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Modelling concept of N×M matrix converter under periodic control for dynamic states

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Abstract: This paper presents a concept of an N×M Matrix Converter (MC) modeling under periodic control strategy patented in Poland. This strategy allows to change an N-phase input system of voltages and current with the frequency f_i to the M-phase output system with the frequency f_o, maintaining both systems symmetrical and providing small distortions of voltage and current waveforms at rather high frequencies. In this paper the control strategy is extended for dynamic states when one of the frequencies is changed. Matrix converter equations have been derived using the constrain matrix, which is determined by the switch states. The equations have the hybrid form of a multi-port circuit. To simplify these equations the symmetrical components of input and output voltages and currents have been applied. As a result, rather simple equations have been found. They can be interpreted to an equivalent scheme. All considerations are illustrated using an exemplary 6×3 matrix converter.

Key words: Matrix converter, control strategy, symmetrical components, equivalent scheme

1. Introduction

The N×M Matrix Converter (MC) is one of the most promising power electronics unit. It is build with fully controlled switches arranged into M columns and N rows as shown in Figure1.

Recently, there have been a lot of papers devoted to its control, possible applications and investigation of properties [5, 7, 9, 10]. The MC is usually modeled as a parametric circuit, in which switches are represented by resistances changing the value for conducting and non-conducting states according control strategy. Models of systems including MC are rather complicated and its simulations are rather time consuming. In this paper is proposed a methodology, which allows reducing number of equations necessary to model MC at any dynamic condition for a special control strategy, named as Periodic Control Strategy (PCS) [1]. Similar methodology has been successfully applied for analysis of steady-states in the MC under the PCS with symmetrical external systems [13, 14].

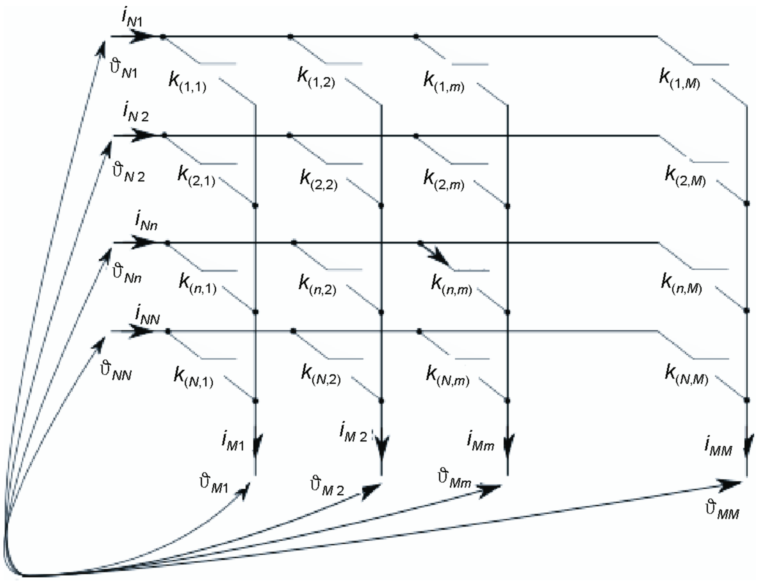

Fig. 1. Topology of N×M matrix converter

2. Periodic Control of MC

Let's the state of each switch of MC in Figure 1 is described by the function $k_{n,m}(t)$, which assumes the value '1' for a conducting state and '0' for a non-conducting one. The subscripts '*n,m*' indicate the switch position in the MC device. Base on it, the state of whole MC can be described by the following matrix $\mathbf{K}(t)$

$$\mathbf{K}(t) = \begin{bmatrix} k_{1,1} & \cdots & k_{1,m} & \cdots & k_{1,\mathrm{M}} \\ \vdots & \ddots & \vdots & \iddots & \vdots \\ k_{n,1} & \cdots & k_{n,m} & \cdots & k_{n,\mathrm{M}} \\ \vdots & \iddots & \vdots & \ddots & \vdots \\ k_{\mathrm{N},1} & \cdots & k_{\mathrm{N},m} & \cdots & k_{\mathrm{N,M}} \end{bmatrix}. \tag{1}$$

