Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 275-282

PAN POLSKA AKADEMIA NAUK

METROLOGY AND MEASUREMENT SYSTEMS

Index 330930, ISSN 0860-8229

www.metrology.pg.gda.pl

M&S

A VELOCITY MEASUREMENT METHOD BASED ON SCALING PARAMETER ESTIMATION OF A CHAOTIC SYSTEM

Lidong Liu, Jifeng Hu, Zishu He, Chunlin Han, Huiyong Li, Jun Li

School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China, (✉liulidong_1982@126.com, +86 02861852218, hujinfeng06@tsinghua.org.cn, zshe@uestc.edu.cn)

Abstract

In this paper, we propose a new method of measuring the target velocity by estimating the scaling parameter of a chaos-generating system. First, we derive the relation between the target velocity and the scaling parameter of the chaos-generating system. Then a new method for scaling parameter estimation of the chaotic system is proposed by exploiting the chaotic synchronization property. Finally, numerical simulations show the effectiveness of the proposed method in target velocity measurement.

Keywords: signal processing, parameter estimation, chaos, synchronization.

1. Introduction

A chaotic signal, generated by nonlinear dynamical circuits and systems, has broadband spectra, noise-like property, good correlation and deterministic feature. These features of a chaotic signal have drawn considerable attention in the radar community [1-11].

In [1], the authors present a processing scheme of chaotic radar signals by exploiting the generating mechanism of transmitted chaotic signals. They find a simple relation between the target parameters (range and velocity) and the system parameters of a special chaotic system, Chua’s circuits (Chaotic system is the system that presents the chaos features, such as initial condition sensitivity, parameter sensitivity, attractors. Chua’s system is one of the chaotic systems). With this relation, the measurement of the target parameters is transformed into estimation of Chua’s circuit parameters. But in [1], the authors focus on Chua’s circuit and they do not show the way how to estimate the parameters in a chaotic system accurately.

In this paper we do further research based on [1] on chaotic radar. The source of chaotic radar is the signal generated by a nonlinear dynamical circuit. Firstly, we offer a relationship between the target parameters and the common chaotic system parameters. It is demonstrated by exploiting the relationship between Doppler shift and the scaling parameter of the chaotic system.

Then, a more accurate method for parameter estimation of the chaotic system is given. Though there are many parameter estimation methods, most of them are focusing on using the optimization algorithm, such as in [12]; an extended word-lifting method is used, in [13,14] particle swarm optimization is applied in parameter estimation. Since chaos is very sensitive to the parameters, the estimation error of the traditional ways is not small enough for parameter estimation in target velocity measurement. The estimation method in this paper makes full use of the synchronization character of chaos. That is, the synchronization error of the chaotic systems is sensitive to the parameters. If there is a small error between the master system parameter and the slave system parameter, there is a distinct synchronization error between them. So an accurate estimation can be obtained by choosing the value which can

Article history: received on Dec. 05, 2011; accepted on Apr. 06, 2011; available online on May 20, 2011; DOI: 10.2478/v10178-011-0009-1.

make the best synchronization performance.

Finally, the parameter estimation method is applied in estimating the scaling parameter of the chaotic system, and the target velocity can be obtained by the relationship between the Doppler shift and the scaling parameter.

This paper is organized as follows. In Section 2 the relation between the target velocity and the scaling parameter of the chaos-generating system is derived. In Section 3 a new parameter estimation method for a chaotic system is proposed. In section 4, numerical simulation is done to verify the effectiveness of the theory. A brief conclusion of this paper is drawn in Section 5.

2. Relating the target velocity and the scaling parameter

In [1], the authors focus on Chua's chaotic system and they proposed the relation between the target velocity and the parameter of Chua's chaos-generating system. In this section, we further develop the relationship between the target velocity and the parameter of a general chaos-generating system. In order to illustrate this, two theorems are given.

