AN ANALYSIS OF DEVIATIONS OF CYLINDRICAL SURFACES WITH THE USE OF WAVELET TRANSFORM

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Abstract

Wavelet transform becomes a more and more common method of processing 3D signals. It is widely used to analyze data in various branches of science and technology (medicine, seismology, engineering, *etc.*). In the field of mechanical engineering wavelet transform is usually used to investigate surface micro- and nanotopography. Wavelet transform is commonly regarded as a very good tool to analyze non-stationary signals. However, to analyze periodical signals, most researchers prefer to use well-known methods such as Fourier analysis. In this paper authors make an attempt to prove that wavelet transform can be a useful method to analyze 3D signals that are approximately periodical. As an example of such signal, measurement data of cylindrical workpieces are investigated. The calculations were performed in the MATLAB environment using the Wavelet Toolbox.

Keywords: wavelet transform, cylindricity profile, decomposition, approximation.

1. Introduction

The wavelet transform is one of the latest calculation techniques used for analyzing measurement signals. It is commonly used in various branches of science and technology, for example in medicine. In [1] the author applies the wavelet transform to reconstruct a 3D signal from a series of two-dimensional sectional images. In the work [2] multiscalar wavelets are used to analyze the topography of wear of orthopaedic implants. In the field of mechanical engineering many researchers dealing with surface texture apply this method to assess surface micro- and nanotopography. In the works [3, 4] authors apply wavelet analysis to assess nanotopography of crystal surfaces. The assessed images were obtained with the use of atomic force microscopes. In the work [5] authors apply lifting wavelets to surface roughness profiles and compare results with the ones obtained through polynomial fitting. The work [6] presents results of the application of wavelet transform to identify features of surfaces of automotive cylinders and femoral heads on a micro- and nanoscale. Some researchers focus on filtering properties of wavelet transform. For example, in the work [7] authors apply wavelet transform to filtering of freeform surfaces.

Wavelet transform is commonly regarded as a very good tool to analyze non-stationary signals. In order to analyze periodical signals most researchers prefer to use well-known methods, like Fourier analysis [8, 9]. However, results of the research on the application of wavelets to analyze periodical signals, such as roundness profiles, presented in [10], show that this method can be very useful to detect irregularities of roundness profiles such as cracks or scratches of the surface. This is the reason why the authors have decided to investigate the application of wavelet transform to the analysis of cylindrical surfaces of machine parts. In author's opinion, the results of the experiments should give an answer if wavelet transform

is useful to analyze deviations of cylindrical surfaces or the traditional approach (*i.e.* Fourier analysis) is sufficient.

In the experiment the authors decided to apply a Matlab package. There are two ways to perform two-dimensional wavelet transform in Matlab: with the use of a graphical user interface called Wavelet toolbox or with the use of instructions entered manually by a user in the command line. Thus, an additional aim of the research work was assessing which method is better to analyze measurement data of cylindrical workpieces.

2. Mathematical fundamentals of two-dimensional wavelet transform

Wavelets $\Psi(t)$ are, as the name suggests, small waves with a limited range and an oscillatory character [11, 12]. The independent variable t is sometimes called a spatial variable. Wavelets can form specific sets of basic functions used for the description of functional spaces. They are particularly suitable for describing non-continuous and irregular functions, which are common in the response of real physical systems. Bases of wavelet functions, which are generally well-localized both in the frequency and time domains, are formed by scaling with the parameter σ and shifting in time, as well as translating the output wavelet, $\Psi(\sigma t + \tau)$, with the parameter τ, which leads to the so-called scaled hierarchical representation of the analyzed function [13, 14].

The continuous wavelet transform (CWT) of the $x(t)$ signal for a given wavelet $\Psi(t)$ is defined as [15]:

$$W(\tau,\sigma) = \int x(t)\Psi_{t,\sigma}^{*}(t)dt \qquad \Psi_{t,\sigma}(t) = \frac{1}{\sqrt{\sigma}}\Psi(\frac{t-\tau}{\sigma}), \qquad (1)$$

where: τ – the time shift, σ – the scale (frequency).

