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MODEL OF RELAXATION OF RESIDUAL STRESSES IN HOT-ROLLED STRIPS

MODEL RELAKSACJI NAPRĘŻEŃ WŁASNYCH W BLACHACH GORĄCO WALCOWANYCH

Residual stresses in hot-rolled strips are of practical importance when the laser cutting of these strip is applied. The factors influencing the residual stresses include the non uniform distribution of elastic-plastic deformations, phase transformation occurring during cooling and stress relaxation during rolling and cooling. The latter factor, despite its significant effect on the residual stress, is scarcely considered in the scientific literature. The goal of the present study was development of a model of residual stresses in hot-rolled strips based on the elastic-plastic material model, taking into account the stress relaxation.

Residual stresses in hot-rolled strips were evaluated using the FEM model for cooling in the laminar cooling line and in the coil. Relaxation of thermal stresses was considered based on the creep theory. Coefficients of elastic-plastic material model and of the creep model for steels S235 and S355 were obtained from the experiments performed on the Gleeble 3800 simulator for the temperatures 35-1100^{o}C. Experiments composed small tensile deformations of the sample (0.01-0.02) and subsequent shutter speed without removing the load. Model of the thermal deformation during cooling was obtained on the basis of the dilatometric tests at cooling rates of 0.057^{o}C/s to 60^{o}C/s.

Physical simulations of the cooling process were performed to validate the model. Samples were fixed in the simulator Gleeble 3800, then heated to the temperature of 1200°C and cooled to the room temperature at a rate of 1-50^{o}C/s. Changes of stresses were recorded. Good agreement between calculated and experimental values of stresses was observed. However, due to neglecting the effect of stress relaxation the stress at high temperatures was overestimated. Due to the change of their stress sign during the unloading process the resulting residual stresses were underestimated.

Simulation of residual stresses in rolling and cooling were performed on the basis of the developed model. It was shown that the effect of stress relaxation and phase transformations on the distribution of residual stresses in strips is essential and neglecting these factors could lead to an underestimation of residual stresses.

Naprężenia resztkowe w blachach walcowanych na gorąco są istotne w przypadku stosowania ciecia laserowego. Głównymi czynnikami wpływającymi na wielkość naprężeń resztkowych jest nierównomierne odkształcenie sprężysto-plastyczne, przemiana fazowa występująca podczas chłodzenia oraz relaksacja zachodząca w trakcie walcowania i chłodzenia[1-2]. Ten ostatni czynnik pomimo znacznego wpływu na naprężenia resztkowe jest niedostatecznie opisany w literaturze. Celem niniejszej pracy jest opracowanie modelu naprężeń resztkowych w blachach walcowanych na gorąco z wykorzystaniem sprężysto-plastycznego modelu materiału z uwzględnieniem relaksacji naprężeń.

Wpływ odkształcenia oraz przemiany fazowej na wielkość naprężeń resztkowych w blach walcowanych na gorąco przedstawiono w pracy [3]. Model MES procesów chłodzenia laminarnego oraz w kręgu wykorzystuje elasto-plastyczny model materiału zależny od temperatury. Relaksacja naprężeń termicznych rozpatrywana jest w oparciu o równania teorii pełzania. Współczynniki modelu materiału oraz modelu pełzania uzyskano z danych eksperymentalnych dla stali S235 i S355 w zakresie temperatur 35-1100°C. W eksperymencie próbkę rozciągano z odkształceniem 0.01-0.02, a następnie wytrzymywano bez zdejmowania obciążenia. Testy przeprowadzono na symulatorze fizycznym GLEEBLE 3800. Model odkształcenia termicznego podczas chłodzenia uzyskano na podstawie testów dylatometrycznych przy prędkościach chłodzenia od 0.057°C/s do 60°C/s.

Fizyczną symulacje naprężeń własnych wykonano celem sprawdzenia poprawność modelu materiału i relaksacji naprężeń. Próbki sztywno zamocowano w symulatorze GLEEBLE, po czym ogrzewano do temperatury 1200°C i chłodzono do temperatury pokojowej z szybkością 1-50°C/s. W trakcie prób rejestrowano zmianę naprężeń. Badania wykazały dobrą zgodność pomiędzy danymi eksperymentalnymi oraz obliczeniami. Pokazano, że pominięcie efektu relaksacji naprężeń w modelu może prowadzić do przeszacowania naprężeń w wysokich temperaturach i niedoszacowania powstałych naprężeń z powodu zmiany ich znaku w trakcie procesu.

