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## DYNAMIC INFLUENCES OF NON-STATIONARY LIQUID FLOWS IN FLUID DRIVES OF HEAVY METALLURGICAL MACHINES ON SYSTEM DYNAMICS AND REACTION FOR SURROUNDINGS

### WYZNACZANIE WPŁYWU CIECZY ZAWARTEJ W UKŁADACH NAPĘDOWYCH MASZYN I URZĄDZEŃ HUTNICZYCH NA ICH WŁASNOŚCI DYNAMICZNE I ODDZIAŁYWANIE NA OTOCZENIE

The influence of liquids contained in hydraulic pipes of drives of heavy metallurgical machines, e.g. forging hammers and presses, on reduced mass and system dynamics and forces and moments of reaction for surroundings, was investigated in the paper.

Most of these calculations were performed on the bases of the volumetric flow rate only, without knowing the rate distribution on the pipe cross-section.

Investigations included hydraulic pipes of variable diameters and arbitrary pathways in the space, for the stationary and non-stationary flows.

*Keywords:* non-stationary flows, dynamics of presses and forging hammers, fluid drives, hydraulic sources of vibrations and noise

W szeregu maszyn ciężkich stosowanych w hutnictwie, szczególnie hydraulicznych pras i młotów kuziennych, zasadnicze znaczenie dla przebiegu procesu roboczego posiada ciecz zawarta w układzie napędowym. Duża długość i średnica przewodów hydraulicznych i gwałtowne zmiany natężenia przepływu powodują, że dynamika i sprawność tych urządzeń zależy w sposób dominujący od własności cieczy zawartej w układach napędowych. Np. ciecz robocza stanowić może nawet [1] do 95% ogółu masy zredukowanej i odpowiadać za ponad połowę poboru mocy typowej prasy kuziennej.

W pracy wyprowadzono zależności pozwalające na ściślejsze wyznaczenie parametrów maszyn o napędzie hydraulicznym z uwzględnieniem cieczy zawartej w przewodach hydraulicznych i wyznaczenie oddziaływania dynamicznego armatury hydraulicznej na otoczenie. Zależności te obejmują przypadek przewodów hydraulicznych o zmiennej średnicy i dowolnym przebiegu w przestrzeni i są słuszne dla przepływu stacjonarnego i nieustalonego.

### 1. Introduction

Forces generated by a liquid flux, especially in long pipelines such as fluid drives, water supply and heating systems in buildings, feeder cables of machines and devices, e.g. mining linings, metallurgic presses [1] and hydraulic hammers [2], etc., can significantly influence dynamic processes in these devices [3] as well as can be sources of harmful vibrations and noises [4]. Due to the fact, that pipe fittings constitute a system of distributed parameters, it means of several resonance frequencies, periodical pressure pulsations exciting stationary and transient resonance vibrations are specially dangerous [5].

Dynamic effects accompanying fluxes, originated from such sources as hydraulic pipelines of drive installations and heat centres, hydrofore workshops, boiler-rooms, intermediate pumping stations, etc. [6,7], constitute one of the basic sources of vibrations and noises in buildings [8] and are subjected to strong standard limitations due to noises generated by them [9].

Performing of relevant calculations based on determinations of energy and momentum integrals [10] and angular momentum [11] is usually rendered difficult by a limited knowledge of a flow character in pipelines and must be made on the bases of a volumetric flow rate  $q$  [m<sup>3</sup>/s] only. It is accompanied with a question whether calculations performed in such way are correct, especially integrals of energy, momentum and angular momentum, which constitute the bases for calculations of dynamic influences of a column of liquids.

This problem will be illustrated on the example of determining a liquid kinetic energy in a hydraulic pipe. Knowledge of this energy constitutes, among others, the basis for using the principle of least action, Lagrange's equations (II-nd kind) and the mass reduction method in calculations. As it is known [13], a stream kinetic energy depends on the character of the rate distribution  $v$  on the pipe cross-section. However, in reality, this dependence is different than the one given in the cited paper (this will be shown below).