The discrete wavelet transform (DWT) can be introduced after discretization of the $x(t)$ signal, assuming that:

$$\sigma = 2^{-s}; \quad \tau = 2^{-s} \cdot l, \qquad (2)$$

where: s – the scale coefficient, l – the shift coefficient.

Taking the above into account, we obtain:

$$W(\tau,\sigma) = W(l \cdot 2^{-s}, 2^{-s}) = W(l,s) = 2^{s/2} \int_{-\infty}^{\infty} x(t)\Psi(2^{s} \cdot t - l)dt =$$

$$= 2^{s/2} \sum_{n=0}^{N-1} x(n)\Psi(2^{s} \cdot n - l). \qquad (3)$$

The wavelet transform is generally used for the decomposition and approximation of measurement signals, then, removal of noise and, finally, the reconstruction of the output signal. Its applications have been described in many works (see [16–20]). These applications include pyramid decomposition and decomposition based on the application of wavelet packets. Many researchers emphasize that it is important to select the right basic wavelet so that the processes of profile decomposition and approximation can be performed properly.

The above algorithm of the wavelet transform of one-dimensional signals, commonly called *2D* profiles, can be extended to signals defined over an n-dimensional space ($n > 1$). In the case of signals describing the surface texture, it is necessary to take into consideration a two-dimensional space ($n = 2$), with the *x*- and *y*- coordinates, over which the function representing irregularities of the surface, called a *3D* profile, registered during measurements of surface topography and form errors, is defined. Such a profile can be represented in the form of a discrete function written on a matrix. Approximation is performed on the rows

and columns of this matrix, corresponding to the horizontal and vertical directions with respect to the orthonormal base consisting of three wavelet functions [13–14]:

$$\Psi^h\left(x, y\right) = \phi(x) \cdot \Psi(y),$$
$$\Psi^v\left(x, y\right) = \Psi(x) \cdot \phi(y),$$
$$\Psi^d\left(x, y\right) = \Psi(x) \cdot \Psi(y),$$

(4)

where: Ψ – the basic wavelet, φ – the scaling function, h – the index denoting the horizontal wavelet, v – the index denoting the vertical wavelet, d – the index denoting the diagonal wavelet.

Figure 1 illustrates the principle of approximation of the output signal f^0 to the second level. The letter a denotes the approximated signal, and the letter d the detail coefficients calculated in the horizontal (h), vertical (v) and diagonal (d) directions.
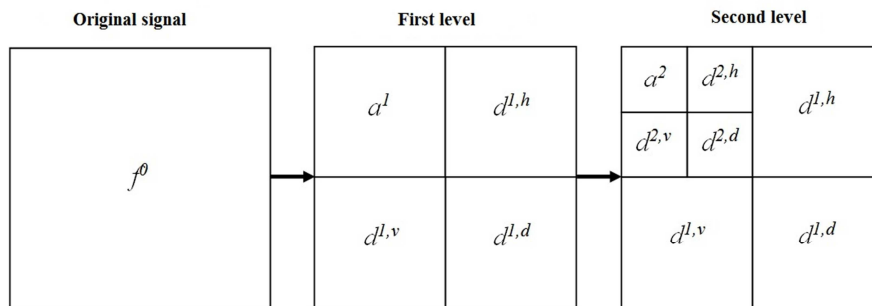

Fig. 1. Signal decomposition defined over a two-dimensional space.

The above procedure was verified for cylindrical profiles, using the software available in the MATLAB Wavelet Toolbox.

3. Decomposition and approximation of cylindrical profiles in the MATLAB environment

3.1. Methodology

In the MATLAB environment, a two-dimensional discrete wavelet transform can be performed [21]:
− using commands provided by the user in the command line;
− from the *Wavelet Toolbox* graphical user interface.
a) Performing a two-dimensional discrete wavelet transform from the command line interface.

When the commands are entered manually into the command line, it is possible to apply two functions: *dwt*2 and *wavedec*2.

The *dwt*2 function enables us to perform a one-level two-dimensional wavelet transform of a given matrix. This operation can be conducted by assuming an arbitrary type of the basic wavelet distinguished by the program or by defining the low-pass and high-pass decomposition filters. The result of the operation of the *dwt*2 function is four sets of data. The first contains approximation coefficients and in the next three consecutive sets there are detail coefficients of decomposition in the horizontal, vertical and diagonal directions.

