis in a frequency domain all quantities in (6), both node potentials, currents, but also conductances, should be represented by the Fourier series (7).

$$i_o^{N-1} = \sum_{k=-\infty}^{\infty} I_{o,k}^{N-1} e^{jk\Omega t} \quad u_o^{N-1} = \sum_{k=-\infty}^{\infty} U_{o,k}^{N-1} e^{jk\Omega t}$$

$$i_i^1 = \sum_{k=-\infty}^{\infty} I_{i,k}^1 e^{jk\Omega t} \quad u_i^1 = \sum_{k=-\infty}^{\infty} U_{i,k}^1 e^{jk\Omega t} \tag{7}$$

$$g_{(N-1)}^{oi} = \sum_{p=-\infty}^{\infty} G_{-1+pN} e^{j(-1+pN)2\Omega t}$$

$$g_{-(N-1)}^{oi} = \sum_{p=-\infty}^{\infty} G_{1+pN} e^{j(1+pN)2\Omega t},$$

where:

$$G_k = \begin{cases} \dfrac{(G_p - G_z)}{N} \dfrac{\sin\left(\dfrac{k\pi}{N}\right)}{\dfrac{k\pi}{N}} & \text{for } k \neq 0 \\[4mm] G_z + \dfrac{(G_p - G_z)}{N} & \text{for } k = 0 \end{cases}, \quad \beta = 2\pi \dfrac{T_0}{T_s},$$

Relations between harmonics of node potentials, currents and conductances can be found balancing both sides of (6). Using the matrix representation of the Fourier series resulting equations take the form (8).

$$\begin{bmatrix} \mathbf{i^o} \\ \mathbf{i^i} \end{bmatrix} = \begin{bmatrix} \mathbf{G^{oo}} & \mathbf{G^{oi}} \\ \mathbf{G^{io}} & \mathbf{G^{ii}} \end{bmatrix} \begin{bmatrix} \mathbf{u^o} \\ \mathbf{u^i} \end{bmatrix}, \tag{8}$$

where:

$$\mathbf{i^o} = \left[\ldots, I_{o,-1+2N}^{N-1}, I_{o,-1}^{N-1}, I_{o,-1-2N}^{N-1}, \ldots \right]^T$$
$$\mathbf{i^i} = \left[\ldots, I_{i,1+2N}^{1}, I_{i,1}^{1}, I_{i,1-2N}^{1}, \ldots \right]^T,$$

$$\mathbf{u^o} = \left[\ldots, U_{o,-1+2N}^{N-1}, U_{o,-1}^{N-1}, U_{o,-1-2N}^{N-1}, \ldots \right]^T$$
$$\mathbf{u^i} = \left[\ldots, U_{i,1+2N}^{1}, U_{i,1}^{1}, U_{i,1-2N}^{1}, \ldots \right]^T,$$

$$\mathbf{G^{oo}} = \mathbf{G^{ii}} = N \cdot \mathrm{diag}[\ldots, G_0, G_0, G_0, \ldots], \ \mathbf{G^{io}} = \mathbf{G^{oi}}^*,$$

$$\mathbf{G^{oi}} = -N \begin{bmatrix} & \vdots & & \\ & G_{-1} & G_{-1+N} & G_{-1+2N} \\ \cdots & G_{-1-N} & G_{-1} & G_{-1+N} & \cdots \\ & G_{-1-2N} & G_{-1-N} & G_{-1} \\ & & \vdots & \end{bmatrix}.$$

This infinite equations set describes the MC in a frequency domain and allows to obtain important relations. First of all, it defines frequencies of higher harmonics that appear in input and output signals (9).

$$f_k = f \cdot |1 + 2pN|, \text{ where } p \in \{\ldots, -1, 0, 1, \ldots\}. \tag{9}$$

It is also possible to obtain an input-output transfer factor for each individual harmonic. To do this, it is enough to assume ideal switching elements ($G_z = 0$, $G_p = \infty$). From the operation point of view, the most important is the transfer factor for the main harmonic, which is given by the formula (10).

$$C_1 = \frac{\sin\left(\dfrac{\pi}{N}\right)}{\dfrac{\pi}{N}} \mathrm{e}^{j\beta}. \tag{10}$$

Furthermore, from these relations (9), (10) there follow significant conclusions concerning the quality of external signals and the control method [12]. The angle β of the main harmonic of transferred voltages can be easily controlled by shifting all state functions by a given time interval T_0 (Fig. 2). Moreover, the transfer factor may be improved by increasing the number of phases of the MC (N).