Let us consider, as an example, an axi-symmetrical flow  $v(r)$ , in case of the Newtonian fluid stationary flow in recti-

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linear hydraulic pipe of a circular cross-section of a diameter  $2R$ . The rate distribution has in this case the paraboloid of revolution character [13]:

$$v(r) = c[1 - (\frac{r}{R})^2] \quad (1)$$

Constant  $c$  can be determined on the basis of the volumetric flow rate  $q$ .

$$q = \int_A v(r)dA = \int_0^R c[1 - (\frac{r}{R})^2]2\pi r dr = \frac{\pi R^2}{2}c \quad (2)$$

Where from:

$$c = \frac{2q}{\pi R^2} = 2\tilde{v} \quad (3)$$

where:  $\tilde{v}$  – average value of a flow rate.

Kinetic energy of a liquid flux of a length  $L$  equals – for each flow:

$$E_L = \int_0^R \frac{1}{2}2\pi r L \rho 4\tilde{v}^2 [1 - (\frac{r}{R})^2]^2 dr = \frac{2}{3}\pi R^2 L \rho \tilde{v}^2 = \frac{2}{3}m\tilde{v}^2 \quad (4)$$

where:  $m$  – mass of a liquid column.

In case of a flow of a homogeneous rate distribution  $v(r) = \tilde{v}$  this energy would be:

$$E_L = \frac{1}{2}m\tilde{v}^2 \quad (5)$$

Thus, the kinetic energy ratio at a laminar flow (4) to the energy (5) – calculated on the basis of the average rate – equals  $4/3$  not  $2$ , as it is given in the cited paper.

It results from the above considerations, that the stream energy depends significantly on the rate distribution and not only on the flow rate. This distribution depends, among others, on a flow character (laminar or turbulent) and on mutual fractions of forces of inertia and internal friction forces in a stream. Since both the flow character and fraction of forces of inertia are – for non-stationary flows – subjected to changes and depend on solutions of equations of motion, which – in turn – depend on a form of expressions of the liquid kinetic energy, it is difficult to estimate *a priori* the distribution rate in a stream, deciding on the kinetic energy of the liquid,  $E_L$ .

## 2. Theorem on the limiting value of the kinetic energy of the liquid

We will presently point out, that **out of all possible rate distributions  $v(r)$  the kinetic energy has the minimum value for the homogeneous distribution:**

$$v(r) = \tilde{v} = \frac{q}{\pi R^2} \quad (6)$$

The proof will be performed by calculus of variations [14] looking for an axi-symmetrical rate distribution in a flux  $v(r)$ , for which energy obtains its extreme value, at a constant flow rate:  $q = \text{const} = q_o$ .

The following functional:

$$E_L = \int dE_l = \int_0^R \frac{1}{2}2\pi r L \rho v^2(r) dr = \pi \rho L \int_0^R r v^2(r) dr \quad (7)$$

is tested at isoperimetric conditions:

$$q = \int_0^R 2\pi r v(r) dr = 2\pi \int_0^R r v(r) dr = q_o = \text{const} \quad (8)$$

An introduction of a multiplier  $\lambda$  allows to build – from integrand expressions – the basic function in a form:

$$H = r v^2(r) + \lambda r v(r) \quad (9)$$

The Euler's condition of extremes existence:

$$\frac{\delta H}{\delta v(r)} - \frac{d}{dr} [\frac{\delta H}{\delta v'(r)}] = 0 \quad (10)$$

leads to a dependence:

$$r2v(r) + \lambda r = 0 \quad (11)$$

After rejection  $r = 0$ , we obtain:

$$v(r) = -\frac{\lambda}{2} \quad (12)$$

Substituting this form into the isoperimetric condition (8) we obtain:

$$2\pi \int_0^R r(-\frac{\lambda}{2}) dr = q_o \text{ from where, after integration:} \quad (13)$$

$\lambda = -\frac{2q_o}{\pi R^2}$  and after substituting to (11), we finally obtain:

$$v(r) = \frac{q_o}{\pi R^2} = \tilde{v} \quad (14)$$

As it is seen, **functional (7) determining a kinetic energy achieves its extreme value for the homogeneous flow with the average rate  $\tilde{v}$  and, as results from the example in item 1, it is the minimum value of this energy.**

Since, in practice, the highest diversity of rates occurs usually in a laminar flow of viscous fluid (Newtonian) –  $v_{\max}(r) = 2\tilde{v}$ , and the smallest in case when damping in the liquid can be omitted in view of forces of inertia fraction ( $v(r) = \tilde{v}$ ), it is possible to determine the range within which the real value of the stream kinetic energy and reduced mass  $m_{red}$  of fluid is included:

$$E_L = (1 \div \frac{4}{3})E_{L\min} \quad (15a)$$

$$E_{L\min} = \frac{1}{2}m\tilde{v}^2 \quad (15b)$$

$$m_{red} = \frac{2E_l}{x^2} \quad (16)$$

where:  $x$  – reduced mass coordinate.