The matrix can be reconstructed by means of the *idwt*2 function, whose input parameters are the matrices of the approximation and detail coefficients as well as the type of the basic wavelet.

Unlike the *dwt*2 function, the *wavedec*2 function makes it possible to perform decomposition at an arbitrary level. As in the case of the *dwt*2 function, this operation can be performed for a given matrix assuming the type of the basic wavelet or defining the low-pass and high-pass decomposition filters. The result of the operation of the *wavedec*2 function is two sets of data. One contains coefficients of approximation as well as horizontal, vertical and diagonal details at the consecutive levels of decomposition. In the other set of data, there is information about the sizes of the particular matrices of coefficients.

The *waverec*2 function is a function inverse to the *wavedec*2 function. It can be used to reconstruct the matrix, by assuming as parameters the sets of data obtained from the operation of the *wavedec*2 function and the type of the basic wavelet or the high- and low-pass filters [17, 21].

b) Performing a two-dimensional discrete wavelet transform from the graphical user interface.

The MATLAB environment is suitable for performing a two-dimensional wavelet transform also from the graphical user interface, which is initiated by entering the *wavemenu* command in the command line. This solution is user-friendly; it does not require knowledge of the particular functions and their parameters. A drawback, however, is its limited application. The graphical user interface can be used to perform a two-dimensional wavelet analysis of three types of data: image, coefficients and decomposition.

It should be emphasized that databases of coefficients and decomposition are also created using a two-dimensional wavelet transform of the image. As a consequence, the graphical user interface cannot be used for performing a two-dimensional wavelet transform of a set of data displayed in a matrix form.

3.2. Calculations for selected profiles

In order to practically verify whether the MATLAB software can be used to perform a two-dimensional wavelet analysis for a cylindrical surface, it was essential to measure selected cylindrical elements by means of radial instruments Talyrond 365 and Talycenta. During the experiment two cylindrical elements were measured. The measurements were conducted at the Laboratory of Computer-Based Measurements of Geometrical Quantities of the Kielce University of Technology. The cylinders were measured using the cross-section strategy based on changes in the radius [22]. The momentary values of the radius measured in certain cross-sections were recorded in columns, creating an $m \times n$ matrix, where m denoted the number of measuring points per cross-section, and n – the number of cross-sections.

Figure 2 shows a 3D graph representing the deviations of the measured cylinders generated on the basis of the measurement data. The surface of cylinder no. 1 was turned, and the surface of the cylinder was turned and additionally scratched in order to simulate local surface defects, *e.g.* cracks.

It is easy to notice that the character of cylinder surfaces shown in Fig. 2 is different. The deviations of the cylinder in Fig. 2a are very regularly distributed on the surface. The cross-sections of this cylinder are triangular, which is quite common for elements whose surface was damaged by a self-centering three-jaws fixture system. Deviations of the cylinder shown in Fig 2b are mostly regular. The character of these deviations is similar to the deviations of cylinder no. 2. However, there are a few significant local deviations on the surface of cylinder no. 2. These irregular deviations correspond to the areas where the cylinder surface was scratched.
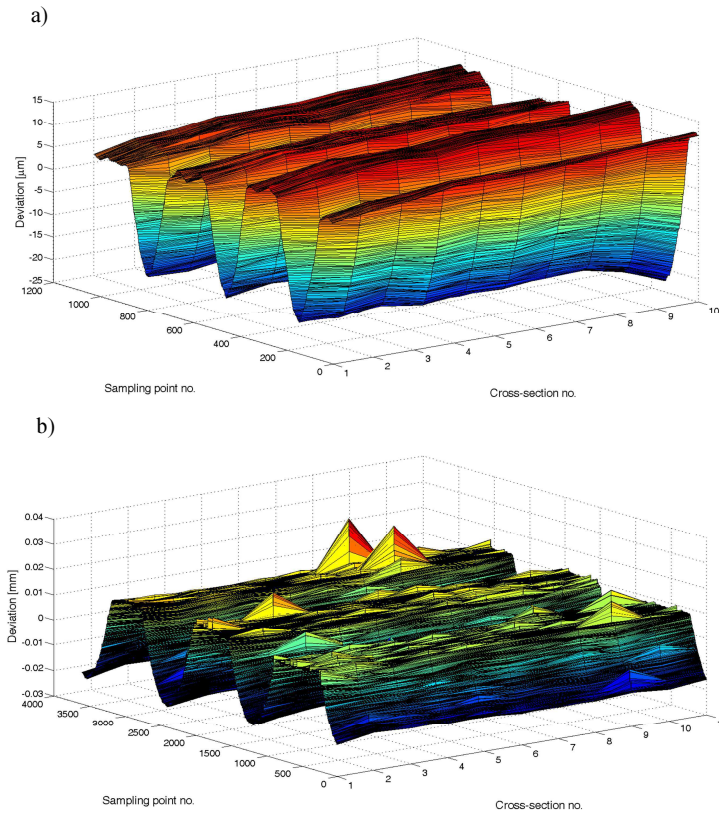
a)

