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Genetic algorithm coupled with Bézier curves applied to the magnetic field on a solenoid axis synthesis

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Abstract: Electromagnetic arrangements which create a magnetic field of required distribution and magnitude are widely used in electrical engineering. Development of new accurate designing methods is still a valid topic of technical investigations. From the theoretical point of view the problem belongs to magnetic fields synthesis theory. This paper discusses a problem of designing a shape of a solenoid which produces a uniform magnetic field on its axis. The method of finding an optimal shape is based on a genetic algorithm (GA) coupled with Bézier curves.

Key words: Bézier curves, genetic algorithms, magnetic field synthesis, nonlinear inverse problems

1. Introduction

Magnetic field generation of a specified distribution has been discussed in numerous papers but it remains an important problem of contemporary research and applications. Electromagnetic arrangements which create a magnetic field of required distribution and magnitude are widely used in electrical engineering, physics and medical imaging. A certain application of such an electromagnetic system can be a compensation of external magnetic fields in order to protect sensitive electronic apparatus from harmful electromagnetic interferences. This technique is called *active shielding* [1, 2]. Usually, sources of a magnetic field are set of coils and various types of solenoids. The development of new accurate designing methods is still a valid topic of technical investigations. From the theoretical point of view the problem belongs to magnetic field synthesis theory, and in mathematics refers to the linear or nonlinear ill-posed inverse problems [3]. However, there is no sufficiently general method allowing for synthesizing arbitrary magnetic fields. In order to solve linear inverse problem simple Tikhonov's regularization method can be applied [3]. The case considered in this paper is nonlinear and to solve it alternative numerical techniques must be applied. Similar magnetic field synthesis problems have been considered in the past as well but objective functions and design variables have been defined in a different way. For example, in [1] the optimization has been performed for assumed constant solenoid's thickness values, while in [2] the magnetic field has been synthesized in the axisymmetric three-dimensional finite region. This paper is a continuation of [1, 2, 4-5] and discusses a problem of designing the shape of a solenoid which produces a uniform magnetic field on its axis. The method of finding the optimal shape is based on a genetic algorithm (GA) coupled with Bézier curves [6-8].

2. Problem description

Suppose it is required to generate a uniform magnetic field in a certain region which lies very close to the solenoid's axis symmetry (z-axis in cylindrical coordinate system). The task is to find the optimal shape of a solenoid represented by unknown $f(z)$ curve, so that the magnetic field is uniform on the z-axis ($\boldsymbol{H} = H_0\,\mathbf{1}_z$). The inner and outer surfaces of the solenoid are created by the rotation of curves $r = f(z)$ and $r = f(z) + t$ around the z-axis, respectively. The solenoid is of height $2l$ and contains a large number N of tightly wound turns of wire with current, then there is an effective current density J within the solenoid, where $J = NI/2lt$. In such a case the actual solenoid can be replaced by a region carrying a constant current density J (the assumption becomes more accurate as N is increased). The considered arrangement is shown in Fig. 1.

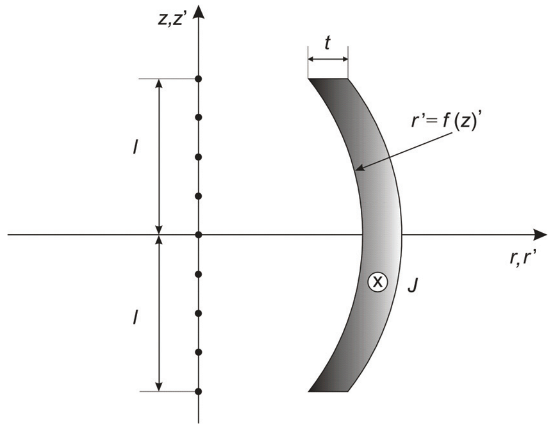

Fig. 1. A definition sketch for a solenoid of unknown shape $r = f(z')$ and solenoid's thickness t