Notations of external currents and node potentials are show in Figure 1, the currents of particular switches are denoted by $i_{n,m}$. Under the Periodic Control Strategy (PCS) the states of MC switches change their state according to the general formula

$$k_{n,m}(t) = k_{1,1}\left(t - (n-1)\cdot\frac{\mathrm{T}}{\mathrm{N}} - (m-1)\cdot\frac{\mathrm{T}}{\mathrm{M}}\right) \tag{2}$$

for $n \in \{1,...\mathrm{N}\}$, $m \in \{1,...\mathrm{M}\}$. It means that the state functions of all switches are shifted according to the formula (2) with respect to the state function $k_{1,1}(t)$ of the switch (1,1), shown in Figure 2, assuming that $\mathrm{N} > \mathrm{M}$.

The states of particular switches are changed periodically with the frequency $\mathrm{f_{sf}} = 1/\mathrm{T}$, which relates the frequencies of main harmonics of input and output voltages as follows

$f_N - f_M = \pm f_{sf}$. Specific features of the MC device under the PCS are described also in [2-4]. Shortly, the advantages of such control are: a good shape of input voltages and relatively small switching frequency f_{sf}.

Fig. 2. The state function $k_{1,1}(t)$

The formula (2) describes the elements of the matrix $\mathbf{K}(t)$ for a steady state. An MC device under the PCS features a constant sequence of states. Each state assumes a certain time interval, which in a transient could be changed. So, for the analysis of MC dynamics, the variable time '*t*' should be replaced be the discrete variable '*z*', which determines a number of time interval during which the state of MC device does not change. The variable '*z*' is more convenient for modeling the dynamics of MCs. Using the discrete variable the formula (2) can be rewritten as follows

$$k_{n,m}(z) = k_{1,1}\left(\left(z-(n-1)-(m-1)\cdot \mathrm{p}\right) \bmod \mathrm{N}\right) \tag{3}$$

where $\mathrm{p} = \mathrm{N/M}$ is an integral. Those functions allow writing down the matrix $\mathbf{K}(z)$. It is periodic with respect to the discrete variable '*z*'

$$\mathbf{K}(z) = \mathbf{K}(z + c\cdot \mathrm{N}) \tag{4}$$

because the states of the MC device under the PCS are repeated after every 'N' states (*c* is an integral). The quotient $\mathrm{p} = \mathrm{N/M}$ at PCS should be integral [3, 4]. In an MC application to a 3-phase power system, both numbers N and M should be a multiple of 3.

Exemplary functions $k_{n,m}(t)$ are shown in Figure 3 for an MC of N = 6 and M = 3 at a transient state when the input frequency (for M = 3) changes. The functions of switches in one column are presented in one graph. The functions of switches in one row share the same color in all graphs. The 6×3 MC have only 6 different states, for which the matrix $\mathbf{K}(z)$ takes the following forms

$$\underset{z=1}{\begin{bmatrix}100\\000\\010\\000\\001\\000\end{bmatrix}} \rightarrow \underset{z=2}{\begin{bmatrix}000\\100\\000\\010\\000\\001\end{bmatrix}} \rightarrow \underset{z=3}{\begin{bmatrix}001\\000\\100\\000\\010\\000\end{bmatrix}} \rightarrow \underset{z=4}{\begin{bmatrix}000\\001\\000\\100\\000\\010\end{bmatrix}} \rightarrow \underset{z=5}{\begin{bmatrix}010\\000\\001\\000\\100\\000\end{bmatrix}} \rightarrow \underset{z=6}{\begin{bmatrix}000\\010\\000\\001\\000\\100\end{bmatrix}} \tag{5}$$