Theorem 1 Consider two n- dimensional chaotic systems:

$$\dot{X}(t) = g(X(t)) \tag{1}$$

$$\dot{\tilde{X}}(t) = \lambda g(\tilde{X}(t)), \tag{2}$$

where $X(t), \tilde{X}(t) \in R^n$, λ is the scaling parameter. If we let $X(t)$, $\tilde{X}(t)$ denote the solution to Eq.(1) and Eq.(2) respectively, then we have

$$\tilde{X}(t) = X(\lambda t) \tag{3}$$

Proof: Assume $X(\tau) = X(\lambda t)$, (where $\tau = \lambda t$) according to Eq.(1):

$$\begin{aligned} \dot{X}(\tau) &= \frac{d}{dt} X(\tau) \\ &= \frac{d[X(\tau)]}{d\tau}\frac{d[\tau]}{dt} = \lambda \frac{d[X(\tau)]}{d\tau} = \lambda g\left(X(\tau)\right). \end{aligned} \tag{4}$$

That is

$$\dot{X}(\lambda t) = \lambda g\left(X(\lambda t)\right) \tag{5}$$

Seen from Eq.(5) and Eq.(2), we have:

$$\tilde{X}(t) = X(\lambda t). \tag{6}$$

Theorem 2 If the transmitted radar signal is generated by $\dot{X}(t)$ defined by Eq.(1), then the returned signal from the moving point target could be simulated by the signal generated by $\dot{\tilde{X}}(t)$ defined by Eq.(2).

Proof: Assume that a point target is located at a distance r_0 far from the radar at time t_0, travelling with a linear velocity of v along the line of sight of the radar. Then the range to target at any time t is:

$$r(t) = r_0 + v(t - t_0). \tag{7}$$

Without the loss of generality, we let the initial time $t_0 = 0$. The delay corresponding to the two-way path will be

$$\tau = \frac{2r(t)}{c} = \frac{2r_0}{c} + \frac{2vt}{c}, \tag{8}$$

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 275-282

where c is the light velocity. Assume that the transmitted radar signal is generated by $x_1(t)$ which is one variable of $X(t)$. (where $X(t)=\left[x_1(t),x_2(t),\cdots,x_n(t)\right]$).Then the returned signal $x_1'(t)$ form the moving target is

$$x_1'(t)=x_1(t-\tau)=x_1\left[\left(1-\frac{2v}{c}\right)\left(t-\left(\frac{2r_0}{c-2v}\right)\right)\right]. \tag{9}$$

Let

$$\lambda=1-(2v/c) \tag{10}$$

$$\tilde{\tau}=2r_0/(c-2v), \tag{11}$$

$x_1'(t)$ can be rewritten as:

$$x_1'(t)=x_1\left(\lambda(t-\tilde{\tau})\right). \tag{12}$$

According to Theorem 1 $x_1'(t)$ is the solution of $\dot{\tilde{X}}(t)$, thus the returned signal from the point moving target could be simulated by the signal generated by $\dot{\tilde{X}}(t)$.

Theorem 2 can be presented by a simple picture, Fig.1. In Fig.1, we can see that the returned signal from the point moving target could be simulated by another signal generated by a similar form chaotic system which multiplied by a scaling parameter.

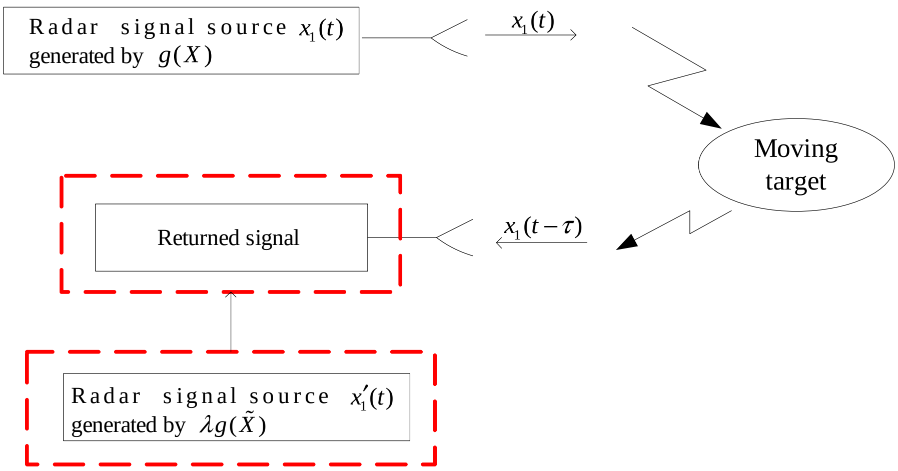

Fig.1 A simple illustration figure of Theorem 2

Theorem 1 and theorem 2 indicate that the scaling parameter λ in fact is the Doppler shift, and if we could estimate the parameter λ in the chaotic system in Eq.(2), then we can get the target velocity by Eq.(13).