By using the formula for the magnetic field intensity on the circular loop axis, it can be shown that the magnetic field intensity vector on the solenoid axis has a simple form (only z-component exists) [1]:

$$H_z(z) = \frac{J}{2}\int_{-l}^{l}\int_{f(z')}^{f(z')+t} \frac{r'^2 dr' dz'}{\left[\sqrt{r'^2 + (z - z')}\right]^3}. \tag{1}$$

Here, primed variables refer to source points within the solenoid, and the unprimed variable z denotes field points on the z-axis. The integral with respect to r' may be evaluated in closed form and the result is [1]:

$$H_z(z) = \frac{J}{2}\int_{-l}^{l}\left\{\ln\frac{f(z')+t+\sqrt{\left(f(z')+t\right)^2+(z-z')^2}}{f(z')+\sqrt{f(z')^2+(z-z')^2}}+\right.$$
$$\left.+\frac{f(z')}{\sqrt{f(z')^2+(z-z')^2}}-\frac{f(z')+t}{\sqrt{[f(z')^2+t]+(z-z')^2}}\right\}dz'. \tag{2}$$

Taking into account that $H_z = H_0$ (H_0 – desired magnetic field intensity on a solenoid's axis), we obtain:

$$\frac{2H_0}{J} = \int_{-l}^{l} K\left[z,\, z',\, f(z')\right] dz' , \tag{3}$$

where: K is a kernel of the integral equation, z', r' are the coordinates of the source points within the solenoid, z are field points on the z-axis.

The equation (3) is a nonlinear Fredholm equation of the first kind with the unknown function $f(z')$, where the form of K is evident from (2). Its solution gives the required shape of the solenoid. The Fredholm equation of the first kind belongs to the class of ill-posed inverse problems as defined by Hadamard. In order to solve (3) well known iteratively regularized Gauss-Newton can be applied [1]. However, such a deterministic approach is possible, when during optimization process t parameter has an assumed constant value. In order to determine the optimal t value alternative method must be used. In this paper genetic algorithm coupled with Bézier curves based method has been utilized. The design parameters are the coordinates of the Bézier curve control points along with t parameter which can be called solenoid's thickness.

A plane Bézier curve can be defined by the following parametric equation [8]:

$$\begin{Bmatrix} r(p) \\ z(p) \end{Bmatrix} = \sum_{i=0}^{n} \begin{Bmatrix} r_i \\ z_i \end{Bmatrix} B_{i,n}(p) , \tag{4}$$

where: r_i, z_i are coordinates of Bézier curve control points $C_i(r_i, z_i)$, $B_{i,n}(p)$ are Bernstein polynomials and $p \in [0, 1]$.

The optimization process is a determination of the parameters r_i, z_i and t which ensure a minimum of the cost function, which is obtained from the desired magnetic field values (equal to H_0) and the calculated ones at n points along the axis of the solenoid. The cost function is given by (root mean square error):

$$F = \sqrt{\sum_{k=1}^{n}\left(H_{desired,k} - H_{calculated,k}\right)^2} . \tag{5}$$

It is known that in a uniform axisymmetric magnetic field only a half of the solenoid has to be optimized. In this case only the upper part of the solenoid is considered for optimization purposes, while the lower part of the solenoid is built taking into account the symmetry plane of the solenoid. The flowchart of the optimization procedure is shown in Fig. 2.