b)

Fig. 2. Deviations of the measured cylinder represented in a 3D graph generated from the measurement data:
a) cylinder no. 1, b) cylinder no. 2.

a) A two-dimensional wavelet analysis from the command line interface.

A two-dimensional wavelet analysis of the measured profile of cylindricality was conducted using the *dwt*2 function and the Daubechies "db5" basic wavelet [23, 24]. The resulting four matrices contain the coefficients of approximation, horizontal detail coefficients, vertical detail coefficients and diagonal detail coefficients, respectively. The values of these coefficients are shown in Fig. 3–10.
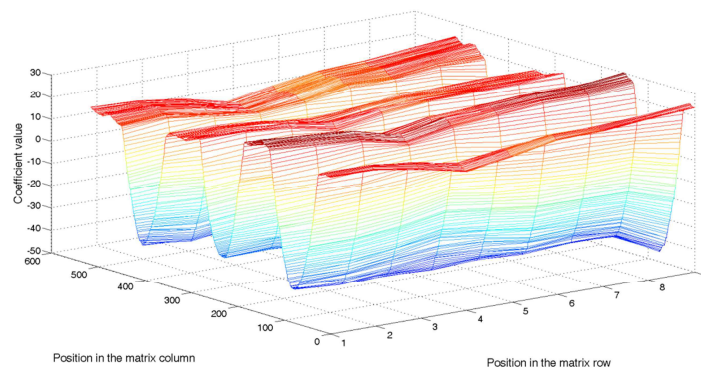
Fig. 3. Coefficients of approximation for cylinder no. 1.
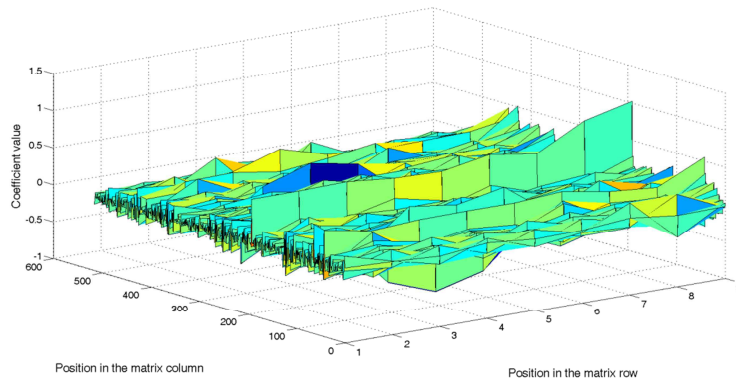
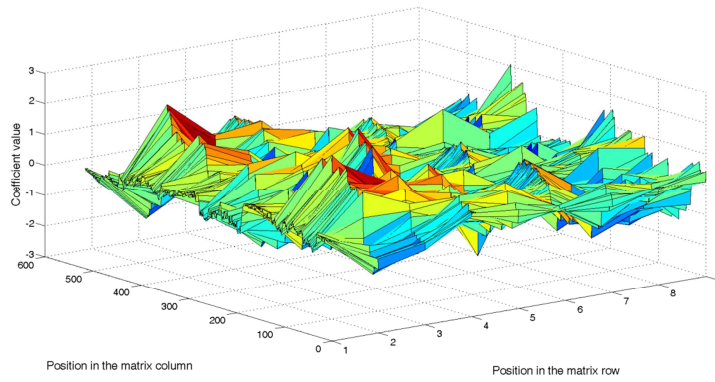Fig. 4. Horizontal detail coefficients of decomposition for cylinder no. 1.