3. Analysis of MC with external systems

As it was shown in the Chapter 2, the analysis of the MC in a frequency domain gives many advantages. To analyse the MC in a system, external objects have to be added. The block diagram presented in the Figure 3 shows the modular approach of a system with the MC.
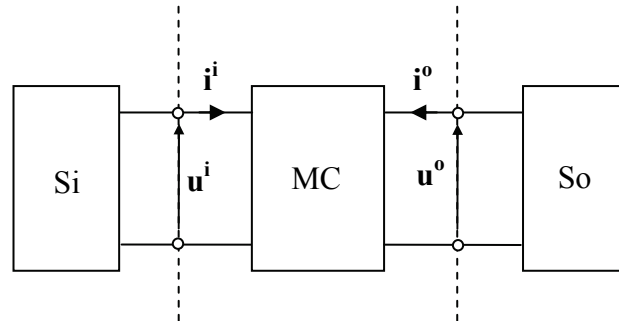
Fig. 3. Modular structure of system with the MC (Si, So – external objects, MC - matrix converter)

Assuming that external objects (Si and So) are symmetrical, what means that they can be described by separate equations for each symmetrical component (linear), the current and voltage signals will contain only higher harmonics generated by the MC (9). Therefore, vectors of input and output current and voltage signals take the form (11).

$$\mathbf{x^r} = \begin{bmatrix} \vdots \\ X^1_{r,1+2N} \\ X^1_{r,1} \\ X^1_{r,1-2N} \\ \vdots \end{bmatrix} \text{ where } \begin{matrix} \mathbf{x} \in \{\mathbf{u},\mathbf{i}\}, X \in \{U,I\} \\ r \in \{i,o\} \end{matrix}. \tag{11}$$

The central object represents the MC, which equations are given in (8). The object called S can be described using the Thevenin approach. Its equivalent circuit contains a Thevenin impedance $\mathbf{Z^{Tr}}$ and a Thevenin voltage source $\mathbf{e^{Tr}}$ (Fig. 4), so it is described by formula (12).
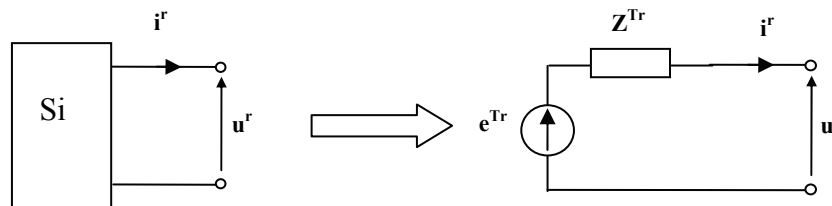

Fig. 4. External object and its Thevenin equivalent circuit

$$\mathbf{u^r} = \mathbf{e^{Tr}} - \mathbf{Z^{Tr}}\mathbf{i^r}. \tag{12}$$

According to the previous assumptions, the matrix $\mathbf{Z^{Tr}}$ will be diagonal but the Thevenin voltage source will contain only one element situated in the middle of the vector (13). This element ($E^1_{Tr,1}$) represents the RMS value of the main harmonic of a positive sequence.

$$\mathbf{Z^{Tr}} = \text{diag}[\ldots, Z_{Tr,1+2N}, Z_{Tr,1}, Z_{Tr,1-2N}, \ldots]$$
$$\mathbf{e^{Tr}} = [\ldots, 0, E^1_{Tr,1}, 0, \ldots]^T. \tag{13}$$

Taking into account two systems (input and output) the equations (14) can be formed.

$$\begin{bmatrix} \mathbf{u^o} \\ \mathbf{u^i} \end{bmatrix} = \begin{bmatrix} \mathbf{e^{To}} \\ \mathbf{e^{Ti}} \end{bmatrix} - \begin{bmatrix} \mathbf{Z^{To}} & \\ & \mathbf{Z^{Ti}} \end{bmatrix} \begin{bmatrix} \mathbf{i^o} \\ \mathbf{i^i} \end{bmatrix}. \tag{14}$$

The equations set of the MC (8) can be easily combine with the equations of external systems (14). As a result, the final equations are obtained (15), which describe the MC with external objects in a frequency domain.