Symulacja naprężeń resztkowych w trakcie walcowania oraz chłodzenia przeprowadzono na podstawie opracowanych modeli. Wykazano, że relaksacja naprężeń oraz przemiana fazowa ma istotny wpływ na rozkład naprężeń resztkowych w blachach gorąco walcowanych a zaniedbanie tych zjawisk może prowadzić do ich niedoszacowania.

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1. Introduction

The problem of residual stresses becomes of practical importance when the laser cutting of strips is applied. High values of these stresses lead to deformation (bending and twisting) of strips during laser cutting. In consequence, it is not possible to get strips with straight edges. For this reason, beyond the demands regarding product microstructure, properties and dimensions, the manufacturers of strips are interested also in reduction of the level of residual stresses. Available methods of experimental determination of these stresses generally cannot be used on-line. Thus, the residual stress measurement requires performing an experimental rolling and cannot be used during the design of technology. For these reasons, methods of calculation of residual stresses in strips became important.

The main factors influencing the residual stresses are: the non uniform distribution of elastic-plastic deformations, phase transformation occurring during cooling and relaxation of the stresses during rolling and cooling [1-3]. The latter factor, despite its significant effect on the residual stress, is scarcely considered in the literature. The problem of calculations the residual stresses in hot-rolled strips which take into account a stress relaxation is considered in the present paper.

Stress relaxation at temperatures of rolling and coiling of the strip influences the results of calculation of the residual stress. Therefore, the stress relaxation must be considered when the elastic-plastic mechanical properties at high temperatures are evaluated. In consequence, the model of stress relaxation will allow to estimate the impact of the rolling process on the residual stresses after the laminar cooling and coiling.

The problem of calculation of residual stresses is generally solved by using the finite element (FE) method. Example of using of ABAQUS software for this purpose is presented in works [2,3]. The current state of knowledge shows that using of three-dimensional model and fine mesh leads to increases in computation time and such a model cannot be used as online control system. On the other hand, using of coarse mesh gives low accuracy of results. Additionally, it was shown in publications [4,5] that only longitudinal stresses have significant influence in rolling process. It allowed to simplify the model and accelerate the calculations [5,6].

Thus, the goal of this study was development of a simple model of residual stresses in hot-rolled strips based on the elastic-plastic material model, taking into account the stress relaxation.

2. Elastic-plastic model of material taking into account the stress relaxation

The schematic representation of strain hardening curve for elastic-plastic material is shown in Fig. 1. The following equations were used to describe strain hardening [7]:

$$\bar{\sigma}(t,\bar{\varepsilon}) = E(t)\bar{\varepsilon} \quad \text{for } \bar{\varepsilon} \le \bar{\varepsilon}_p(t) \tag{1}$$

$$\bar{\sigma}(t,\bar{\varepsilon}) = \bar{\sigma}_p(t) + E_p(t)\left(\bar{\varepsilon} - \frac{\bar{\sigma}_p(t)}{E(t)}\right) \quad \text{for } \bar{\varepsilon} > \bar{\varepsilon}_p(t) \tag{2}$$

$$\bar{\varepsilon}_p(t) = \frac{\bar{\sigma}_p(t)}{E(t)} \tag{3}$$

where: $\bar{\varepsilon}$ - effective strain, $\bar{\varepsilon}_p(t)$ - boundary of plasticity, $E_p(t)$ - modulus of plasticity, $\bar{\sigma}_p(t)$ - yield stress, $E(t)$ - Young modulus, t - temperature.

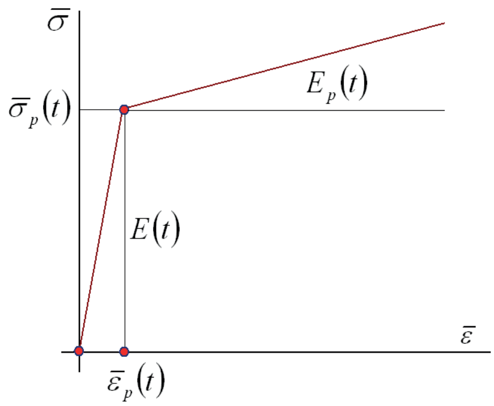


Fig. 1. Schematic representation of material model

According to these equations, the material is characterized by three parameters $E(t)$, $\bar{\sigma}_p(t)$ and $E_p(t)$, each of which depends on the temperature. Equations (2) and (3) can be written in incremental form:

$$\Delta\bar{\sigma}(t,\bar{\varepsilon}) = E(t)\Delta\bar{\varepsilon} \tag{4}$$

$$\Delta\bar{\sigma}(t,\bar{\varepsilon}) = E_p(t)\Delta\bar{\varepsilon} \tag{5}$$

