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A MEASURING METHOD FOR GYRO-FREE DETERMINATION OF THE PARAMETERS OF MOVING OBJECTS

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Abstract

The paper presents a new method for building measuring instruments and systems for gyro-free determination of the parameters of moving objects. To illustrate the qualities of this method, a system for measuring the roll, pitch, heel and trim of a ship has been developed on its basis. The main concept of the method is based, on one hand, on a simplified design of the base coordinate system in the main measurement channel so as to reduce the instrumental errors, and, on the other hand, on an additional measurement channel operating in parallel with the main one and whose hardware and software platform makes possible performing algorithms intended to eliminate the dynamic error in real time. In this way, as well as by using suitable adaptive algorithms in the measurement procedures, low-cost measuring systems operating with high accuracy under conditions of inertial effects and whose parameters (intensity and frequency of the maximum in the spectrum) change within a wide range can be implemented.

Keywords: adaptive measuring systems, dynamic error, dynamic measurements, Kalman filter, micro-electromechanical systems, roll, pitch, heel, trim.

1. Introduction

Measurements in the dynamic mode are becoming more and more topical in today's metrological theory and practice. It is linked, to a great extent, with the continuous improvement of modern means of transport (ships, aircraft, road transport, *etc.*) in relation to the speed of motion, maneuverability, economy, safety, comfort and so on [1]. Nowadays, in this sense, development and improvement of the measuring equipment for determining the parameters that characterize the space-temporal position, mode of motion, *etc.*, of the above mentioned means of transport are of great significance. The control effectiveness of those moving objects depends on the quality (accuracy, reliability, form and rate of presentation) of the measurement information.

However, measurements in the dynamic mode are more complicated and burdened with a number of problems of metrological nature. This holds true especially for measurements of dynamic quantities carried out under conditions of inertial effects [2]. The existing instruments for measuring the parameters of moving objects are distinguished for their sophisticated design intended to provide high dynamic accuracy. Nevertheless, this leads to an increase in the value of the instrumental error, as well as to a number of other disadvantages [3].

Therefore, one of the main tasks of metrology in this area refers to development of methods and instruments providing high measurement accuracy under conditions of inertial effects whose parameters (intensity and frequency of the maximum in the spectrum) change in a wide range. The purpose of the presented paper can be formulated in this context, namely to present

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a method for developing measuring systems for determining the space-temporal position of moving objects in relation to the local vertical.

2. A block diagram of the measuring method

The key quality indicator of measuring instruments is their accuracy. A distinguishing feature of the measuring instruments considered in this paper is that they operate under conditions of dynamic actions created either by motion of moving objects (shaking of ships, fluctuations of aircraft and road means of transport) or by vibrations existing in the location of the measuring instruments [4]. These motions generate inertial forces and moments which act upon the measuring instruments and systems causing a dynamic error in the measurement result [5]. When no appropriate solutions exist in the metrological chains and procedures of the measuring instruments, the dynamic error can be considerable, which leads to high inaccuracy of the measurement result.

To reduce the influence of the inertial forces and moments in the existing measuring instruments, gyro-stabilized systems are mainly used. They model the base coordinate system with regard to which the position of a moving object is measured when it rotates around its centre of mass and moves along with it [6–8]. In addition, the set motion direction is kept. This complicated model for reproducing the direction of the vertical results in a number of disadvantages [3]. The major ones are as follows: a sophisticated design which leads to increasing the value of the instrumental error, less reliability under extreme conditions, requirement of special systems ensuring the gyro-vertical operation, large sizes, high prices of the instrument or system, *etc*.

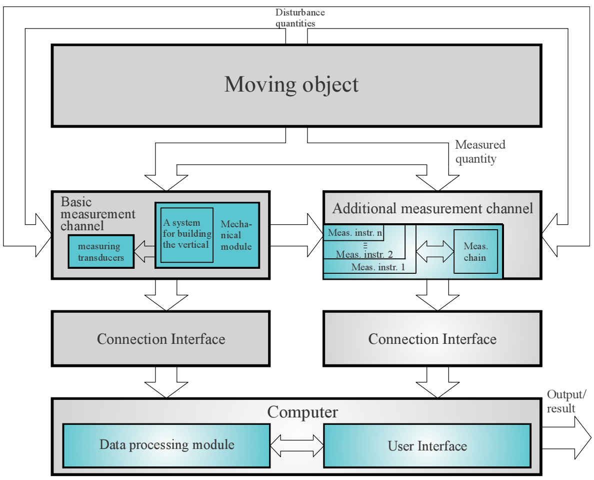

Fig. 1. A block diagram illustrating the principle of modelling measuring systems.