$$\begin{bmatrix} \mathbf{E + G^{oo}Z^{To}} & \mathbf{G^{oi}Z^{Ti}} \\ \mathbf{G^{io}Z^{To}} & \mathbf{E + G^{ii}Z^{Ti}} \end{bmatrix} \begin{bmatrix} \mathbf{i^o} \\ \mathbf{i^i} \end{bmatrix} = \begin{bmatrix} \mathbf{G^{oo}} & \mathbf{G^{oi}} \\ \mathbf{G^{io}} & \mathbf{G^{ii}} \end{bmatrix} \begin{bmatrix} \mathbf{e^{To}} \\ \mathbf{e^{Ti}} \end{bmatrix}, \tag{15}$$

where $\mathbf{E}$ – infinite unitary matrix.

Presented equations confirm that the MC with the one-periodic control keeps symmetry in input and output systems of currents.

4. Numerical tests

The description in a frequency domain gives precise information about the quality of signals, what was shown in the previous chapters. However, it can be also used to the quantitative analysis. To do this, the Equation (15) has been solved for exemplary parameters of 115 kV transmission line and $S_{sc} = 1150$ MW, $k_{XR} = 10$. For this, its dimension has to be reduced to the finite one, but sufficiently high to obtain the satisfactory results. A proper choice of the values G_z and G_p is also very important, what can be done only by numerical tests.

The RMS values of particular harmonics are the direct result of calculations so the steady state characteristic any of them is easy to obtain. In order to compare results, obtained from this analysis (frequency domain) and results obtained from the variable structure method (time domain), figures present signal waveforms in a time domain. This verification is presented on example of the simple system structure shown in the Figure 5. Due to the fact that inductances appear on both sides obtained currents will contain discontinuity. This unreal case is analysed to show the numerical convergence for the worst situation from the numerical calculations point of view.
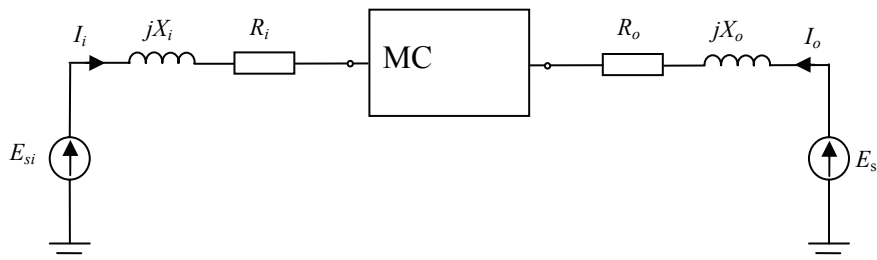

Fig. 5. MC within a transmission line

In this case the *N*-phase symmetrical voltage sources on both sides of the MC are considered. The Thevenin impedances take a simple form (16).

$$Z_{Tr,k} = R_r + j\Omega_k L_r \ \text{ for } k = 1 \pm 2pN.$$ (16)

The Figure 6 presents exemplary waveforms of currents obtained from the above mentioned methods. It is visible that these waveforms differ in the range of low harmonics.
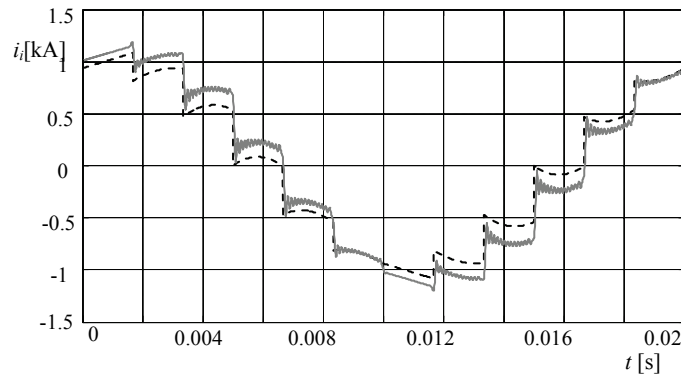

Fig. 6. Input current waveforms obtained from the frequency analysis
(full line) and time analysis (dotted line) for $k \in \{-210, \ldots, 210\}$

Fig. 7. Input current waveforms obtained from the frequency analysis (full line)
and time analysis (dotted line) for $k \in \{-540, \ldots, 540\}$

Better numerical convergence may be obtained by increasing the dimension of the equations set (15). Result of increasing the dimension of almost three times is shown in the Figure 7. The difference between these two waveforms is almost invisible. Despite the fact that Equation (15) is ill-conditioned, using the Gaussian elimination algorithm, with row pivoting strategies, gave satisfactory results.