Equation (4) was used for modeling of stress in elastic zone and during unloading process. Equation (5) was used for modeling of stress during an active load in the elastic-plastic zone. Developing a model of residual stress needed to create elastic-plastic material model, taking into account material parameters depending on the temperature and phenomena of stress relaxation. Parameters of this elastic-plastic model were approximated by the following equations:

$$\sigma_p = \frac{\sigma_{p20}}{k_r(\bar{\theta},\tau)}\exp(b_1\bar{\theta})(1 + b_2\bar{\theta} + b_3\bar{\theta}^2 + b_4\bar{\theta}^3) \tag{6}$$

$$E = \frac{E_{20}}{k_r(\bar{\theta},\tau)}\exp(a_1\bar{\theta})(1 + a_2\bar{\theta} + a_3\bar{\theta}^2 + a_4\bar{\theta}^3) \tag{7}$$

$$k_r(\bar{\theta},\tau) = \frac{\sigma}{\sigma_0} \tag{8}$$

$$E_p = E_{p20}\exp\left(c_1\overline{\theta}\right)\left(1 + c_2\overline{\theta} + c_3\overline{\theta}^2 + c_4\overline{\theta}^3\right) \tag{9}$$

$$\overline{\theta} = \frac{t-20}{1000} \tag{10}$$

Where σ_{p20} E_{20} - yield stress and Young module in temperature 20°C, $k_r\left(\overline{\theta},\tau\right)$ - coefficient, which takes into account the stress relaxation during testing of material, *a, b, c* - empirical coefficients which were obtained by processing of tensile tests, σ_0 - mean stress.

During the experimental tests which are used to determine yield stress and Young modulus material is relaxing (especially at high temperature). Therefore, equations (6) and (7) contain correction factor k_r defined by equation (8), which depends on temperature of the test was and on the time at which value of E and σ_p were measured (time measured from the start of the test corresponds to the relaxation time). The phenomenon of material relaxation was taken into account using creep deformation theory, according to which the rate of creep ξ_c is expressed in terms of the relationship:

$$\xi_c = B\left(\overline{\theta}\right)\overline{\sigma}^{n\left(\overline{\theta}\right)} \tag{11}$$

$$B\left(\overline{\theta}\right) = r_{B0} + r_{B1}\overline{\theta} + r_{B2}\overline{\theta}^2 + r_{B3}\overline{\theta}^3 + r_{B4}\overline{\theta}^4 + r_{B5}\overline{\theta}^5 \tag{12}$$

$$n\left(\overline{\theta}\right) = r_{n0} + r_{n1}\overline{\theta} + r_{n2}\overline{\theta}^2 + r_{n3}\overline{\theta}^3 + r_{n4}\overline{\theta}^4 + r_{n5}\overline{\theta}^5 \tag{13}$$

$$\overline{\sigma}_{\tau+\Delta\tau} = \overline{\sigma}_{\tau} - E\left(\overline{\theta}\right)B\left(\overline{\theta}\right)\overline{\sigma}_{\tau}^{n\left(\overline{\theta}\right)}\Delta\tau \tag{14}$$

Where $\overline{\sigma}$ - current value of residual stress, τ - time of relaxation, r_B, r_n - empirical coefficients which were determined on the basis of experimental relaxation curves.

Material model given by equations (6) - (14) was incorporated into the fast model of residual stress during laminar cooling and cooling in coils.