Unlike the above mentioned, the paper presents a new method for developing measuring systems in this area. It is based on a different concept which aims to eliminate, in real time, the dynamic error caused by the deviation of the vertical in the inertial space rather than to stabilize it. The applied method overcomes the disadvantages of the existing measuring instruments as it is based, on one hand, on a very simplified mechanical module, and – on the other hand – on capabilities of modern microprocessor and computer equipment. Furthermore, it is based on successfully integrated processing algorithms intended to eliminate the dynamic error. In this way the system algorithm for defining the result is sub-adjusted automatically by adapting to changes of the measured quantity and the system operating conditions.

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The block diagram in Fig. 1 shows the general concept of the proposed method. The generalized block diagram of the measuring system, developed according to the proposed method, consists of the main measurement channel, an additional channel, and interfaces to computer and software modules for processing and presenting the measurement information. The main channel provides information about the values of the measured quantities. Due to its simplified design and lack of systems for stabilizing the base coordinate system, the signal at the output of this channel contains a dynamic error as a result of deviations of the elements modeling the local vertical from its actual position. The procedure referring to obtaining the measurement information required for defining the current values of the dynamic error is implemented in the additional channel (Fig. 1). The latter operates in parallel with the main one, which enables to eliminate the dynamic error from the measurement result in real time. The structure of the additional channel and the type of the instruments included in it are specified on the basis of the model chosen for determining the current values of the dynamic error and the algorithm for correcting the signal from the main measurement channel.

3. A measuring system for determining the roll, pitch, heel and trim of a ship

3.1. The main measurement channel

To illustrate the features of the proposed method, the characteristics of a specific measuring system developed in compliance with the above mentioned concept are presented. The system is designed for measuring the roll, pitch, heel and trim of a ship. The main channel is intended to measure the current values of the roll and pitch. For this purpose the angles of deviation of the frame are determined along the measuring coordinates of the heel and trim with regard to the vertical modeled by a physical pendulum having two degrees of freedom.

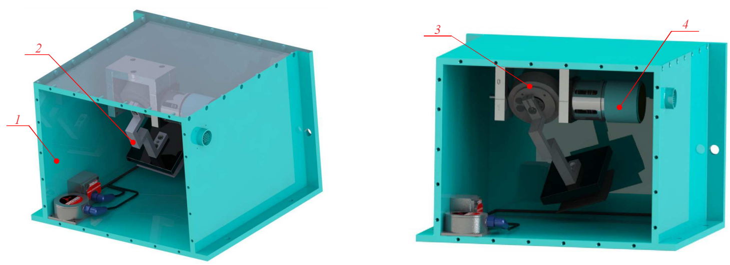

Fig. 2. The design model of the main measurement channel: 1 – frame; 2 – pendulum; 3 and 4 – absolute encoders

The 3-D model of the main channel is shown in Fig. 2. A physical pendulum 2 is attached to the frame 1 so as to have two degrees of freedom with regard to the coordinates which will be defined in the mathematical model below. The angles of rotation of the frame 1 with regard to the pendulum 2 along the heel and trim coordinates are recorded by two absolute encoders 3 and 4. The encoders are intended to convert angular displacements and positions into coded electrical signals corresponding to the absolute position between the frame 1 and the shaft 2. Application of differential parallel scanning of each bit of the rotating scale in Gray code eliminates errors due to interferences and provides a wide operating temperature range. The absolute encoders are distinguished for their high accuracy, high noise immunity, fast response, wide range of supply voltage and small size. In this particular case absolute encoders of 2^{13} bit resolution are used. An additional bit is provided in the encoders. It is analogous to the least-significant one but de-phased by 90 electrical degrees. This makes possible increasing the resolution to 2^{16} bits by analogue interpolation.

3.2. The mathematical model of the dynamic error

In the presented paper the model of the dynamic error is a theoretical basis not only for analysis of the dynamic accuracy. Actually, the functional and structural organization of this error underlies the above mentioned method. Therefore, the mathematical model of the dynamic error is an important tool for solving the problem related to synthesis of the measuring system.