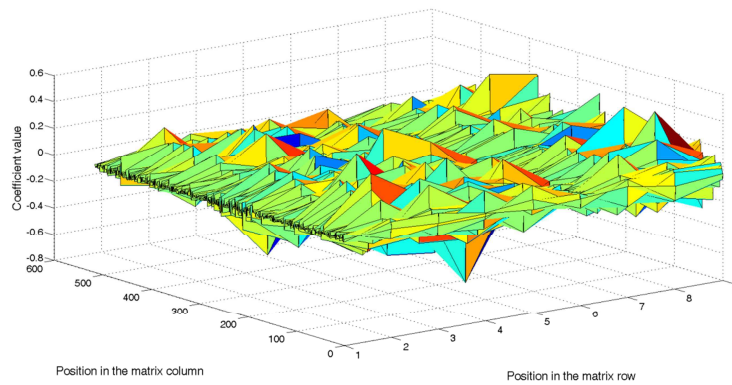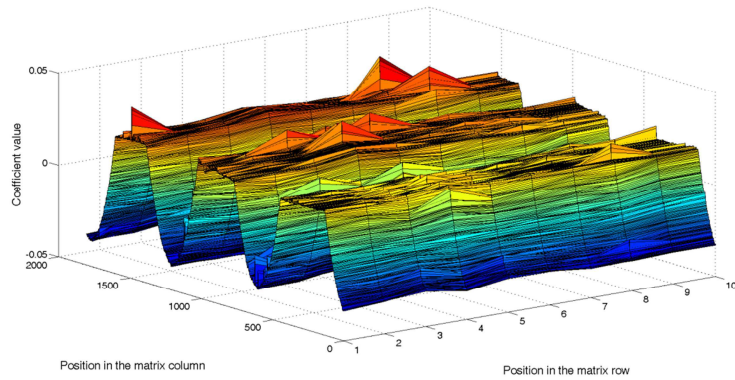Fig. 5. Vertical detail coefficients of decomposition for cylinder no. 1.

Fig. 6. Diagonal detail coefficients of decomposition for cylinder no. 1.

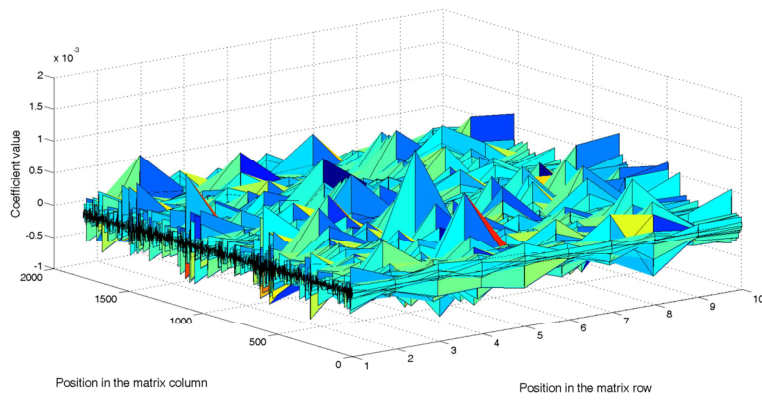Fig. 7. Coefficients of approximation for cylinder no. 2.

Fig. 8. Horizontal detail coefficients of decomposition for cylinder no. 2.