$$v=\frac{c}{2}(1-\lambda). \tag{13}$$

3. A New Method for Estimating the Scaling Parameter

In this section, a new method for parameter estimation is given. It is based on chaotic synchronization character, but not using the optimization algorithm in the traditional way. Since the chaotic synchronization parameter is sensitive, the method in this paper is more accurate.

In section 2, we derived the relationship between the target velocity and the scaling parameter of the chaotic system. Seen from Eq.(13), in order to get the target velocity, the scaling parameter λ should be estimated accurately. If there is a 10^{-6} error in λ, the estimation error in target velocity could be 150m/s. Thus an accurate parameter estimation method is needed here. Though there are many parameter estimation methods in a chaotic system, such as [4, 5, 12-15], the estimation is not accurate enough. So in this section, a more accurate estimation method is proposed based on the parameter sensitivity of chaotic synchronization.

The main idea of the proposed estimation method is as follows. A special interval of the scaling parameter should be computed firstly. This is easy to be accomplished, since in practice the target velocity is limited, so the estimated parameter is restricted in a small interval according to Eq.(10). Then uniform sampling is made through the special interval with a small sampling interval. Let the returned signal from the moving target be the master system and let the sampled value in the special interval be the scaling parameter of the slave system. Because of the parameter sensitivity of the chaotic system, only the master and the slave parameter are nearly matched, the synchronization performance is the best. So the value which can make the smallest synchronization error is chosen in the interval and let it be the estimated scaling parameter value.

In order to illustrate the method, some notation should be given. The master chaotic system, which is simulated by the returned signal from the moving target, is shown as Eq.(2). The slave system is as Eq.(14).

$$\dot{\hat{X}}(t) = \gamma f(\hat{X}(t)). \tag{14}$$

where γ is a constant and $\gamma \in T = [1-(2v_{\max}/c), 1+(2v_{\max}/c)]$, $v_{\max}$ is the top limit of the target velocity and $\hat{X}(t) = [\hat{x}_1(t), \hat{x}_2(t), \cdots \hat{x}_n(t)]$. $\hat{X}(t)$ is similar to $\tilde{X}(t)$ which is defined by Eq.(2). The difference is that they have a different scaling parameter. The scaling parameter of $\hat{X}$ is λ and the scaling parameter is γ.

The step of the new method for estimation of the parameter is as follows:

1) Define the sampling interval l and sample γ_i (i= 1,2,…N) in the special interval T, where N is the total sampling number in the small interval. Let $\gamma = \gamma_i$.

2) Use one component of $\hat{X}(t)$, which is $\hat{x}_1(t)$, as the driven signal to drive the system defined by Eq.(14).

3) Compute the synchronization error and choose the value $\gamma \in T$ which can make the smallest synchronization error as the estimation value $\hat{\lambda}$ by using Eq.(15).

$$\hat{\lambda} = \gamma = \arg \inf_{\gamma \in T} \left\| \tilde{\mathbf{x}}_1(t) - \hat{\mathbf{x}}_1(t) \right\|, \tag{15}$$

where inf(f(x)) denotes the infimum of f(x), and $\left\|\mathbf{x}(t)\right\| = \sqrt{\left\langle \mathbf{x}(t), \mathbf{x}^*(t) \right\rangle}$.

4. Numerical Simulation

In order to verify the effectiveness of the theory in this paper, simulations have been done in this section.