The block diagram developed in [5] offers a new approach to modeling and examining the dynamic error. It is based on the mathematical model of the function of the dynamic error as an independent component having particular characteristics and participating in the formation of the measurement result. In this way the chosen concept enables not only to examine the characteristics of the dynamic error but also to develop algorithms [9–11], methods [12] and instruments [13, 14] for discriminating it as an independent component.

As it has already been said, developing the main measurement channel is focused on a simplified design of the vertical in the form of a physical pendulum. The current values of the angles defining the heel and trip of a ship, as well as their dynamic analogues – roll and pitch, are measured with regard to the base coordinate system established with the help of the physical pendulum. Hence, each deviation of the physical pendulum from the actual position of the local vertical leads to an error in the measurement result. The deviations of the pendulum are due to the inertial effects caused by the dynamic characteristics of the measured quantity and the interference effects. Determined in the time domain, the deviations are defined as stationary random processes, and in a mathematical form, depending on their degrees of freedom, they are obtained as solutions to one or more differential equations.

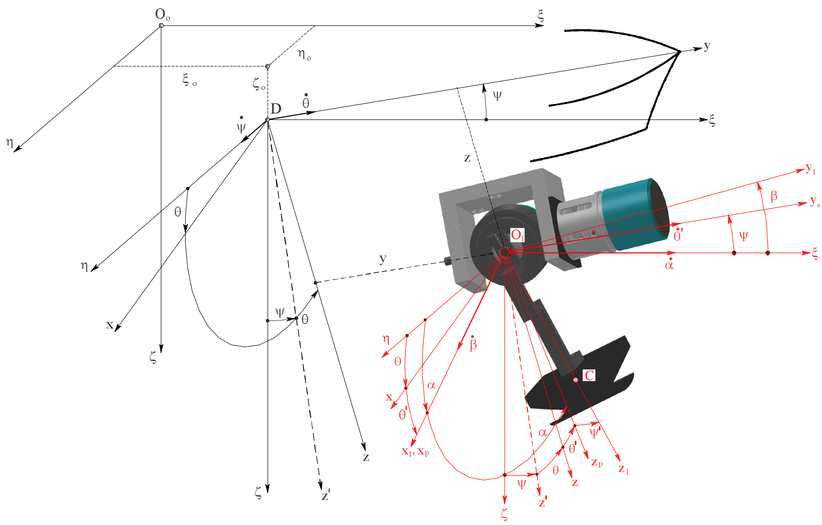

Fig. 3. The dynamic system for working out differential equations.

The system of differential equations is worked out on the basis of both Lagrange's equations of the second kind and the dynamic system presented in Fig. 3. The motions of a ship are defined as angular and linear fluctuations of a rigid body around or along with its centre of gravity. The moving object (the ship), to which the coordinate system *Oxyz* is assigned, changes its position randomly in relation to the Earth's coordinate system $O_o\xi\eta\zeta$. The measuring instrument is mounted on the ship and its sensor (the physical pendulum) is connected to the coordinate system Cx_1yz_1. The suspension point O_1 of the instrument sensor coincides with the diametrical plane of the ship and its position related to the centre of gravity of the moving object *O* is defined by the *z* and *y* coordinates. The coordinate system $O_1\xi\eta\zeta$, which is analogous to $O_o\xi\eta\zeta$

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but shifted in the suspension point, is connected to point O_1. Actually, the system $O_1\xi\eta\zeta$ is the base coordinate system whose axis $O_1\zeta$ determines the direction of the local vertical. The position of the moving object in relation to the Earth's coordinate system $O_o\xi\eta\zeta$ is set by three coordinates of its centre of gravity $O - \xi_o$, η_o, ζ_o and the matrix $A = \|a_{ij}\|$ $(i,j = 1,2)$ of the given angle cosines between the axes of the systems $O_1\xi\eta\zeta$ and $Oxyz$.

The physical pendulum has two degrees of freedom and its generalized coordinates are α and β. They define the angular displacement of the pendulum from the vertical when it rotates around axes $O_1\xi$ and O_1x_1. The choice of those generalized coordinates enables to define the dynamic error that forms upon measuring the trim and pitch by means of the β coordinate, as well as to define the dynamic error that forms in the measurement channel for the heel and roll by means of the α coordinate. The angular coordinates ψ' and θ' determine the position of the pendulum related to the ship's coordinate system $Oxyz$. The angles of the ship's trim and heel, which are determined on the basis of the measured functions of the pitch $\psi(t)$ and roll $\theta(t)$, are denoted by ψ and θ, respectively.