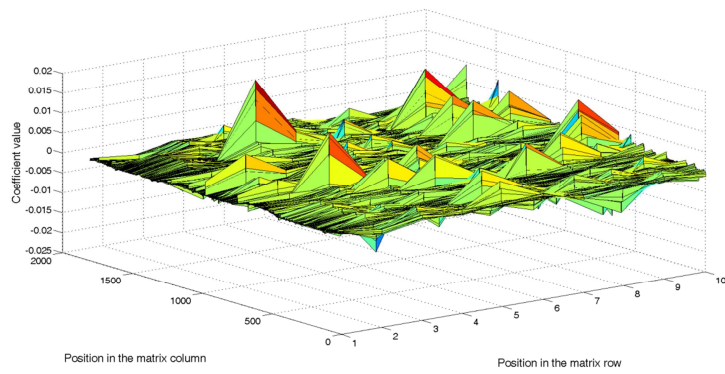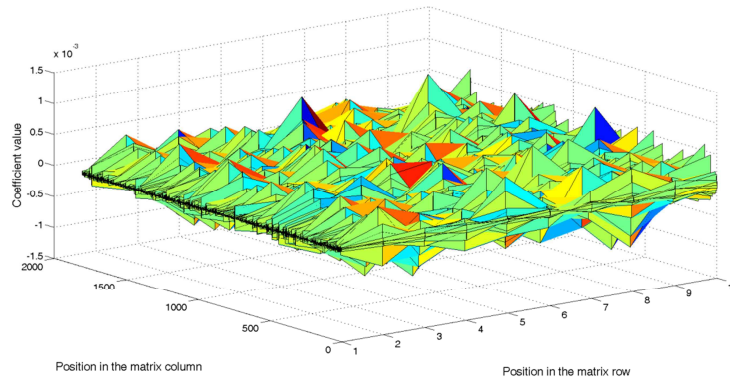Fig. 9. Vertical detail coefficients of decomposition for cylinder no. 2.

Fig. 10. Diagonal detail coefficients of decomposition for cylinder no. 2.

b) A two-dimensional wavelet analysis from the graphical user interface.

As was already mentioned, the output parameters to be used to perform a two-dimensional wavelet transform from the graphical user interface are the image or the data of coefficients and decomposition generated from this image. Unfortunately, the graphical user interface Wavelet Toolbox does not allow users to load and analyze numerical data.

Since it was impossible to perform a wavelet analysis based on the data of the surface included in the matrix, first contour plots of the measured cylindrical surfaces were generated, then they were saved as graphical files. The plots are shown in Fig. 11.

The images were then loaded by means of the *Load image* graphical user interface option used for a two-dimensional wavelet analysis by means of the *db*5 wavelet. Decomposition level was equal to one. Fig. 12 shows results of the two-dimensional wavelet transform of the images with the use of the graphical interface.
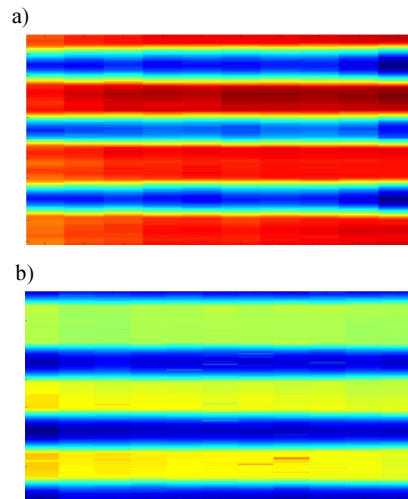
a)

b)

Fig. 11. Values of the deviations of a measured cylindrical surface in the form of a contour plot:
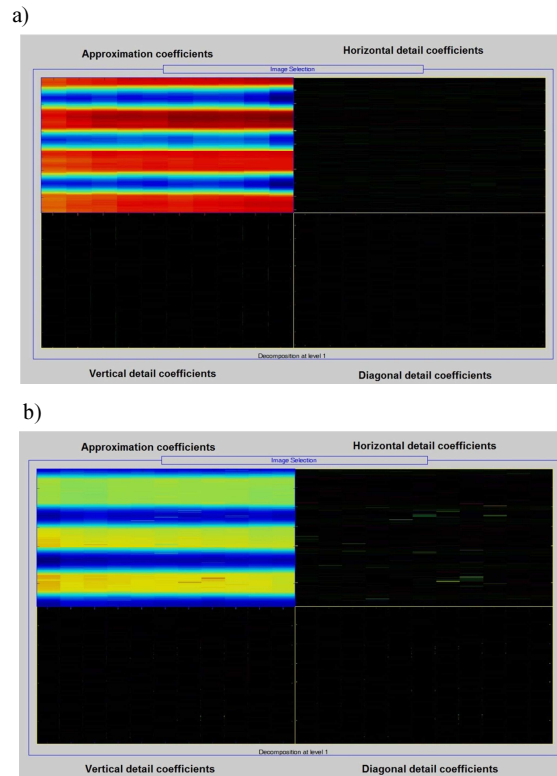a) cylinder no. 1, b) cylinder no. 2.