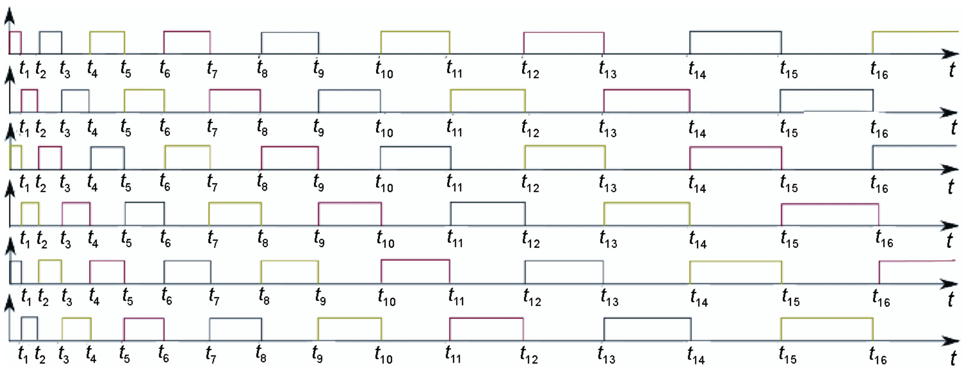

Fig. 3. Example of successive values of functions $k_{n,m}(t)$ in matrix $\mathbf{K}(t)$ for dynamic MC operations

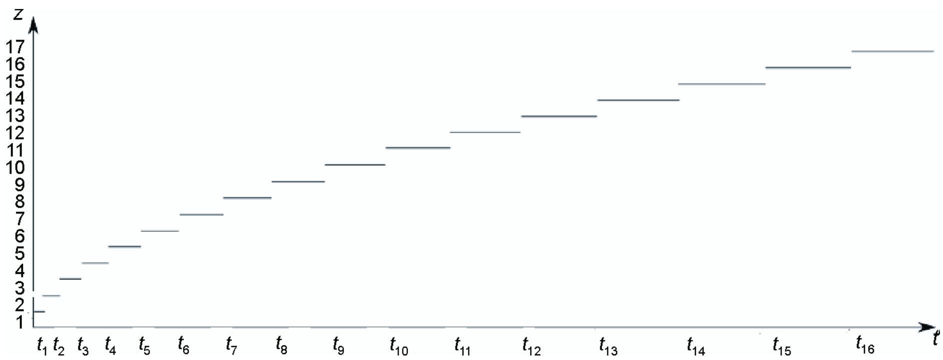

Fig. 4. Assignment variable 'z' to individual time intervals

3. Description of an MC under the PCS

A) Equations in natural coordinates

For the mathematical description the following vectors of external potentials and currents are defined (see Fig. 1):

$$\mathbf{v}_{\mathrm{N}} = \left[v_{\mathrm{N},1}(t)\ v_{\mathrm{N},2}(t) \cdots v_{\mathrm{N},\mathrm{N}}(t)\right]^T,$$

$$\mathbf{i}_{\mathrm{N}} = \left[i_{\mathrm{N},1}(t)\ i_{\mathrm{N},2}(t) \cdots i_{\mathrm{N},\mathrm{N}}(t)\right]^T,$$

$$\mathbf{v}_{\mathrm{M}} = \left[v_{\mathrm{M},1}(t)\ v_{\mathrm{M},2}(t) \cdots v_{\mathrm{M},\mathrm{M}}(t)\right]^T,$$

$$\mathbf{i}_{\mathrm{M}} = \left[i_{\mathrm{M},1}(t)\ i_{\mathrm{M},2}(t) \cdots i_{\mathrm{M},\mathrm{M}}(t)\right]^T$$

The external currents are related to the switch currents $i_{n,m}(t)$

$$i_{\mathrm{N},n}(t)=\sum_{m=1}^{\mathrm{M}} i_{n,m}(t)\;;\; i_{\mathrm{M},m}(t)=\sum_{n=1}^{\mathrm{N}} i_{n,m}(t)$$

Let the elementary switch $n,\ m$ is represented by a circuit revealed in Figure 5, in which $\mathrm{R_c}$ denotes the switch resistance in a conducting state, $\mathrm{G_n}$ is the switch conductance in a non-conducting state and $k_{n,m}$ is an ideal switch.