Assume two leaving targets with the velocity v_1=30m/s , v_2=150m/s respectively. According to Eq.(13).

$$\lambda_1 = 1-(2v_1/c) = 1-2\times10^{-7},\ \lambda_2 = 1-(2v_2/c) = 1-10^{-6}.$$

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 275-282

According to the theory in section 2 we could use the signal generated by Eq.(2) to simulate the returned signal from the moving target when the transmitted signal is generated by Eq.(1). Here the typical Lorenz chaotic system is used for illustration and it is shown in Eq.(16). The radar signal is generated by x_1 component in the Lorenz chaotic system. The synchronization error is defined as Eq.(17).

$$\begin{cases} \dot{x}_1 = -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 = r x_1 - x_2 - x_1 x_3 \\ x_3 = x_1 x_2 - b x_3, \end{cases} \tag{16}$$

where σ=10, r = 28 and b=8/3.

$$E(t) = \tilde{x}_1(t) - \hat{x}_1(t)\,. \tag{17}$$

Next, the method proposed in section 3 is used to estimate the parameter $\lambda_i (i = 1,2)$. In this paper we focus on the target with the velocity in $[0(m/s) - 600(m/s)]$ which is the interval which contains the velocity of most targets (cars, planes). Thus, we let $v_{\max} = 600 m/s$. So the small interval T = [1– 4 x 10^{-6}, 1+ 4 x 10^{-6}]. We let the sampling interval l = 10^{-8}. Using the method in section 3 we get $\hat{\lambda}_1 = 1 - 2.1 \times 10^{-7}$, $\hat{\lambda}_2 = 1 - 0.98 \times 10^{-6}$ and the synchronization error simulation is shown in Fig.2 and Fig.3. According to Eq. (10) we use $\hat{v} = \frac{c}{2}(1 - \hat{\lambda})$ to get $\hat{v}_1 = 31.5 m/s$, $\hat{v}_2 = 147 m/s$. The estimation error is small.

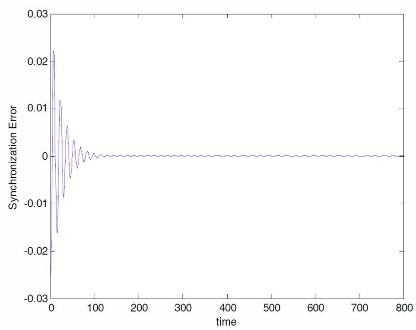

Fig.2 The synchronization error of the driven-response system; the driven system with parameter λ_1 and the response system with parameter $\hat{\lambda}_1$.

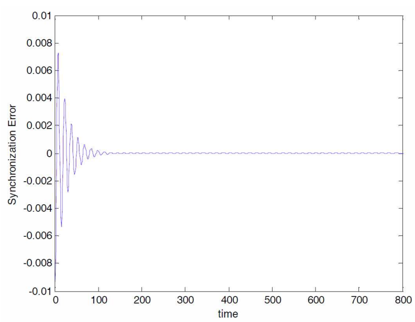

Fig.3 The synchronization Error of the driven-response system; the driven system with parameter λ_2 and the response system with parameter $\hat{\lambda}_2$.

For comparison, we also use other methods for estimating the scaling parameter. This is shown in Tab.1. From Tab.1, we can see that the estimation method in this paper is more accurate and suitable for getting the target velocity. The reason is that the estimated parameter is restricted in a special interval, thus, we can make full use of the chaotic synchronization parameter sensitivity (when the parameters in the driven and response systems are not matched well, the synchronization error is large.) to get the estimated parameter in the special interval, while other methods are focusing on using the optimization algorithm to estimate the parameter.

Table 1. The average error of the methods in the estimation scaling parameter and the target velocity; 20 simulation experiments have been done in each method

ESTIMATION METHODS	AVERAGE ESTIMATION ERROR	AVERAGE VELOCITY ESTIMATION ERROR
The method in [4]	2.3×10^{-7}	34.5m/s
The method in [13]	3.6×10^{-6}	540m/s
The method in [14]	1.7×10^{-6}	255m/s
The method in this paper	1.3×10^{-8}	1.95m/s

5. Conclusions

In this paper we derived the relation between the scaling parameter of a general chaotic system and the target velocity. What is more, a new method for parameter estimation of the chaotic system is proposed. We can get the target velocity by estimating the scaling parameter of the returned signal. Numerical simulation shows the effectiveness of the method proposed in this paper. Notice that in this paper the noise is not considered. Here we just offer a principle of velocity measurement and the noise can be reduced by other methods such as the way in [16]. How to obtain the target velocity in engineering will be discussed later.