The next example (Fig. 8) presents a more complex system which consists of: low pass filters (F), transformers converting 3-phase system into 6-phase system (T), transmission lines (L) and power grids (G). Its modular structure is shown in the Figure 9.
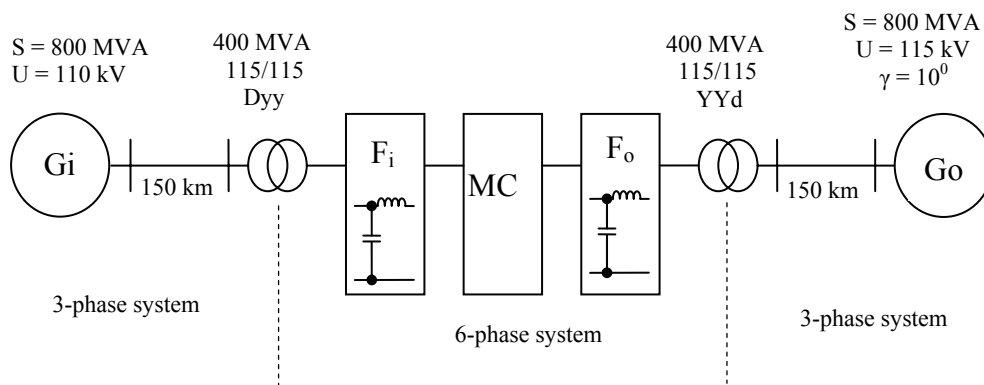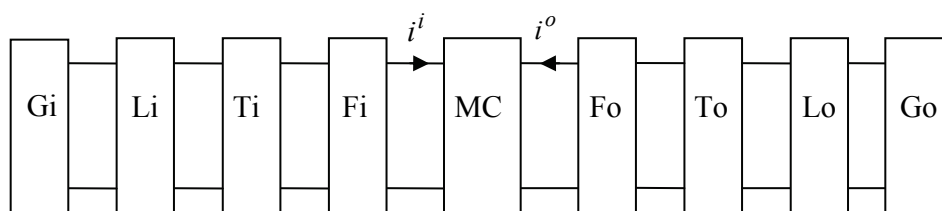
Fig. 8. Complex system with the MC

Fig. 9. Modular structure of the system from Figure 8

In this case the external system S consists of several objects. To introduce them into the MC description it is enough to describe each of them by cascade equations in the form (17).
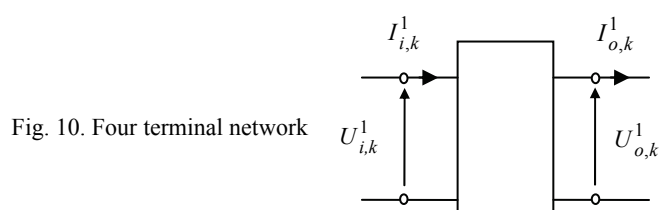

Fig. 10. Four terminal network

$$\begin{bmatrix} U_{i,k}^1 \\ I_{i,k}^1 \end{bmatrix} = \begin{bmatrix} T_{r,k}^{11} & T_{r,k}^{12} \\ T_{r,k}^{21} & T_{r,k}^{22} \end{bmatrix} \begin{bmatrix} U_{o,k}^1 \\ I_{o,k}^1 \end{bmatrix} \tag{17}$$

Below, equivalent circuit models and cascade matrices, for a given frequency Ω_k, of all objects are presented.

The *T*-section low pass filter consists of capacitor (*C*) and inductors (L_1, L_2) what is shown in the Figure 11.
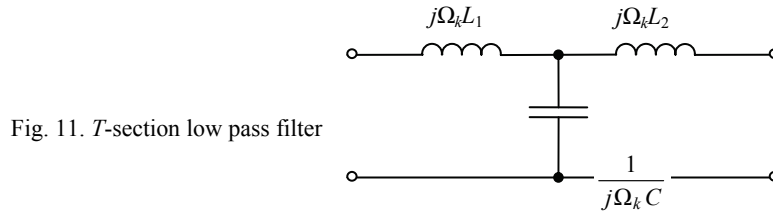
Fig. 11. *T*-section low pass filter

Its cascade matrix is given in (18).