3. Model of residual stresses in hot-rolled strips during laminar cooling and cooling

In the previous author's work [7] FE model of heat transfer in the strip during rolling was developed. This model was based on the Fourier equation taking into account the heat deformation and phase transformation of a cross section of the rolled strip. The model of residual stresses was based on the assumption that all components of the stress tensor except tension along the length of the strip are zero. The strip was presented as a system of rods. In addition to the thermal deformation of each rod, all the rods were exposed to the average strain of the strip $\overline{\varepsilon}_m$ that is a result of the changing of the length of the strip in the cooling process. Thus, if in the rod i an increment of temperature Δt and the corresponding increment of thermal deformation $\Delta\overline{\varepsilon}_t$ appeared, the total increment of the deformation of the rod was equal to:

$$\Delta\overline{\varepsilon}_i = \Delta\overline{\varepsilon}_m - \Delta\overline{\varepsilon}_t - \xi_c\Delta\tau \tag{15}$$

The rate of creep ξ_c was determined by equation (11), and the current strain of the rod i is

$$\overline{\varepsilon}_i = \sum_{n=1}^{n}\Delta\overline{\varepsilon}_i \tag{16}$$

where: n - number of increments of time.

The increment of the thermal deformation was determined taking into account phase transformations, as described in works [7,8]. This relation can be represented in a general form:

$$\Delta\overline{\varepsilon}_t = f\left(t,\Delta t\right) \tag{17}$$

Dilatometric curves for steels S235 and S355, which were used for modeling the $\Delta\overline{\varepsilon}_t$ values were obtained for the cooling rates of 0.057-60°C/s on DIL 805 dilatometer. Increment of stress can be defined by equations (4) - (17) and a generalized form of this increment is:

$$\Delta\overline{\sigma}_i = \overline{\sigma}\left(t,\overline{\varepsilon}_i,\Delta\overline{\varepsilon}_i\right) \tag{18}$$

In the considered method $\overline{\varepsilon}_m$ is an unknown quantity (or $\Delta\overline{\varepsilon}_m$, if the problem is solved in increments). Determination of $\Delta\overline{\varepsilon}_m$ was carried out on the basis of the conditions of equilibrium of the system of *k* rods:

$$\sum_{i=1}^{k}\overline{\sigma}_i\left(t,\overline{\varepsilon}_i,\Delta\overline{\varepsilon}_t,\Delta\overline{\varepsilon}_m\right)S_i = 0 \tag{19}$$

where: k – the number of rods, S_i – cross section area of the rod i, $\overline{\sigma}_i\left(t,\overline{\varepsilon}_i,\Delta\overline{\varepsilon}_t,\Delta\overline{\varepsilon}_m\right)$ - elastic-plastic model of rod.

A numerical solution of the nonlinear equation (19) was performed next and $\Delta\overline{\varepsilon}_m$ was determined [9].

4. Experimental procedure and calibration of materials model

The parameters of relaxation model (11) were determined using tests on the physical simulator Gleeble 3800. The scheme of the experiment is shown in Fig. 2. During the experiment, the sample was deformed with a strain of 0.02 and after stop of deformation the stress relaxation curve was recorded. Equations (4), (7) and (11)-(14) were used for modeling of relaxation, which was considered as unloading process. The parameters of these equations were calculated by the least squares method and the goal function was based on difference between measured and calculated stresses during relaxation. Experiments were performed for temperatures from 35°C to 1000°C.

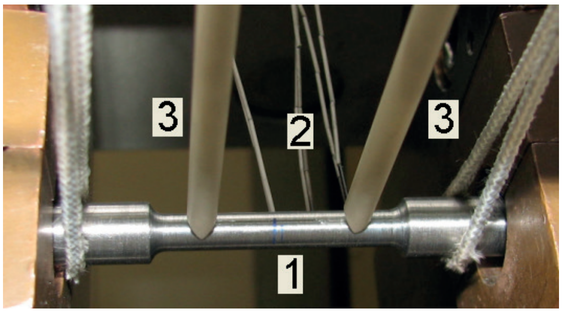
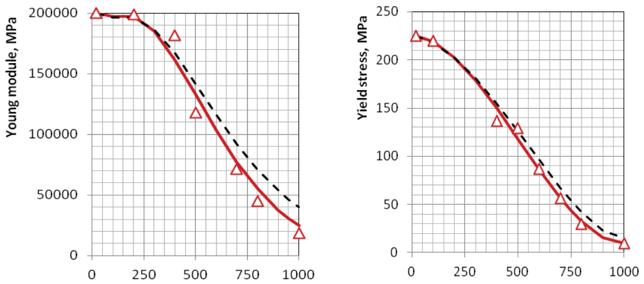