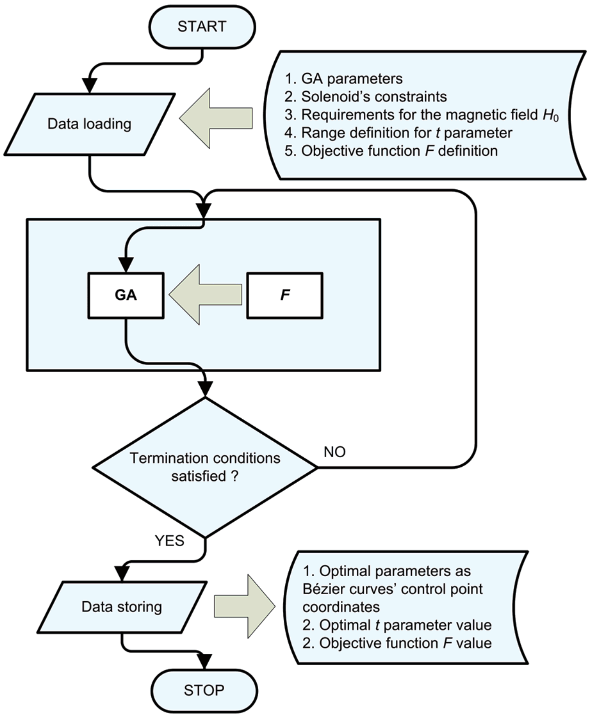

Fig. 2. The flowchart of the optimization procedure based on genetic algorithms and Bezier curves

Table 1. Search space (minimal and maximal values of all parameters under consideration)

Parameter		Lowest value (m)	Highest value (m)
t		0.005	0.015
C_0	r_0	0.020	0.150
	z_0	0.075	0.150
C_1	r_1	0.020	0.150
	z_1	0.060	0.105
C_2	r_2	0.020	0.350
	z_2	0.045	0.090
C_3	r_3	0.200	0.600
	z_3	0.030	0.090
C_4	r_4	0.200	0.600
	z_4	0.015	0.060
C_5	r_5	0.200	0.600
	z_5	0	0.045

At the beginning of the optimization process, GA parameters, requirements for the magnetic field, cost function F, and constraints of the solenoid are defined. Next, the initial population is randomly generated. Cost functions are calculated for all the candidates. A selection process then takes place. It is based on roulette-wheel selection. After the crossover process, mutation takes place. This relies on random changes in candidates. The aim of the mutation is to improve candidates, making them into better solutions. The mutation coefficient should not be high and in all the cases has been set to 0.2. Promising candidates go to the next generation and become a new set of candidate solutions. This process repeats until it stops after reaching a maximum number of iterations.

There are total 13 design parameters in the current optimization problem: r_i, z_i, where $i = 0, 1, 2, 3, 4, 5$ and t parameter. All possible values of the parameters under consideration are shown in Table 1. The total number of generations was equal to 4000.

4. Numerical results

Within the solenoid, the constant current density J equal to 10 A/mm^2 was assumed. The exemplary calculations have been performed for the desired magnetic field H_0 equal to 35, 45, 55 and 75 kA/m, respectively. Each optimization case is represented by two figures. For example, in Fig. 3, on the left, the objective function F values after succeeding generations of GA have been shown, while the relative error δ for the required magnetic field H_0 (in this case equal to 35 kA/m) has been presented in the same figure, on the right. The optimal shape of the solenoid is shown in Fig. 4. Figures 5, 7, 9, on the left, show the objective function F values for the successive desired magnetic fields H_0, and on the right relative errors. Figures 6, 8, 10 present the optimal shapes of the solenoids. In each case the optimal t value (called t_{opt}) is determined.

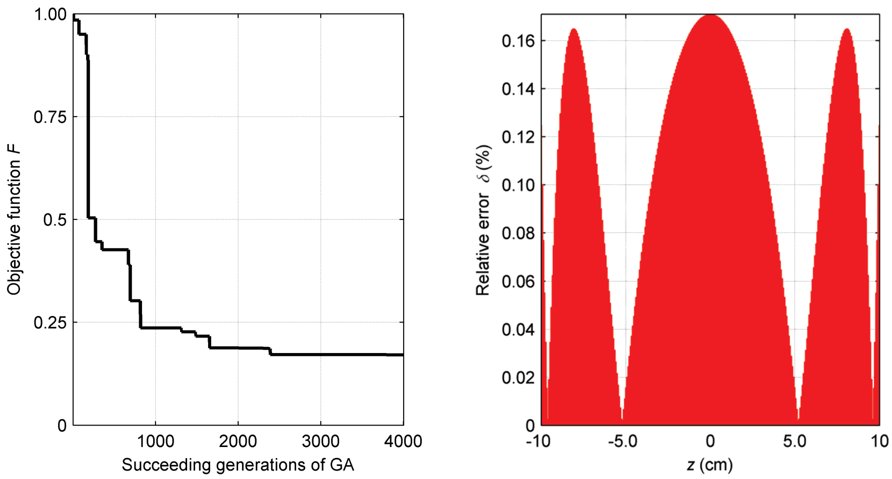

Fig. 3. Objective function F values after succeeding generations of GA for desired magnetic field $H_0 = 35$ kA/m – on the left; relative error δ – on the right