$$
\begin{bmatrix}
1 - \Omega_k^2 L_2 C & -j\Omega_k \left(L_1 + L_2 - \Omega_k^2 L_1 L_2 C \right) \\
-j\Omega_k C & 1 - \Omega_k^2 L_1 C
\end{bmatrix}.
\tag{18}
$$

Transmission line of a length x is represented by the Telegraph Equations. The equivalent circuit of the 3-phase system for signals of positive component in a steady state is presented in the Figure 12.
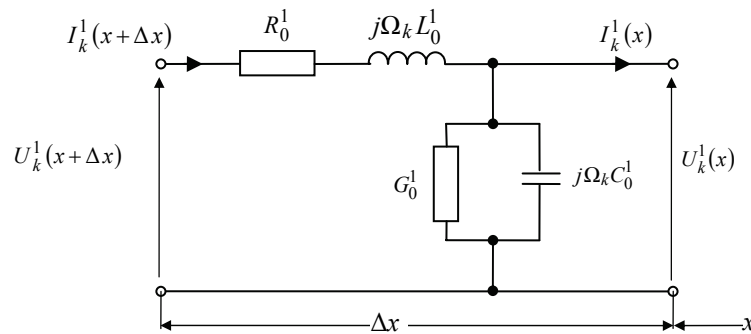

Fig. 12. An equivalent circuit of 3-phase transmission line for a positive sequence

Line parameters: R_0^1 – resistance, L_0^1 – inductance, C_0^1 – capacitance and G_0^1 – conductance are specified per unit length. The cascade matrix is defined by the formula (19).

$$
\begin{bmatrix}
\cosh(\gamma_k x) & Z_{c,k}^1 \sinh(\gamma_k x) \\
\dfrac{1}{Z_{c,k}^1} \sinh(\gamma_k x) & \cosh(\gamma_k x)
\end{bmatrix},
\tag{19}
$$

where:

$$
Z_{c,k}^1 = \sqrt{\dfrac{R_0 + j\Omega_k L_0}{G_0 + j\Omega_k C_0}} \quad \text{– characteristics impedance,}
$$

$$
\gamma_k = \sqrt{\left(R_0 + j\Omega_k L_0 \right)\left(G_0 + j\Omega_k C_0 \right)} \quad \text{– propagation constant.}
$$

The description of coupling transformer depends on its structure. Figure 13 presents the case when the secondary windings are connected in a double star.
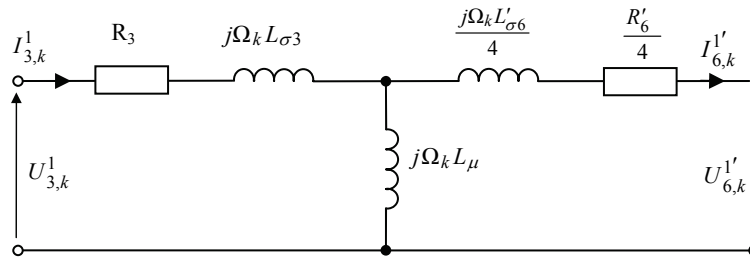

Fig. 13. An equivalent circuit of 3 to 6-phase transformer for positive sequence signals

Parameters of 3-phase primary windings are the resistance R_3 and the leakage reactance $L_{\sigma 3}$. Secondary 6-phase winding is characterized by the resistance R'_6 and the leakage reactance $L'_{\sigma 6}$, which are referred to the primary side by the scaling factor. The magnetizing inductance is marked as L_μ. The formula (20) shows the cascade matrix of this transformer.

$$\begin{bmatrix} R_3 + j\Omega_k\left(L_{\sigma 3} + L_\mu\right) & -j\Omega_k L_\mu \\ j\Omega_k L_\mu & -\dfrac{R'_6 + j\Omega_k L'_{\sigma 6}}{4} - j\Omega_k L_\mu \end{bmatrix} \tag{20}$$

The power grid is represented by a simplified scheme which contains the impedance $Z_{r,k}$ and the voltage source $E^1_{r,1}$ (Fig. 14) related in the formula (21).
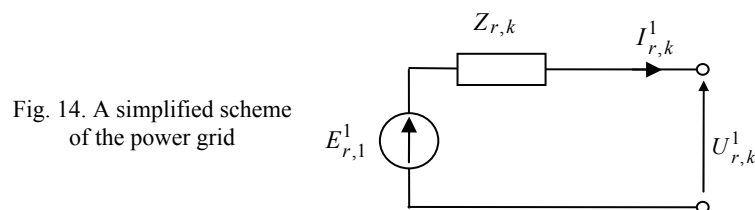