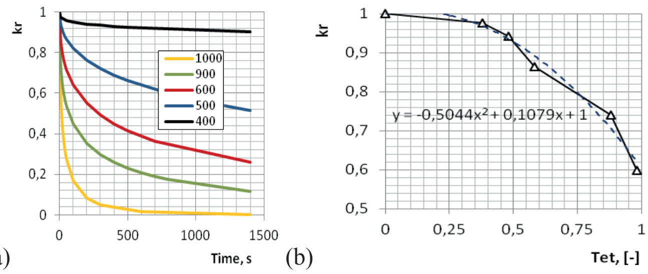
Fig. 2. Scheme of the experiment to determine the material model parameters: 1 - sample, 2 - thermocouple, 3 - extensometers

The temperature dependence of Young modulus and yield stress of steel S235 is presented in Fig. 3. Using experimental data correction curves (equation (8)) for different temperatures were designed (Fig 4a). In next step dependence of k_r on temperature after 7 seconds (time of the experiment) was calculated and it is shown in Fig 4b. The characteristics of yield stress and Young modulus after taking into account of influence of relaxation are presented in Figs 3a and 3b by dashed lines. The values of empirical parameters of material model (4)-(14), which takes into account the stress relaxation process for steels S235 and S355, are presented in table 1.



(a) (b)

Fig. 3. Dependence of Young modulus (a) and yield stress (b) on temperature for steel S235, Δ - experiment, solid line - direct approximation of experiment data, dashed line - model (6)-(8) which takes into account the relaxation of stresses



(a) (b)

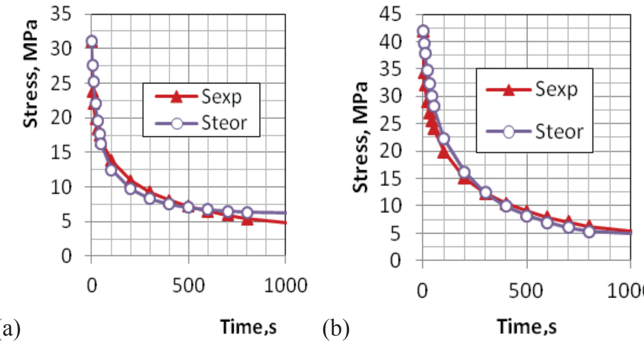
Fig. 4. Dependence of relaxation coefficient k_r on time for different temperatures (a) and on relative temperature for time of relaxation 7 s (b) for steel S235

TABLE 1

Coefficients of material model for steels S235 and S355.

	S235	S355
E20 [MPa]	200000	194000
a_1	4,40296	-6,102609985
a_2	3,639778	4,846877613
a_3	18,60568	34,51050758
a_4	-13,9485	1,25248E-05
a_{kc1}	0,108	7,37E-02
a_{kc2}	-0,5044	-3,75E-01
σ_{p20} [MPa]	225	600
b_1	-0,00162	3,470151943
b_2	-0,10455	-3,189488129
b_3	-2,76118	3,416779656
b_4	1,908628	-1,227081334
r_{b0}	4,598	0
r_{b1}	21,64677	-0,038612545
r_{b2}	-185,937	0,316597117
r_{b3}	255,6452	-0,971835351
r_{b4}	0	1,252213983
r_{b5}	0	-0,48294
r_{n0}	15,82367	5,240989
r_{n1}	-37,4838	4,863380869
r_{n2}	45,65633	-13,62225156
r_{n3}	-20,7	1,178518699
r_{n4}	0	4,441280867
r_{n5}	-0,48294	0

Calibration of material model was done using relaxation data for steels S235 and S355 in the temperature range 400°C–1000°C. Comparison between experimental data and calculation of stress relaxation in temperature 900°C for both analyzed steels is presented in Fig 5. It is clear from those charts that the parameters of materials model were correctly designed.



(a) (b)

Fig. 5. Experimental data and theoretical prediction of stress during relaxation in temperature 900°C for steels S235 (a) and S355 (b)