Since the origin of the moving coordinate system $Cx_1y_1z_1$ coincides with the pendulum centre of mass, the following formula can be used to determine the kinetic energy of the system:

$$E_k = \frac{1}{2}\cdot m_1 \cdot V_C^2 + \frac{1}{2}\cdot J_{C\omega}\cdot\omega^2, \tag{1}$$

where: m_1 – the pendulum mass; $\vec{V}_C$ – the absolute velocity of the centre of mass C; $J_{C\omega}$ – the body's moment of inertia with regard to the moment axis $C\omega$ through the body's centre of mass; ω – the body's angular velocity.

The absolute velocity of the centre of mass C is determined by the formula:

$$V_C = \sqrt{[\dot{\xi}_C(t)]^2 + [\dot{\eta}_C(t)]^2 + [\dot{\zeta}_C(t)]^2}, \tag{2}$$

where ξ_C, η_C and ζ_C are the coordinates of point C in the coordinate system $O_o\xi\eta\zeta$.

As the coordinate system $Cx_1y_1z_1$ is permanently linked to the physical pendulum, it follows that its inertial characteristics remain constant in time. In this case the mass moments of inertia of the sensor remain constant with regard to the coordinate axes of $Cx_1y_1z_1$. Consequently, the following is obtained for the second addend in (1), which characterizes the pendulum rotary motion:

$$T_r = \frac{1}{2}\cdot J_{C\omega}\cdot\omega^2 = \frac{1}{2}\cdot\Big(J_{x_1}\cdot\omega_{x_1}^2 + J_{y_1}\cdot\omega_{y_1}^2 + J_{z_1}\cdot\omega_{z_1}^2 - 2\cdot J_{x_1y_1}\cdot\omega_{x_1}\cdot\omega_{y_1} - 2\cdot J_{x_1z_1}\cdot\omega_{x_1}\cdot\omega_{z_1} - 2\cdot J_{y_1z_1}\cdot\omega_{y_1}\cdot\omega_{z_1}\Big), \tag{3}$$

where: $J_{x_1}, J_{y_1}, J_{z_1}$ are the pendulum mass moments of inertia with regard to the respective axes of system $Cx_1y_1z_1$; $J_{x_1y_1}, J_{x_1z_1}, J_{y_1z_1}$ are the centrifugal mass moments of inertia with regard to the respective axes of system $Cx_1y_1z_1$; ; $\omega_{x_1}, \omega_{y_1}, \omega_{z_1}$ – the projections of the vector of absolute angular velocity $\vec{\omega}$ on the axes of system $Cx_1y_1z_1$.

The following dependences are obtained for the projections of the vector of absolute angular velocity $\vec{\omega}$:

$$\begin{aligned} \omega_{x_1} &= \dot{\beta} + \dot{\psi}\cdot\cos\alpha, \\ \omega_{y_1} &= \dot{\alpha}\cdot\cos\beta + \dot{\theta}\cdot\cos(\beta-\psi) + \dot{\psi}\cdot\sin\alpha\cdot\sin(\beta-\psi), \\ \omega_{z_1} &= \dot{\alpha}\cdot\sin\beta + \dot{\theta}\cdot\sin(\beta-\psi) - \dot{\psi}\cdot\sin\alpha\cdot\cos(\beta-\psi). \end{aligned} \tag{4}$$

The dynamic characteristics upon the translational motion of system $Cx_1y_1z_1$, whose origin coincides with the centre of mass C, are presented by the absolute velocity V_C which, expressed by its projections on $O_o\xi\eta\zeta$, has the form (2). The parametric functions of coordinates ξ_C, η_C and ζ_C of point C in system $O_o\xi\eta\zeta$ can be defined by Fig. 3, where the functions being sought are of the form:

$$\begin{aligned} \eta_C &= \eta_o - z\cdot\sin\theta\cdot\cos\psi + y\cdot\sin\theta\cdot\sin\psi - l\cdot\cos\beta\cdot\sin\alpha, \\ \xi_C &= \xi_o + z\cdot\cos\theta\cdot\sin\psi + y\cdot\cos\psi + l\cdot\cos\alpha\cdot\sin\beta, \\ \zeta_C &= \zeta_o + z\cdot\cos\theta\cdot\cos\psi - y\cdot\sin\psi\cdot\cos\theta + l\cdot\cos\alpha\cdot\cos\beta, \end{aligned} \tag{5}$$

where: ξ_o, η_o and ζ_o are the coordinates of the ship's centre of mass in the coordinate system $O_o\xi\eta\zeta$; y and z are the coordinates defining the position of the suspension point of the physical pendulum with regard to the ship's centre of mass D; l – the distance from the suspension point O_1 to the pendulum centre of mass C.