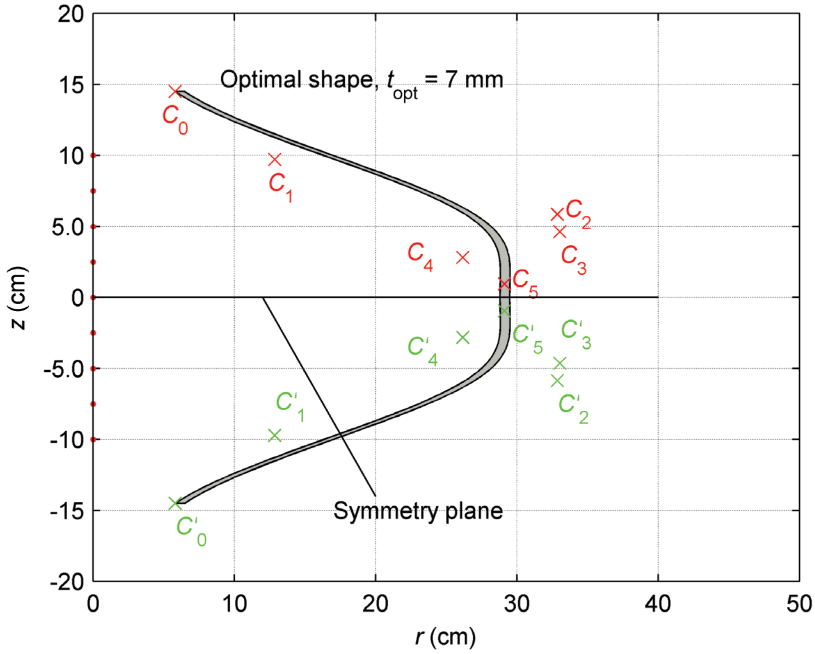

Fig. 4. Optimal shape of the solenoid for desired magnetic field H_0 = 35 kA/m with determined t_{opt} = 7 mm

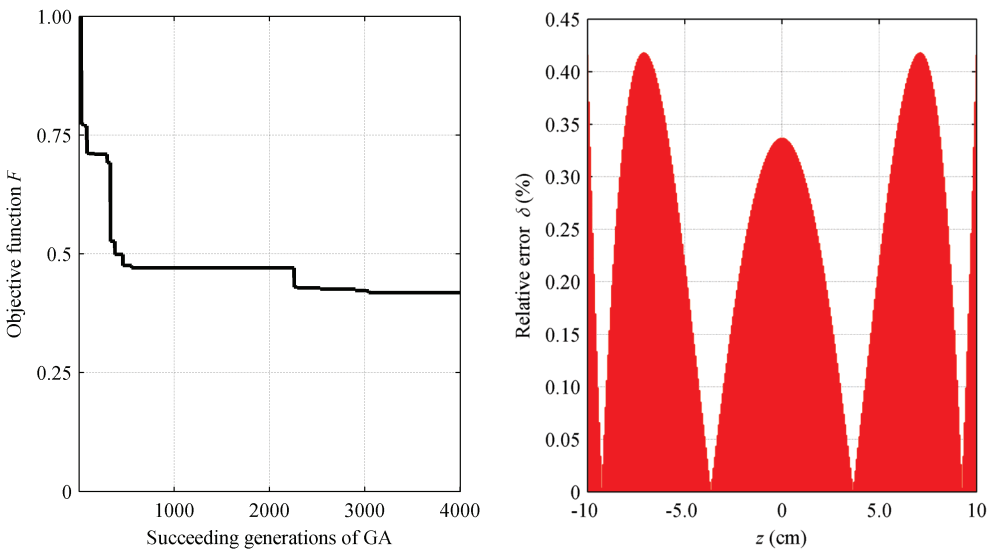

Fig. 5. Objective function F values after succeeding generations of GA for desired magnetic field H_0 = 45 kA/m – on the left; relative error δ – on the right