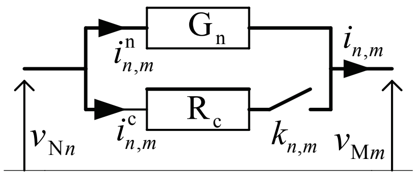

Fig. 5. Representation of an elementary switch

For such a switch representation the MC can be divided into two connected parts in parallel, the first one with the conductances $\mathrm{G_n}$, representing an MC at non-conducting states and the second with the ideal switches $k_{n,m}$ and resistances $\mathrm{R_c}$ representing an MC in the ON condition. Introducing the vectors of the currents conducting and non-conducting parts of an MC

$$\mathbf{i}^{\mathrm{n}}_{\mathrm{N}}=\left[i^{\mathrm{n}}_{\mathrm{N},1}(t)\ i^{\mathrm{n}}_{\mathrm{N},1}(t)\cdots i^{\mathrm{n}}_{\mathrm{N,N}}(t)\right]^T,$$

$$\mathbf{i}^{\mathrm{n}}_{\mathrm{M}}=\left[i^{\mathrm{n}}_{\mathrm{M},1}(t)\ i^{\mathrm{n}}_{\mathrm{M},1}(t)\cdots i^{\mathrm{n}}_{\mathrm{M,M}}(t)\right]^T,$$

$$\mathbf{i}^{\mathrm{c}}_{\mathrm{N}}=\left[i^{\mathrm{c}}_{\mathrm{N},1}(t)\ i^{\mathrm{c}}_{\mathrm{N},1}(t)\cdots i^{\mathrm{c}}_{\mathrm{N,N}}(t)\right]^T,$$

$$\mathbf{i}^{\mathrm{c}}_{\mathrm{M}}=\left[i^{\mathrm{c}}_{\mathrm{M},1}(t)\ i^{\mathrm{c}}_{\mathrm{M},1}(t)\cdots i^{\mathrm{c}}_{\mathrm{M,M}}(t)\right]^T,$$

The vectors of currents fulfill the relations

$$\mathbf{i}_{\mathrm{N}}=\mathbf{i}^{\mathrm{c}}_{\mathrm{N}}+\mathbf{i}^{\mathrm{n}}_{\mathrm{N}},\qquad \mathbf{i}_{\mathrm{M}}=\mathbf{i}^{\mathrm{c}}_{\mathrm{M}}+\mathbf{i}^{\mathrm{n}}_{\mathrm{M}}$$

$$i^{\mathrm{c}}_{\mathrm{N},n}(t)=\sum_{m=1}^{\mathrm{M}} i^{\mathrm{c}}_{n,m}(t)\quad i^{\mathrm{c}}_{\mathrm{M},m}(t)=\sum_{n=1}^{\mathrm{N}} i^{\mathrm{c}}_{n,m}(t) \tag{6}$$

$$i^{\mathrm{n}}_{\mathrm{N},n}(t)=\sum_{m=1}^{\mathrm{M}} i^{\mathrm{n}}_{n,m}(t)\quad i^{\mathrm{n}}_{\mathrm{M},m}(t)=\sum_{n=1}^{\mathrm{N}} i^{\mathrm{n}}_{n,m}(t).$$

The part with the conductances $\mathrm{G_n}$ has a constant structure and can be described using the node potential method

$$\begin{bmatrix}\mathbf{i}^{\mathrm{n}}_{\mathrm{N}}\\ \mathbf{i}^{\mathrm{n}}_{\mathrm{M}}\end{bmatrix}=\mathrm{G_n}\cdot\begin{bmatrix}\mathrm{M}\cdot\mathbf{E}_{\mathbf{N}} & -\mathbf{O}^{\mathbf{T}}_{\mathbf{NM}}\\ -\mathbf{O}_{\mathbf{NM}} & \mathrm{N}\cdot\mathbf{E}_{\mathbf{M}}\end{bmatrix}\begin{bmatrix}\mathbf{v}_{\mathbf{N}}\\ \mathbf{v}_{\mathbf{M}}\end{bmatrix}, \tag{7}$$

where $\mathbf{E}_{\mathbf{N}}$, $\mathbf{E}_{\mathrm{M}}$ are unit matrices with respective dimensions and $\mathbf{O}_{\mathbf{NM}}$ is a matrix with ‘N’ columns and ‘M’ rows, having all elements equal to ‘1’.