Acknowledgment

The authors would like to thank for the suggestions of the anonymous reviewers. And the authors would like to thank for the support of Defense Pre-research Fund (9140A07011609DZ0216), the Fundamental Research Funds for the Central Universities (ZYGX2009J011, ZYGX2009J015, ZYGX2010J015, 103.1.2-E022050205) and Doctoral Fund (200806141026).

Reference

[1] Liu, Z., Zhu, X., Hu, W. (2007). Principles of chaotic signal radars. *International Journal of Bifurcation and Chaos*, 17 (5),1735-1739.

[2] Leung, H., Shanmugam, S., Xie, N.(2006). An ergodic approach for chaotic signal estimation at low SNR with application to ultra-wide-band communication. *IEEE Trans. Signal Process.*, 54 (5), 1091–1103.

[3] Venkatasubramanian, V., Leung, H. (2005). A novel chaos-based high-resolution imaging technique and its application to through-the-wall imaging. *IEEE Signal Processing Letters*, 12 (6), 528-531.

[4] Ghosh, D. (2008). Adaptive scheme for synchronization-based multiparameter estimation from a single chaotic time series and its applications. *Phys. Rev. E*, 78, 056211(1)-056211(5).

[5] Wang, K. et al (2008). Symbolic Vector Dynamics Approach to Initial Condition and Control Parameters Estimation of Coupled Map Lattices. *IEEE Trans. Circuits and Systems I: Regular Papers*, 55 (4), 1116-1124.

[6] Thayaparan, T. et al (2008). Editorial Signal Processing in Noise Radar Technology. *IET Radar, Sonar and Navigation*, 2 (4), 229–232.

[7] Narayanan, R.M., Dawood, M. (2000). Doppler estimation using a coherent ultrawide-band random noise radar. *IEEE Trans. Antennas and Propagation*, 28 (6), 868-878.

Metrol. Meas. Syst., Vol. XVIII (2011), No. 2, pp. 275-282

[8] Carroll, T. (2005). Chaotic system for self-synchronizing Doppler measurement. *Chaos*, 15, 013109.1-013109.5.

[9] Shi, Z ., Qiao, S., Chen, K.S. (2007). Ambiguity functions of direct chaotic radar employing microwave chaotic Colpitts oscillator. *Progress In Electromagnetics Research*, 77, 1–14.

[10] Susek, W., Stec, B. (2010). Broadband microwave correlation of noise signals. *Metrology and Measurement System*, 17 (2), 289–299.

[11] Pecora, L., Carroll, T. (1990). Synchronization in chaotic systems. *Phys. Rev. Lett.*, 64 (6), 821-825.

[12] Wang, K., Pei, W., He, Z. (2007). Estimating initial conditions in coupled map lattices from noisy time series using symbolic vector dynamics. *Phys. Lett. A*, 367 (6), 316-321.

[13] He, Q ., Wang, L., Liu, B. (2006). Parameter estimation for chaotic systems by particle swarm optimization. *Chaos, Solitons & Fractals*, 34 (2), 654–661.

[14] Gao, F., Lee, J., Li, Z. (2009). Parameter estimation for chaotic system with initial random noises by particle swarm optimization. *Chaos, Solitons and Fractals*, 42, 1286-1291.

[15] Fostin, H., Woafo, P. (2005). Adaptive synchronization of a modified and uncertain chaotic Van der Pol-Duffing oscillator based on parameter identification. *Chaos, Solitons & Fractals*, 24 (12), 1363–1371.

[16] Travassos X. L., Vieira D., Palade V. (2009). Noise Reduction in a Non-Homogenous Ground Penetrating Radar Problem by Multiobjective Neural Networks. *IEEE Trans. Magnetics*, 45 (3), 1454-1457.