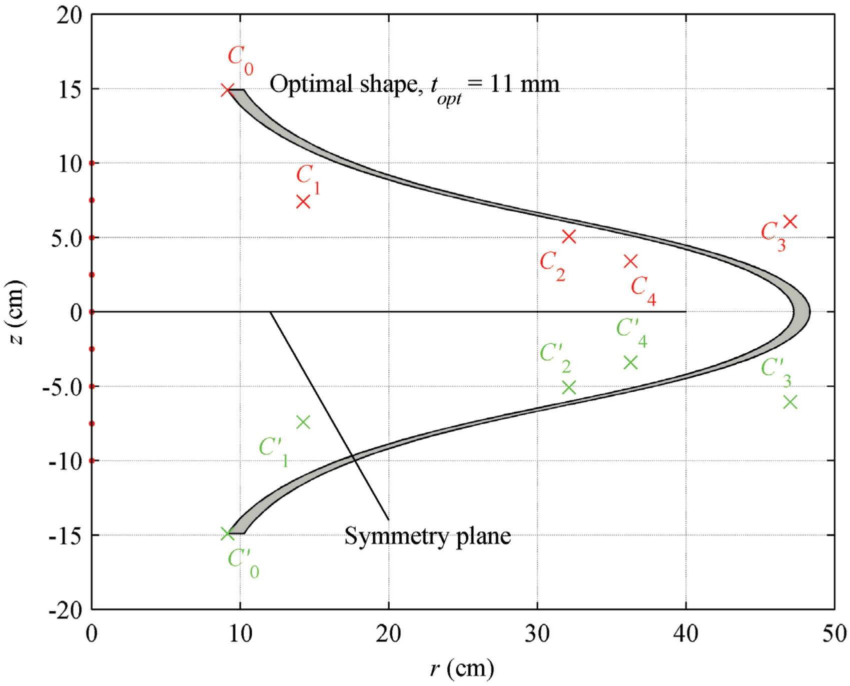

Fig. 6. Optimal shape of the solenoid for desired magnetic field $H_0 = 45$ kA/m with determined $t_{opt} = 11$ mm

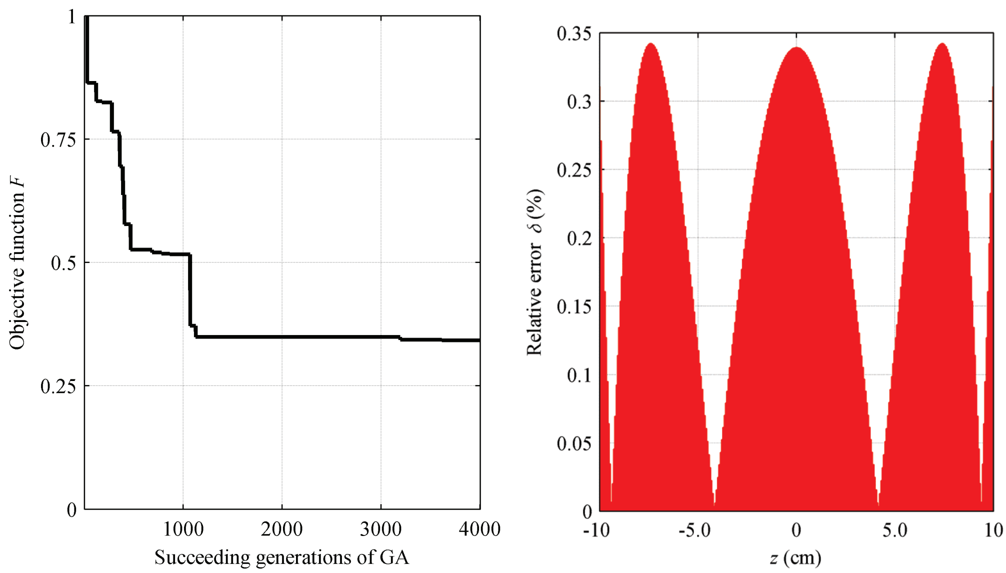

Fig. 7. Objective function F values after succeeding generations of GA for desired magnetic field $H_0 = 55$ kA/m – on the left; relative error δ – on the right