To describe the second part of MC, with the resistances $\mathrm{R_c}$ and switches $k_{n,m}$, the matrix $\mathbf{K}(z)$ can be used. It should be noticed that while under at the PCS in that matrix in each column it appears no more than one element equal to '1' and also in each row there is no more than one element '1'. It means that the matrix $\mathbf{K}(z)$ is in fact the constrain matrix for a given MC state and it relate the currents $\mathbf{i}_\mathrm{N}^\mathrm{c}(t)$ to the currents $\mathbf{i}_\mathrm{M}^\mathrm{c}(t)$

$$\mathbf{i}_\mathrm{N}^\mathrm{c} = \mathbf{K}(z)\cdot\mathbf{i}_\mathrm{M}^\mathrm{c}. \tag{8}$$

So, external currents on the 'M' side are generalized variables of that second part of an MC, at $\mathrm{N = p \cdot M}$. On the other hand, the voltage over the switch '*n,m*' in a conducting state can be determined based on the circuits in Figure 5.

$$v_{\mathrm{N},n}(t) - v_{\mathrm{M},m}(t) = \mathrm{R_c}\cdot i_{\mathrm{M},m}^\mathrm{c}(t) \tag{9}$$

The relationships (8) and (9) lead to the following equations for the second MC part

$$\begin{bmatrix}\mathbf{i}_\mathrm{N}^\mathrm{c}\\ \mathbf{v_M}\end{bmatrix} = \begin{bmatrix}\mathbf{0} & \mathbf{K}(z)\\ (\mathbf{K}(z))^T & -\mathrm{R_c}\cdot\mathbf{E_M}\end{bmatrix}\begin{bmatrix}\mathbf{v_N}\\ \mathbf{i}_\mathrm{M}^\mathrm{c}\end{bmatrix} \tag{10}$$

Using such hybrid description the external currents and voltages of an MC are only linked by the matrix $\mathbf{K}(z)$.

B) Equations in multiphase symmetrical components

An $\mathrm{N\times M}$ MC under the PCS maintains the symmetry of topologies for each state generated by switches. So, it can be expected that the application for description symmetrical components should simplified the Equation sets (7) and (10). Multiphase symmetrical components are defined by the matrix

$$\mathbf{S}_\mathrm{R} = \frac{1}{\sqrt{\mathrm{R}}}\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ 1 & \mathrm{b_R} & (\mathrm{b_R})^2 & \cdots & (\mathrm{b_R})^{\mathrm{R}-1}\\ 1 & (\mathrm{b_R})^2 & (\mathrm{b_R})^4 & \cdots & (\mathrm{b_R})^{2(\mathrm{R}-1)}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 1 & (\mathrm{b_R})^{\mathrm{R}-1} & (\mathrm{b_R})^{2(\mathrm{R}-1)} & \cdots & (\mathrm{b_R})^{(\mathrm{R}-1)(\mathrm{R}-1)}\end{bmatrix} \tag{11}$$

where

$$\mathrm{b_R} = \mathrm{e}^{j\frac{2\pi}{\mathrm{R}}}.$$

New variables are defined as:

- symmetrical components of voltage vectors

$$\mathbf{v}^\mathrm{R} = \mathbf{S}_\mathrm{R}\cdot\mathbf{v}_\mathrm{R} \quad \text{for} \quad \mathrm{R}\in\{\mathrm{N,M}\} \tag{12}$$

- symmetrical components of current vectors

$$\mathbf{i}^{\mathrm{s,R}} = \mathbf{S}_\mathrm{R}\cdot\mathbf{i}_\mathrm{R}^\mathrm{s} \quad \text{for} \quad \mathrm{R}\in\{\mathrm{N,M}\} \text{ and } \mathrm{s}\in\{\mathrm{n,c}\} \tag{13}$$

The individual symmetrical components of all vectors, both voltages and currents, and are denoted as follows

$$\mathbf{x}^{\mathrm{R}} = \left[x^{0}(t)\ x^{1}(t) \cdots x^{\mathrm{R}-1}(t)\right]^{T}. \tag{14}$$

New current vectors keep the relations relative to (6)

$$\mathbf{i}^{\mathrm{N}} = \mathbf{i}^{\mathrm{c,N}} + \mathbf{i}^{\mathrm{n,N}}, \quad \mathbf{i}^{\mathrm{M}} = \mathbf{i}^{\mathrm{c,M}} + \mathbf{i}^{\mathrm{n,M}} \tag{15}$$