5. Experimental validation of model of residual stresses

In the experiment, the sample (rod, d = 8 mm, steel S235) with fixed both ends was heated from the temperature 20ºC to 1200ºC and then it was cooled to the temperature 20ºC. During the experiment the change of stress as a function of time and resulted residual stress were monitored. Two variants of the simulation were considered. The developed model of residual stress described by equations (1) - (18) was used. In the first variant A of calculation, the thermal deformation was calculated on the basis of the thermal expansion coefficient of 10^{-5} and stress relaxation were not taken into account. In the second variant B the developed model of stress relaxation and model of phase transformation [3] were added. The results presented in Fig. 6 allow to conclude that including the relaxation process in the calculation has a large impact on the value of stress at high temperatures and substantially changes the stress values in the temperature range of 800ºC÷1200ºC. Taking into account the relaxation resulted in an increase of residual stresses after the cooling process. It is related to the fact that the residual stresses were developed as a result of the unloading processes during the elastic-plastic thermal deformation. Relaxation at high temperatures reduces the level of compressive stress at the beginning of the unloading process, which resulted in the formation of tensile stresses inside the material. Taking into account the nonlinear dependence of the thermal deformation on the temperature has a significant impact on the stress evolution in the form of wave formation (Fig. 6b) on the stress chart (Fig. 6a, b).

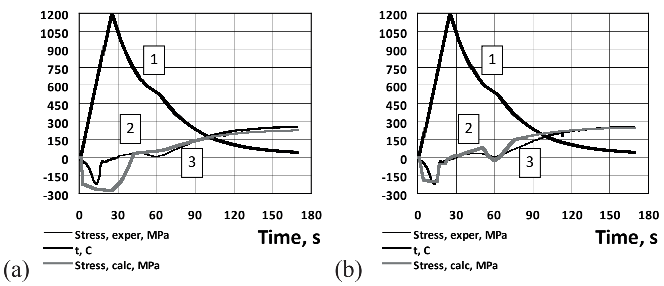


Fig. 6. Validation of developed model: (a) – variant A, (b) – variant B, 1 – change in temperature, 2 – the result of stress modeling, 3 – result of stress measurement (the similar scale for stress and temperature was used)

6. Simulation of residual stresses during hot rolling and cooling

An assessment of the distribution of residual stresses in the strip after rolling and laminar cooling was performed next. The process of the rolling of 1.48 mm thick strip from steel S235 in 11 stands was selected as an example. The mill contains two groups of stands. The first five of the stands are a roughing train and the remaining stands form a finishing train. The time of transport of the strip between the mill groups was about 60 seconds. Schedule of deformation during rolling is shown in Table 2. Detailed description of the rolling technology is given in publication [10].

Simulation of changes in temperature and longitudinal thermal stress was performed. Results of calculations are presented for four points, which are schematically shown in Fig. 7. The results of temperature calculations are shown in Fig. 8. Simulation takes into account the main features of the thermal state of the cooled metal in mill line and thermal effects of deformation. Cooling with water sprays was applied after rolling in the last stand.

The corresponding calculation of thermal stresses is shown in Fig. 9(a) for the linear dependence of thermal deformation on the temperature (variant A). The values of obtained stresses are the result of uneven temperature in the width and thickness of the strip. During the rolling the edges of the strip are a subject to tensile thermal stresses. The center of the strip is dominated by compressive stresses, except the surface. After rolling during cooling and temperature equalization, unloading of thermal stress occurred. However, since in the initial stages of cooling plastic deformation occurred, unloading leads to residual stresses of the opposite sign development. This is seen in all the curves in Fig. 9 after 100 s cooling. High values of compressive stresses at the edges of the strip (up to 200 MPa) and tensile stresses in the central volumes are obtained in the final product. Attention is drawn to the fact that the entire surface of the strip has compressive residual stresses (curves 2s and 4s in Fig. 9), which are balanced by tensile stresses inside the strip (curve 3s in Fig. 9). This result may explain the contradictory data on the distribution of stress across the strip width, as described in work [8]. The answer may lie in the fact that by using the three-dimensional solution based on the finite element method [2,8] it is impossible to use fine FE mesh through the thickness of the strip. This can lead to the homogenization of tensile and compressive stresses along the thickness and erroneous results.

Accounting for relaxation (variant B) leads to the same effect, which was observed in modeling heating and cooling of the rod (Fig. 6b). Fig. 9b shows the effect of reducing stress at high temperatures, which leads to increase of residual stresses after cooling of the strip. Effect of phase transformations on residual stresses is more significant than the effect of relaxation. Fig. 9 shows the result of a calculation, which take into account relaxation and dilatometric curves (variant C). Wave stress changes similar to those observed in the experiments for the rod (Fig. 6b) were evident.

The time of the calculations on a computer DELL Intel i7 26-30QM CPU 2GHz was 6 seconds, which is 40 times less than the time of the process itself. Thus, it can be concluded that the possibility of using this model in the control system is possible.