The lower limiting value  $E_{L\min}$  corresponds to the energy of a stream of a mass  $m$  and of a homogeneous rate distribution (14).

### 3. Influences related to energy losses in liquids

Apart from inertial influences, the column of liquid exerts – on cooperating elements (servo-motor pistons, etc.) – influences resulting from flow losses. These losses can be taken into account when calculations either of a generalised or of a reduced force – depending on the applied method – are performed.

Thus, e.g. calculation of power of losses  $N$  in a liquid, on the basis of the flow nominal parameters can be performed as follows [12]:

$$N = q \cdot \left[ \frac{128}{\pi} \rho v \sum_{i=1}^I q \frac{L_i}{d_i^4} + \frac{1}{2} \rho \sum_{j=1}^J \frac{\xi_j}{A_j^2} q^2 \operatorname{sgn}(q) + \frac{1}{2} \rho \sum_{k=1}^K \frac{\lambda L_k}{d_k A_k^2} q^2 \operatorname{sgn}(q) \right] \quad (17)$$

where: **sum of  $i$**  contains pressure losses on pipe elements and on fasteners (treated as additional pipe segments), in which a laminar flow occurs,

**sum of  $j$**  contains pressure losses on resistance elements, while

**sum of  $k$**  contains pressure losses on pipe elements, in which a turbulent flow occurs.

$\rho$  – liquid density,

$v$  – coefficient of kinematic viscosity,

$L, d, A$  (with indices) – denote length, diameter and pipe cross-sectional area – respectively,

$\xi$  – coefficient of local losses,

$\lambda$  – coefficient of linear losses,

– remaining notations as before.

### 4. Momentum of the stream of liquid

The momentum of an elementary segment of the stream of liquid of a length  $dl$  in a pipe of cross-sectional area  $A$  is determined by integral:

$$d\bar{p} = \int_A (\rho \cdot \bar{v} \cdot dl) \cdot dA = \bar{i}_\rho \cdot dl \int_A v \cdot dA = \rho \cdot q \cdot \bar{i} dl \quad (18)$$

where  $\bar{i}$  – versor of a pipe axis

$v$  – rate distribution on a pipe cross-section.

As it results from (18), **the rate distribution on the pipe cross-section does not change the elementary stream momentum, which can be expressed by a flow rate  $q$ .**

The hydraulic pipe example is presented in Fig. 1.

$l \in [0, L]$  – distance of the tested cross-section  $A(l)$  from the pressure source  $A_o$ , measured along the pipe axis,

$A_o, A_L$  – initial and final cross-section area,

$x$  – coordinate determining a piston location,

$O\xi\eta\zeta$  – absolute coordination system of versors:  $\bar{i}_\xi, \bar{i}_\eta, \bar{i}_\zeta$ ,

$\bar{i}(l)$  – pipe axis versor.

The momentum of the liquid contained in the hydraulic pipe between the pump piston and the free surface of the liquid in the hydropneumatic accumulator can be determined by means of the integral:

$$\bar{p}_a = \int_{0L} d\bar{p} = \rho \cdot q \int_0^L \bar{i} dl \quad (19)$$

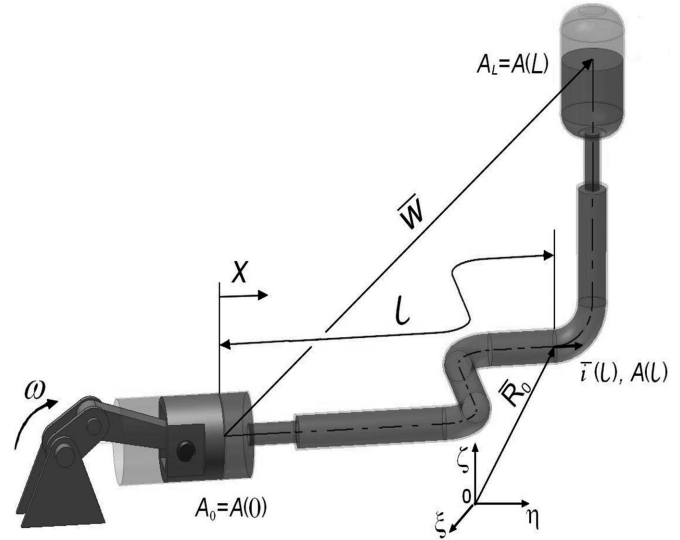


Fig. 1. Computational diagram

As can be seen in Figure 1, the integral occurring in (19) equals vector  $\bar{w}$  running from the piston centre and turning to the centre of the liquid free surface in the accumulator, while the total momentum of the column of liquid equals:

$$\bar{p}_a = \rho \cdot q \cdot \bar{w} = \rho \cdot A_o \cdot \dot{x} \cdot \bar{w} \quad (20)$$

where:  $\dot{x}$  means the piston rate, while  $A_o$  – its surface area.

Thus, the interaction force of the liquid with the hydraulic fittings and piston can be determined from the dependence:

$$\bar{P}_n = \frac{d\bar{p}_a}{dt} = \rho \frac{dq}{dt} \bar{w} = \rho \cdot A_o \cdot \ddot{x} \cdot \bar{w} \quad (21)$$

However, expression (21) does not describe the total change of the momentum. In order to determine this total change, the change of the momentum resulting from the change – in time  $dt$  – of the momentum of the layer being pressed by the piston to the fittings into the momentum of the layer increasing the liquid level in the accumulator, should be found additionally. The elementary momentum increase, resulting from the above considerations equals:

$$d\bar{p}_s = \rho \cdot q \cdot dt \cdot \bar{i}_l \cdot \frac{A_o}{A_L} \frac{dx}{dt} - \rho \cdot q \cdot dt \cdot \bar{i}_0 \cdot \frac{dx}{dt} = \rho \cdot q \cdot dt \cdot (\bar{i}_L \frac{A_o}{A_L} - \bar{i}_0) \frac{dx}{dt} \quad (22)$$

where  $\bar{i}_o, \bar{i}_L$  means versor of the pipe axis at the beginning (piston) and at the end (accumulator) of fittings – respectively, while  $A_o$  and  $A_L$  mark cross-section areas of fittings (initial and final).

Thus, the component of force – with which fittings are influencing the liquid – resulting from this increase equals:

$$\bar{P}_s = \frac{d\bar{p}_s}{dt} = \rho \cdot q \cdot (\bar{i}_L \frac{A_o}{A_L} - \bar{i}_0) \frac{dx}{dt} = \rho \frac{q^2}{A_o} (\bar{i}_L \frac{A_o}{A_L} - \bar{i}_0) \quad (23)$$

$$\begin{aligned} \bar{P} &= -(\bar{P}_n + \bar{P}_s) = -\rho \cdot A_o \cdot \ddot{x} \cdot \bar{w} - \rho \cdot q \cdot (\bar{i}_L \frac{A_o}{A_L} - \bar{i}_0) \dot{x} = \\ &= -\rho \frac{dq}{dt} \bar{w} - \rho \frac{q^2}{A_o} (\bar{i}_L \frac{A_o}{A_L} - \bar{i}_0) \end{aligned} \quad (24)$$

## 5. Angular momentum of the stream of liquid

Let us choose pole  $O$  for the reduction of forces in a certain point of fittings.

The elementary angular momentum of the stream sector of a thickness  $dl$  versus pole  $O$ , can be written as:

$$d\bar{K}_{nO} = \int_A (\bar{R} \times \bar{v} \cdot \rho \cdot dl) dA = \rho \cdot dl \cdot \int_A (\bar{R} \times \bar{v}) dA \quad (25)$$

where distribution of the rate vector  $\bar{v}$  on the cross-section  $A$  is arbitrary, while  $\bar{R}$  is the vector running from pole  $O$  to the elementary sector  $dA$  of the pipe cross-section area  $A(l)$ .