After recalculations the equations (7) take form

$$\begin{bmatrix} \mathbf{i}^{\mathrm{n,N}} \\ \mathbf{i}^{\mathrm{n,M}} \end{bmatrix} = \mathrm{G_n} \begin{bmatrix} \mathrm{M} \cdot \mathbf{E_N} & -\sqrt{\mathrm{N}\cdot\mathrm{M}} \cdot (\mathbf{O^{NM}})^{T} \\ -\sqrt{\mathrm{N}\cdot\mathrm{M}} \cdot \mathbf{O^{NM}} & \mathrm{N} \cdot \mathbf{E_M} \end{bmatrix} \begin{bmatrix} \mathbf{v^N} \\ \mathbf{v^M} \end{bmatrix}$$

where the matrix $\mathbf{O^{NM}}$ with 'N' columns and 'M' rows takes the form

$$\mathbf{O^{NM}} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & & 0 \end{bmatrix}.$$

So, the set of Eq. (16) is decomposed into equations for individual symmetrical components

$$i_{\mathrm{N}}^{\mathrm{n},n}(t) = \mathrm{M} \cdot \mathrm{G_n} \cdot v_{\mathrm{N}}^{n}(t) \quad \text{for} \quad n \in \{1,2,\ldots,\mathrm{N}-1\} \tag{16a}$$

$$i_{\mathrm{M}}^{\mathrm{n},m}(t) = \mathrm{N} \cdot \mathrm{G_n} \cdot v_{\mathrm{M}}^{m}(t) \quad \text{for} \quad m \in \{1,2,\ldots,\mathrm{M}-1\} \tag{16b}$$

The equations for zero symmetrical components are coupled

$$i_{\mathrm{N}}^{\mathrm{n},0}(t) = \mathrm{M} \cdot \mathrm{G_n} \cdot v_{\mathrm{N}}^{0}(t) - \sqrt{\mathrm{N}\cdot\mathrm{M}} \cdot \mathrm{G_n} \cdot v_{\mathrm{M}}^{0} \tag{16c}$$

$$i_{\mathrm{M}}^{\mathrm{n},0}(t) = \mathrm{N} \cdot \mathrm{G_n} \cdot v_{\mathrm{M}}^{0}(t) - \sqrt{\mathrm{N}\cdot\mathrm{M}} \cdot \mathrm{G_n} \cdot v_{\mathrm{N}}^{0} \tag{16d}$$

Equations (16a, b) represent elementary conductances. Circuit representations of Equations (16c, d) for zero components are presented in Figure 6.

Introducing symmetrical components the Equations (10) take the form

$$\begin{bmatrix} \mathbf{i}^{\mathrm{c,N}} \\ \mathbf{v}^{\mathrm{M}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{K}^{\mathrm{S}}(z) \\ \left(\overset{*}{\mathbf{K}}{}^{\mathrm{S}}(z)\right)^{T} & -\mathrm{R_c} \cdot \mathbf{E}_{\mathrm{M}} \end{bmatrix} \begin{bmatrix} \mathbf{v}^{\mathrm{N}} \\ \mathbf{i}^{\mathrm{c,M}} \end{bmatrix}. \tag{17}$$

The matrix $\mathbf{K}^{\mathrm{S}}(z)$ is obtained as a product of the three matrices

$$\mathbf{K}^{\mathrm{S}}(z) = \mathbf{S}_{\mathrm{N}} \cdot \mathbf{K}(z) \cdot (\overset{*}{\mathbf{S}}_{\mathrm{M}})^{T}.$$

The elements of this matrix are calculated according to the expression

$$k^{n,m}(z) = (\mathrm{b_N})^{(z-1)\cdot m} \cdot \sum_{r=1}^{\mathrm{M}} (\mathrm{b_M})^{(r-1)\cdot(m+n)},$$

which means that non-zero elements appear only when $\mathrm{b_N}$

$$n + m \in \{\ldots,-2\mathrm{M},-\mathrm{M},0,\mathrm{M},2\mathrm{M},\ldots\}.$$