TABLE 2

Rolling schedule

Stand	1	2	3	4	5	6	7	8	9	10	11
Initial thickness, mm	168.0	142.0	90.0	50.0	33.0	19.0	9.12	5.46	3.37	2.31	1.76
Final thickness, mm	142.0	90.0	50.0	33.0	19.0	9.12	5.46	3.37	2.31	1.76	1.48
Rolling velocity, m/s	0.70	1.07	1.10	1.30	1.88	1.42	2.37	3.83	5.60	7.37	8.73

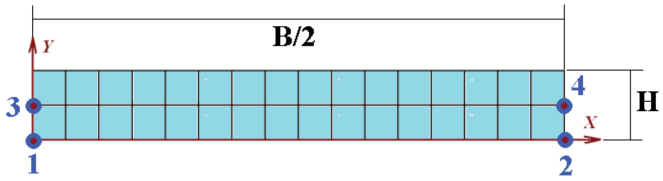


Fig. 7. Position of control points 1-4 at the cross section of strip

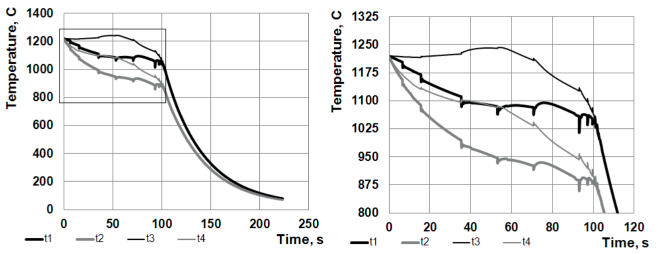


Fig. 8. The simulation results - temperature in control points

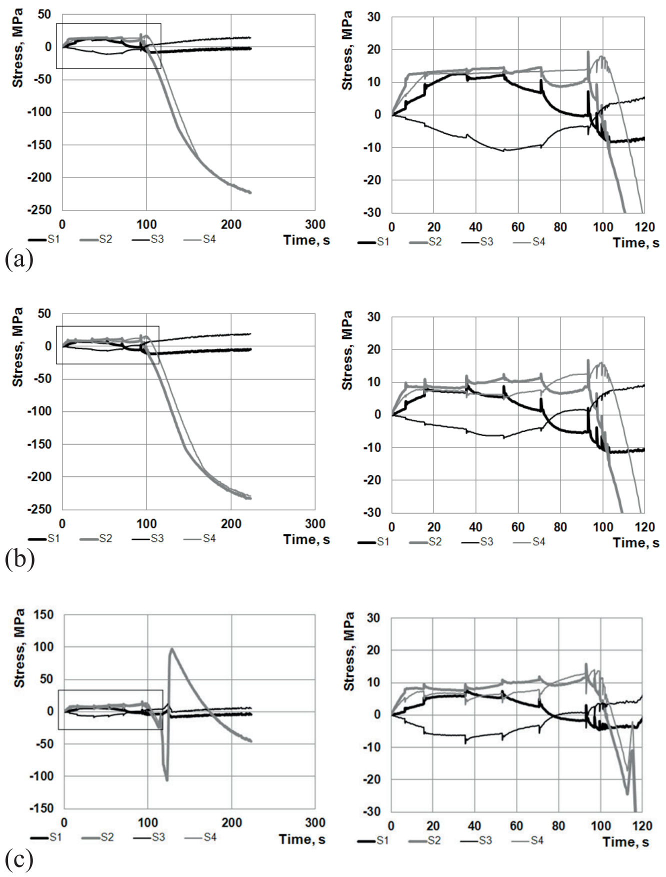


Fig. 9. Distribution of longitudinal stresses in control points during rolling and cooling: (a) - variant A; (b) variant B, (c) variant C

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7. Conclusions

Based on a simplified representation of the stress tensor, the numerical-analytical model of the thermal residual stresses in hot rolled strip was proposed.

It was found that even with a small strip thickness compressive residual stresses can be predicted on the surface of the strip. Compressive residual stresses occur at the edges of the strip through its thickness. Tensile residual stresses are inside the volume of the central part of the strip.

The stress relaxation phenomenon at high temperatures increases the residual stresses after the final cooling of the strip.

Effect of phase transformations on residual stresses is very important and neglecting this effect leads to discrepancies of the model predictions with the experimental data.

The performance of the developed model and the program confirmed its applicability to the control system.

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