The form of equation (25) indicates, that **the vector of the elementary angular momentum, in general, depends on the rate distribution on the pipe cross-section.**

If (e.g. apart from a pipe curvature) the flow can be considered as being **axi-symmetrical of a rate distribution in axial direction**  $\bar{v}(r)$ , where  $r$  is a distance  $dA$  from the cross-section  $A$  centre (axis), the following occurs:

$$\begin{aligned} d\bar{K}_{nO} &= \rho \cdot dl \cdot \int_A (\bar{R} \times \bar{v}) dA = \rho \cdot dl \cdot \int_A [\bar{R}_0 \times \bar{i}] v(r) dA = \\ &= \rho \cdot [\bar{R}_0 \times \bar{i}] \cdot dl \cdot \int_A v(r) dA = \rho [\bar{R}_0 \times \bar{i}] q \cdot dl \end{aligned} \quad (26)$$

The angular momentum of the axi-symmetrical stream sector of a rate  $\bar{v}$  constitutes a sum of the centre of mass momentum of this sector and the angular momentum in a relative motion around the centre of mass of this sector. The last one equals  $0$  on rectilinear sectors.

$\bar{R}_0$  – is the vector running from pole  $O$  to the axis of the tested cross-section

$\bar{i}$  – is the versor of the pipe axis in the given place.

Thus, it can be seen that, **the elementary angular momentum vector, for the axi-symmetrical flow, does not depend on the rate distribution  $v(r)$  on the cross-section.**

The total angular momentum of the liquid contained inside pipes is determined by the following integral:

$$\begin{aligned} \bar{K}_{n0} &= \int_0^L \rho \cdot q \cdot [\bar{R}_0 \times \bar{i}] dl = \rho \cdot A_0 \cdot \dot{x} \cdot \int_0^L [\bar{R}_0(l) \times \bar{i}(l)] dl = \\ &= \rho \cdot q \cdot \int_0^L [\bar{R}_0(l) \times \bar{i}(l)] dl \end{aligned} \quad (27)$$

The external momentum influencing - from the liquid side - the fittings and piston, related to the angular momentum change of the system due to the flow rate change, is of the following form:

$$\begin{aligned} \bar{M}_n &= \frac{d\bar{K}_n}{dt} = \rho \cdot A_0 \cdot \dot{x} \cdot \int_0^L [\bar{R}_0(l) \times \bar{i}(l)] dl = \rho \cdot A_0 \cdot \dot{x} \cdot \bar{I} = \\ &= \rho \cdot \frac{dq}{dt} \cdot \bar{I} \end{aligned} \quad (28)$$

The computational process of the vector integral  $\bar{I}$  occurring in (28) can be proposed as follows:

– co-ordinate system  $O\xi\eta\zeta$  of the centre in pole  $O$  should be chosen,

– axis pathway of fittings should be approximated by functions  $\xi_R(l), \eta_R(l), \zeta_R(l)$  continuous by intervals with the first derivative, to obtain directly vector coordinates:

$$\bar{R}_0[\xi_R(l), \eta_R(l), \zeta_R(l)] \quad (29)$$

– versor coordinates should be determined as functions  $l$ :

$$\bar{i}[\xi_i(l), \eta_i(l), \zeta_i(l)] \quad (30)$$

where, as can be shown:

$$\xi_i(l) = \frac{1}{\sqrt{\left(\frac{d\xi_R}{dl}\right)^2 + \left(\frac{d\eta_R}{dl}\right)^2 + \left(\frac{d\zeta_R}{dl}\right)^2}} \frac{d\xi_R}{dl} \quad (31a)$$

$$\eta_i(l) = \frac{1}{\sqrt{\left(\frac{d\xi_R}{dl}\right)^2 + \left(\frac{d\eta_R}{dl}\right)^2 + \left(\frac{d\zeta_R}{dl}\right)^2}} \frac{d\eta_R}{dl} \quad (31b)$$

$$\zeta_i(l) = \frac{1}{\sqrt{\left(\frac{d\xi_R}{dl}\right)^2 + \left(\frac{d\eta_R}{dl}\right)^2 + \left(\frac{d\zeta_R}{dl}\right)^2}} \frac{d\zeta_R}{dl} \quad (31c)$$