Matrices $\mathbf{K}^{\mathrm{S}}(z)$ for the exemplary 6×3 MC take the following forms

$$\begin{gathered}
\rightarrow \frac{1}{\sqrt{2}} \overset{z=1}{\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \\ 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}} \rightarrow \frac{1}{\sqrt{2}} \overset{z=2}{\begin{bmatrix} 1 & & \\ & \mathrm{b_N} & \\ & & (\mathrm{b_N})^2 \\ (\mathrm{b_N})^3 & & \\ & (\mathrm{b_N})^4 & \\ & & (\mathrm{b_N})^5 \end{bmatrix}} \rightarrow \\
\rightarrow \frac{1}{\sqrt{2}} \overset{z=3}{\begin{bmatrix} 1 & & \\ & (\mathrm{b_N})^2 & \\ & & (\mathrm{b_N})^4 \\ (\mathrm{b_N})^6 & & \\ & (\mathrm{b_N})^8 & \\ & & (\mathrm{b_N})^{10} \end{bmatrix}} \rightarrow \frac{1}{\sqrt{2}} \overset{z=4}{\begin{bmatrix} 1 & & \\ & (\mathrm{b_N})^3 & \\ & & (\mathrm{b_N})^6 \\ (\mathrm{b_N})^9 & & \\ & (\mathrm{b_N})^{12} & \\ & & (\mathrm{b_N})^{15} \end{bmatrix}} \rightarrow \qquad (18) \\
\rightarrow \frac{1}{\sqrt{2}} \overset{z=5}{\begin{bmatrix} 1 & & \\ & (\mathrm{b_N})^4 & \\ & & (\mathrm{b_N})^8 \\ (\mathrm{b_N})^{12} & & \\ & (\mathrm{b_N})^{16} & \\ & & (\mathrm{b_N})^{20} \end{bmatrix}} \rightarrow \frac{1}{\sqrt{2}} \overset{z=6}{\begin{bmatrix} 1 & & \\ & (\mathrm{b_N})^5 & \\ & & (\mathrm{b_N})^{10} \\ (\mathrm{b_N})^{15} & & \\ & (\mathrm{b_N})^{20} & \\ & & (\mathrm{b_N})^{25} \end{bmatrix}}
\end{gathered}$$

Matrices $\mathbf{K}^{\mathrm{S}}(z)$ have quite different forms than matrices $\mathbf{K}(z)$ for a 6×3 MC as shown in (5). Now, the nonzero elements lay in constant positions on the main diagonal of the 3×3 hyper matrices. In a general case, the structure of those matrices depends on the quotient N/M and diagonal matrices with dimensions $\mathrm{M}\times\mathrm{M}$ are repeated N times. Such structures of the matrix $\mathbf{K}^{\mathrm{S}}(z)$ allows for the decomposition of the set of Equations (17) onto M subsets, separately for each of the symmetrical component on the 'M' side.

In the exemplary 6×3 case the MC equations are divided onto three subsets

$$\begin{bmatrix} i_N^{c,0}(t) \\ i_N^{c,3}(t) \\ v_M^0(t) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & (b_N)^{3(z-1)} \\ 1 & (b_N)^{-3(z-1)} & -\sqrt{2}R_p \end{bmatrix} \begin{bmatrix} v_N^0(t) \\ v_N^3(t) \\ i_M^{c,0}(t) \end{bmatrix}, \tag{19a}$$

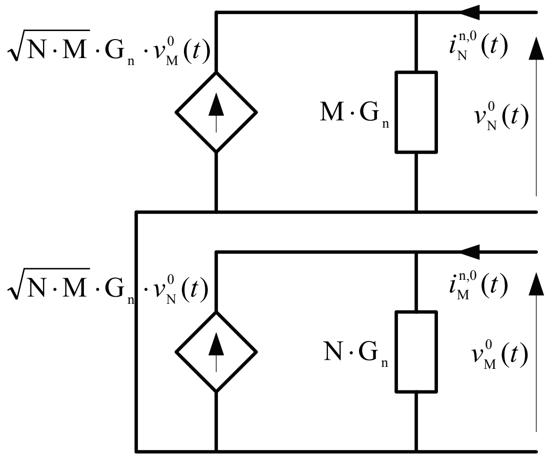

Fig. 6. Equivalent scheme for symmetrical components '0' of non-conductingpart of MC