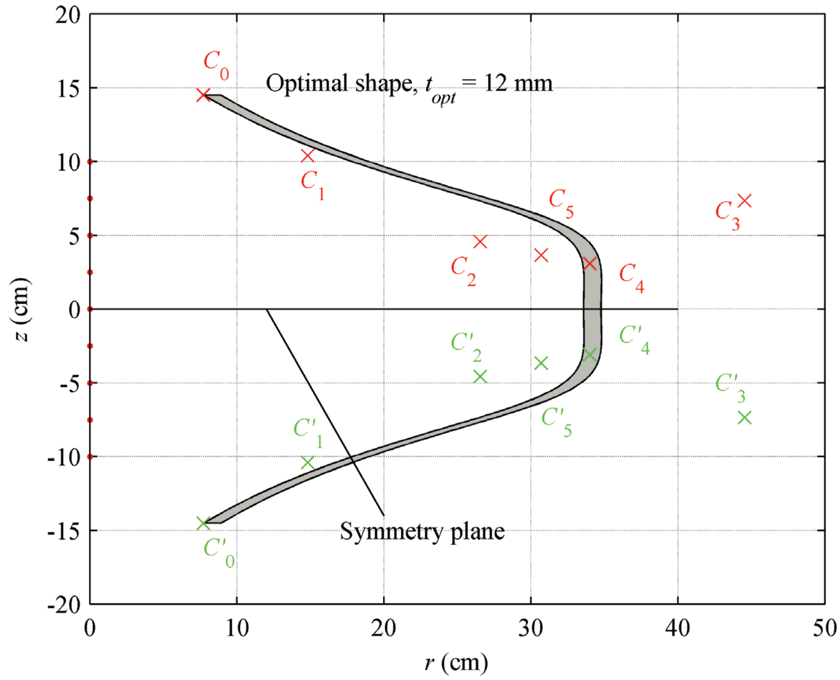

Fig. 8. Optimal shape of the solenoid for desired magnetic field $H_0 = 55$ kA/m with determined $t_{opt} = 12$ mm

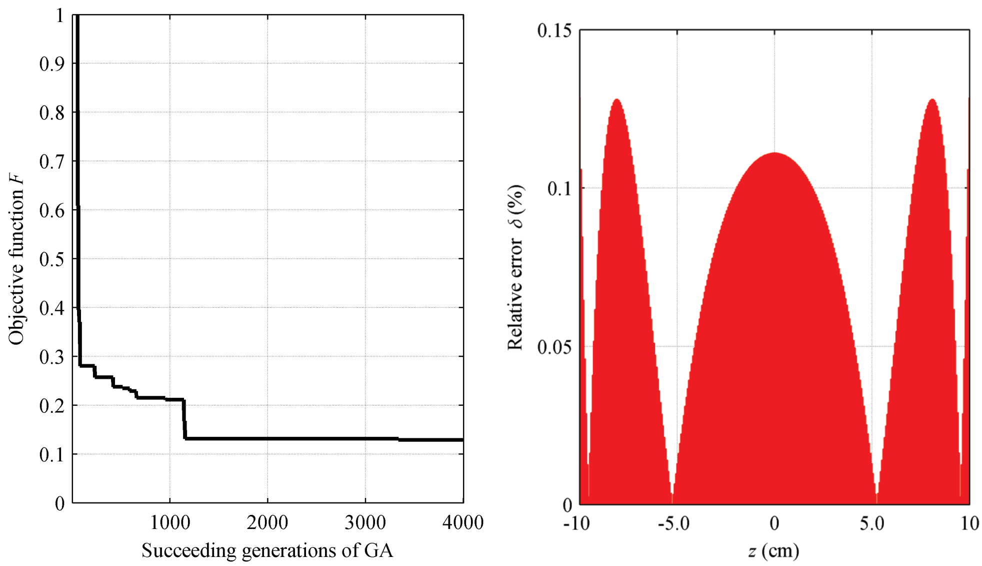

Fig. 9. Objective function F values after succeeding generations of GA for desired magnetic field $H_0 = 75$ kA/m – on the left; relative error δ – on the right