Then, the vector integral  $\bar{I}$  in (28) can be calculated as:

$$\begin{aligned} \bar{I} &= \int_0^L [\bar{R}_0(l) \times \bar{i}(l)] dl = \int_0^L \begin{vmatrix} \bar{i}_\xi & \bar{i}_\eta & \bar{i}_\zeta \\ \xi_R & \eta_R & \zeta_R \\ \xi_i & \eta_i & \zeta_i \end{vmatrix} dl = \\ &= \bar{i}_\xi \int_0^L (\eta_R \zeta_i - \zeta_R \eta_i) dl + \bar{i}_\eta \int_0^L (\zeta_R \xi_i - \xi_R \zeta_i) dl + \bar{i}_\zeta \int_0^L (\xi_R \eta_i - \eta_R \xi_i) dl \end{aligned} \quad (32)$$

where:  $\bar{i}_\xi, \bar{i}_\eta, \bar{i}_\zeta$  – versors of the system:  $O\xi\eta\zeta$ .

In the special case, as e.g. the example of fittings being in one plane, discussed in paper [11], the author proposed a simple, geometrical interpretation for the determination of the above integral.

In a similar fashion as in case of the momentum increase (see item 4), also the angular momentum has – apart from the determined above increase originated from a non-stationary flow character – an *increasing component related to the change of the angular momentum of the elementary mass entering fittings into the angular momentum of the elementary mass leaving the analysed sector of a network*:

$$\begin{aligned} d\bar{K}_s &= \bar{R}_{0L} \times \bar{i}_L \frac{q}{A_L} \cdot \rho \cdot q \cdot dt - \bar{R}_{00} \times \bar{i}_0 \frac{q}{A_0} \cdot \rho \cdot q \cdot dt = \\ &= \rho \cdot \frac{q^2}{A_0} \cdot dt \cdot [\bar{R}_{0L} \times \bar{i}_L \cdot \frac{A_0}{A_L} - \bar{R}_{00} \times \bar{i}_0] \end{aligned} \quad (33)$$

Thus, the external forces momentum causing this angular momentum change equals:

$$\bar{M}_s = \frac{d\bar{K}_s}{dt} = \rho \cdot \frac{q^2}{A_0} \cdot [\bar{R}_{0L} \times \bar{i}_L \cdot \frac{A_0}{A_L} - \bar{R}_{00} \times \bar{i}_0] \quad (34)$$

**The total momentum with which the liquid influences fittings** (including the piston), can be written as:

$$\bar{M} = -(\bar{M}_n + \bar{M}_s) = -(\rho \cdot \frac{dq}{dt} \cdot \bar{I} + \rho \cdot \frac{q^2}{A_0} \cdot [\bar{R}_{0L} \times \bar{i}_L \cdot \frac{A_0}{A_L} - \bar{R}_{00} \times \bar{i}_0]) \quad (35)$$

where:  $\bar{i}_0, \bar{i}_L$  – versors of the pipe axis in the initial and final point of the installation,

$\vec{R}_{00}, \vec{R}_{0L}$  – leading vector, running from pole  $O$  to centres of the initial and final cross-sections, respectively,

$A_0, A_L$  – cross-section areas of the hydraulic pipe in its initial and final points, respectively,

– remaining notations, as above.

**The force applied in pole  $O$ , given by dependence (24) as well as the momentum determined by (35) describe the total dynamic influence of liquids present in the installation on the surroundings.**

## 6. Conclusions

1. The possibility of calculations of integrals of energy, momentum and angular momentum and of determinations – on these bases – of forces and moments of reaction for surroundings as well as of the liquid reduced mass, was investigated in the paper. These calculations were performed on the bases of the volumetric flow rate only, without knowing the rate distribution on the pipe cross-section.
2. Dependences allowing to determine a dynamic influence of liquids, contained in hydraulic pipes of machines and devices, on surroundings were derived. The case of hydraulic pipes of variable diameter and arbitrary pathways in space for a stationary and non-stationary flows was also investigated.
3. It was shown that the kinetic energy of liquids contained in pipes, it means also their reduced mass, obtains minimum for axi-symmetrical flows for the rate homogeneous distribution on the cross-section.

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