$$\begin{bmatrix} i_N^{c,1}(t) \\ i_N^{c,4}(t) \\ v_M^1(t) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & (b_N)^{(z-1)} \\ 0 & 0 & (b_N)^{4(z-1)} \\ (b_N)^{-(z-1)} & (b_N)^{-4(z-1)} & -\sqrt{2}R_p \end{bmatrix} \begin{bmatrix} v_N^1(t) \\ v_N^4(t) \\ i_M^{c,1}(t) \end{bmatrix}, \tag{19b}$$

$$\begin{bmatrix} i_N^{c,2}(t) \\ i_N^{c,5}(t) \\ v_M^2(t) \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & (b_N)^{2(z-1)} \\ 0 & 0 & (b_N)^{5(z-1)} \\ (b_N)^{-2(z-1)} & (b_N)^{-5(z-1)} & -\sqrt{2}R_p \end{bmatrix} \begin{bmatrix} v_N^2(t) \\ v_N^5(t) \\ i_M^{c,2}(t) \end{bmatrix}. \tag{19c}$$

The set (19b) is most important as it contain symmetrical components of the order '1', both on the 'N' side and the 'M' side, which appear at symmetrical operation of MC. Two first equations in (19b) can be interpreted as current sources $i_N^{c,1}(t)$ and $i_N^{c,4}(t)$ on the 'N' side, controlled by the current $i_M^{c,1}(t)$ on the 'M' side. The third equation describes the voltage source $v_M^1(t)$ on the 'M' side with the internal resistance R_c, which is controlled by the voltages $v_N^1(t)$ and $v_N^4(t)$ on the 'N' side. Equations (19b) together with the respective Equations (16a, b) can be interpreted as an equivalent MC scheme, shown in Figure 7.

This equivalent scheme shows that an MC under the PCS at symmetrical operations can be described by the first symmetrical components of currents and voltages on both sides. That scheme allows for the addition of any external object (generator, motor, power system etc.) using their representations for the first symmetrical component.

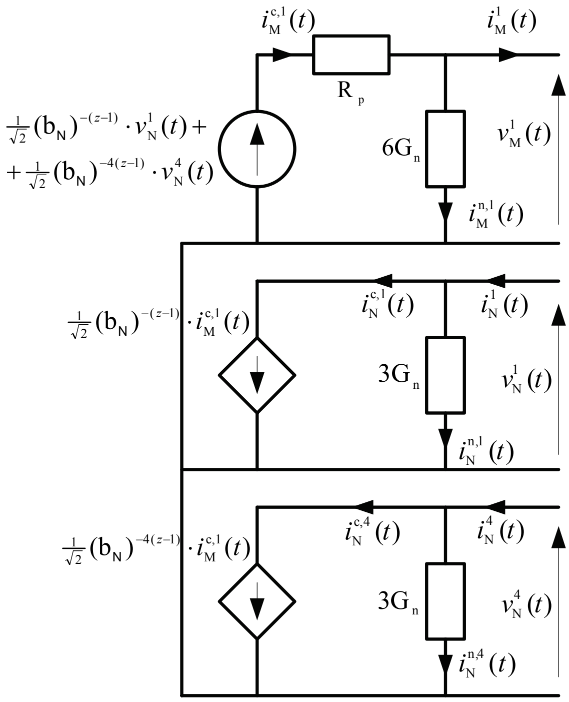

Fig. 7. Equivalent scheme of MC for symmetrical components '1'

4. Conclusions

The paper describes an $N \times M$ matrix converter under the, so called, periodic control for dynamic performances, when the switches change the states non-periodically but the sequence of a matrix converter states is maintained. To omit the problem of aperiodicity of states a discrete variable has been introduced, replacing time, which determines uniquely an actual state of the matrix converter. Specific features of the periodic control scheme allow to introduce constrain matrices relating external currents and voltages on both sides of an $N \times M$ matrix converter for $N = p \cdot M$. Based on these matrices, the hybrid equations of the $N \times M$ matrix converter can be created. Application of multi-phase symmetrical components decomposes those equations onto M sub-sets. The example of a 6×3 matrix converter shows all steps of analysis.

To summarize, in the paper, the most important equations for the symmetrical components '1' on the side 'N' and the side 'M', have been interpreted as an equivalent scheme of matrix converter. It could be useful for modeling the matrix converter as an element of a drive or a power system.

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