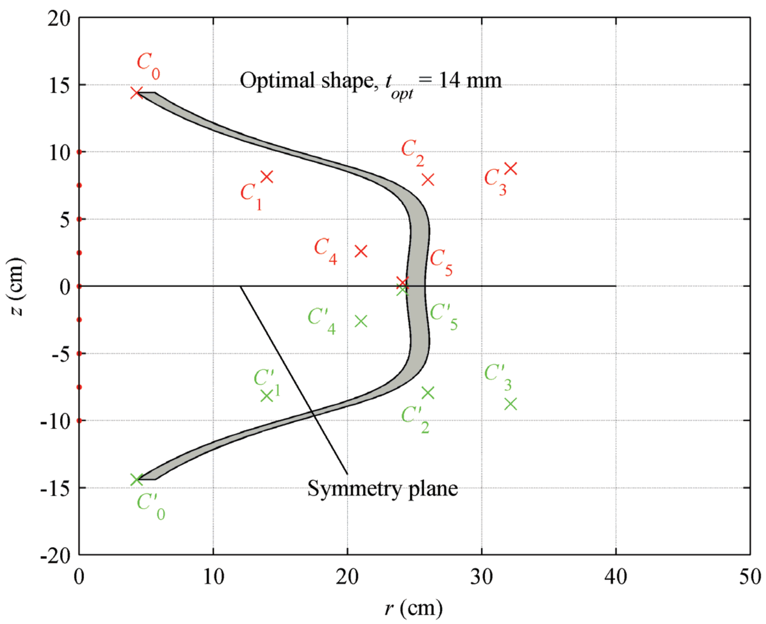

Fig. 10. Optimal shape of the solenoid for desired magnetic field H_0 = 75 kA/m with determined t_{opt} = 14 mm

5. Conclusions

In this paper the synthesis of the uniform magnetic field on the solenoid's axis has been performed. The proposed optimization method utilizes a genetic algorithm as a global minimizer and a rotated Bézier curve as surfaces of the solenoid. For given constraints the relative errors δ and the optimal shapes of the solenoid have been presented. The proposed method allows also to determine the optimal t_{opt} parameter. Although the optimization has been performed on solenoid's axis only, the field uniformity is also valid in a certain volume close to the axis.

References

[1] Ziolkowski M., *Comparison of two methods for magnetic field synthesis on a solenoid's axis*, Archiv fur Elektrotechnik (Electrical Engineering), 93(4): 227-235 (2011).
[2] Ziolkowski M., *Topology optimization of magnetic field in three-dimensional finite region*, Engineering Optimization, 44(6): 673-684 (2012).
[3] Tikhonov A., Goncharsky A., Stepanov V., Yagola A., *Numerical Methods for the Solution of Ill-posed Problems*, Kluwer Academic Publishers, Dordrecht (1995).
[4] Adamiak K., *Tichonov's regularization method applied to the magnetic field on a solenoid axis synthesis*, Archiv fur Elektrotechnik, 59: 19-22 (1977).
[5] Di Barba P., Gottvald A., Savini A., *Global optimization of Loney's solenoid: a benchmark problem*, International Journal of Applied Electromagnetics and Mechanics, 6: 273-276 (1995).

[6] Ziolkowski M., Gratkowski S., *Genetic algorithm and Bezier curves based shape optimization of conducting shields for low frequency magnetic fields*, IEEE Transactions on Magnetics, 44(6): 1086-1089 (2008).
[7] Haupt R., Haupt S., *Practical Genetic Algorithms*, 2nd ed, John Wiley & Sons, Inc., New York (2004).
[8] Piegl L., *Fundamental developments of computer aided geometric design*, CA: Academic Press, San Diego (1994).