Fig. 14. A simplified scheme of the power grid

$$U^1_{r,k} = E^1_{r,1} - Z_{r,k} I^1_{r,k}. \tag{21}$$

To obtain the Thevenin parameters, all objects described by the cascade matrices (18, 19, 20) have to be joined using the cascade approach and represented by one four terminal network (Fig. 10). Combining equations of this network with the description of power grid (21) leads to the Thevenin impedance $Z_{Tr,k}$ and the Thevenin voltage source $E^1_{Tr,1}$ of the external complex systems (22).

$$Z_{Tr,k} = \frac{Z_{r,k}T^{22}_{r,k} + T^{12}_{r,k}}{Z_{r,k}T^{21}_{r,k} + T^{11}_{r,k}} \qquad E^1_{Tr,1} = \frac{E^1_{r,1}}{Z_{r,1}T^{21}_{r,1} + T^{11}_{r,1}}. \tag{22}$$
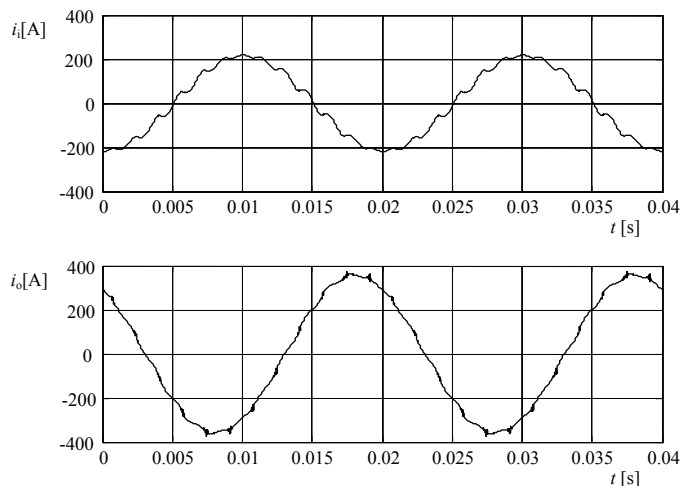
Fig. 15. Current waveforms of the MC system given in the Figure 8
(obtained from the frequency analysis)

The current waveforms for the exemplary parameters ($L_1 = L_2 = 10$ mH, $C = 50\ \mu\text{F}$, $R_0^1 = 0.06\ \Omega\,/\,\text{km}$, $L_0^1 = 1.3$ mH $/$ km, $C_0^1 = 8.5$ pF/km $G_0^1 = 50$ pS/km, $R_3 = R_6' = 0.1\ \Omega$, $L_3 = L_6' = 7$ mH, $L_\mu = 10$ H, $Z_{r,\,k} = (5 + j50k)\ \Omega$) of the analysing system (Fig. 8) are given in the Figure 15.

Using this methodology it is possible to obtain currents (in the form of Fourier coefficients) for any points of the system in its modular structure (Fig. 8). This can be done by rearranging matrices in the main equation (15) according to the chosen points on both sides of the MC.

5. Conclusions

The paper presents abilities and advantages following from the use of the harmonic balance method for an analysis of steady state of circuit with periodically variable parameters on example of the MC. Due to the analysis in a frequency domain the analytical relations between parameters for particular harmonics and signal waveforms in a steady state were determined. The numerical calculation results of the linear equations set were verified by the variable structure method, by comparison of current waveforms in a time domain. The proper choice of parameters and transformations of equations set allow to obtain the numerical convergence.

Such a description of circuit in a frequency domain requires the detailed analysis and also a number of transformations, therefore it might be complicated. Assuming that external systems of the MC are symmetrical, it is simple way to describe the whole system by using the Thevenin method approach. The external systems may consist of several objects (filter, transmission line, power transformer, synchronous generator). In this case the Thevenin parameters can be obtained by describing all objects using cascade equations. Such approach is presented

for the exemplary system and the calculations results in the form of time domain waveforms of currents are shown.

This method is not limited to the analysis of square MC (the same number of input and output phases), this assumption was introduced only to simplify the description in the paper. As the latest investigations showed, it is also possible to model commutation condition changes using the frequency analysis [2]. The above mentioned advantages make this description valuable in systems with the MC.

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