a)

b)

Fig. 12. Results of the two-dimensional wavelet transform with the use of the graphical interface:
a) cylinder no. 1; b) cylinder no. 2.

3.3. Discussion

An analysis of values of approximation and detail coefficients obtained for the cylinder no. 1 shows that wavelet transform did not give any significant information about the surface investigated. It is obvious that the reason is that deviations of the cylinder no. 1 are approximately periodical. In such cases a much better solution is to apply Fourier analysis instead of wavelet transform. Comparing diagrams showing an output signal (Fig. 2a) and approximation coefficients (Fig. 3) it is easy to notice the effect of profile smoothing. This confirms very good filtration properties of the wavelet transform for primary form profiles and justifies the investigations on its implementation in the analysis of all types of 3D surface texture profiles. However, it should be noted that nowadays there are also other advanced filtration techniques that can be applied in such cases, *e.g.* those described in works [25–27].

Deviations of the cylinder shown in Fig 2b are also mostly regular. It is easy to notice that the character of these deviations is similar to the deviations of cylinder no. 2. The main difference is that there are a few significant local irregular deviations on the surface of cylinder no. 2. The character of the regular deviations can be easily identified by Fourier analysis. However, it is not possible to detect local irregular deviations of the surface by this method. Results obtained in the experiment show that these local defects can be detected and localized very easily with the use of wavelet transform. It is confirmed by diagrams showing values of approximation and detail coefficients. One can note that for the cylinder no. 2 differences between approximation and detail coefficients are smaller than for cylinder no. 1. Moreover, on the contour map of coefficients shown in Fig. 12b it is easy to see that changes

of the coefficients correspond to the location of the surface defects (see Fig. 11b). Thus, wavelet transform can be a very useful method that supplements the Fourier transform in an analysis of periodical signals, such as roundness or cylindricality profiles.

Two-dimensional wavelet transform can be also applied as a method of filtration of local surface defects. Results of the research work presented in [28] and [29] show that significant local defects can be easily filtered with the use of robust filters, too. But thanks to the application of wavelet transform we can find out where surface defects are located, whereas an application of the robust filter does not give such information.

The study concerning the application of the MATLAB program to a two-dimensional wavelet transform shows that performing a wavelet analysis from the graphical user interface is simple and easy, even for an inexperienced user. The method, however, can only be used to analyze an image. Entering certain functions in the command line appears to be a better solution. It is possible, for instance, to conduct a two-dimensional wavelet analysis on a matrix containing measurement data in a numerical form. That is why this method of conducting two-dimensional decomposition is more suitable for analyzing data obtained with a measuring instrument; it is better than the graphical user interface option.

4. Conclusions

The paper has dealt with the principles of decomposition and approximation of 3D signals, which are signals described in a two-variable function over a two-dimensional space. The calculations were performed in the MATLAB environment and they illustrate the decomposition of cylindricality signals.

It has been shown that in the case of surfaces whose deviations are regular, wavelet transform does not provide any significant information. In such cases the traditional approach, *i.e.* Fourier transform, is a much more useful method. However, if there are some local defects of the surface, Fourier analysis is not sufficient, since it cannot be used to determine the location of such defects. In such cases wavelet transform can be a very good supplement of Fourier transform.

The issues to be focused on in further studies are:
− assessing the variability of basic parameters characterizing a 3D profile during decomposition for different types of surface irregularities (form errors, waviness and roughness);
− determining the influence of the form of the basic wavelet on the course and result of approximation;
− establishing the admissible level of decomposition of the output profile with the aim of determining the limitations and criteria of the calculation procedure.

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