After defining the kinetic energy according to (1) and doing substitutions in Lagrange's equations, we obtain the final form of the system of two differential equations describing the motion of the physical pendulum with regard to the local vertical. The matrix format of the system has the following form:

$$\begin{aligned} &\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{Vmatrix} + \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \dot{\alpha}^2 \\ \dot{\beta}^2 \end{Vmatrix} + \begin{Vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \dot{\alpha} \\ \dot{\beta} \end{Vmatrix} + \begin{Vmatrix} d_1 \\ d_2 \end{Vmatrix}\cdot\dot{\alpha}\cdot\dot{\beta} + \\ &+ \begin{Vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \alpha \\ \beta \end{Vmatrix} = \begin{Vmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \ddot{\theta} \\ \ddot{\psi} \end{Vmatrix} + \begin{Vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \dot{\theta}^2 \\ \dot{\psi}^2 \end{Vmatrix} + \\ &+ \begin{Vmatrix} n_1 \\ n_2 \end{Vmatrix}\cdot\dot{\theta}\cdot\dot{\psi} + \begin{Vmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \dot{\theta}\cdot\dot{\alpha} \\ \dot{\theta}\cdot\dot{\beta} \end{Vmatrix} + \begin{Vmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{Vmatrix}\cdot\begin{Vmatrix} \dot{\psi}\cdot\dot{\alpha} \\ \dot{\psi}\cdot\dot{\beta} \end{Vmatrix} + \begin{Vmatrix} s_1 \\ s_2 \end{Vmatrix}\cdot\ddot{\xi}_o + \\ &+ \begin{Vmatrix} t_1 \\ t_2 \end{Vmatrix}\cdot\ddot{\eta}_o + \begin{Vmatrix} y_1 \\ y_2 \end{Vmatrix}\cdot\ddot{\zeta}_o. \end{aligned} \tag{6}$$

The elements of the separate matrices in (6) are functions of the parameters defining the geometric and mass inertial characteristics of the measuring instrument and quantities determining the position of the ship and physical pendulum with regard to their equilibrium positions. The differential (6) are formed under the influence of a number of parameters which, depending on the deciding factors, can be divided into the following groups: parameters determining the characteristics of rough waters; parameters defining the form, size, geometric and mass characteristics of the ship; parameters determining the position of the ship with regard to the direction of the waves and kinematic parameters characterizing its motion; hydrodynamic parameters characterizing interaction between water and the ship; geometric and mass parameters characterizing the design of the instrument, and parameters defining its position related to the ship's centre of gravity.

Equation (6) has the linearized form:

$$\left|\begin{aligned} &\left(J_{y_1} + m_1\cdot l^2\right)\cdot\ddot{\alpha} + b_1\cdot\dot{\alpha} + m_1\cdot g\cdot l\cdot\alpha = m_1\cdot l\cdot\ddot{\eta}_o - \left(J_{y_1} + m_1\cdot l\cdot z\right)\cdot\ddot{\theta}, \\ &\left(J_{x_1} + m_1\cdot l^2\right)\cdot\ddot{\beta} + b_2\cdot\dot{\beta} + m_1\cdot g\cdot l\cdot\beta = -m_1\cdot l\cdot\ddot{\xi}_o - \left(J_{x_1} + m_1\cdot l\cdot z\right)\cdot\ddot{\psi}. \end{aligned}\right. \tag{7}$$

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The mathematical model defined by the (7) describes the linearized form of interrelation between the dynamic error and the values of the measured quantity, the design parameters and the influencing quantities. Availability of the mathematical models (6) and (7) enables to predict the measurement results when the system operates under different running conditions, as well as to optimize its structure and parameters as their choice is subjected to the conditions assuring the minimum error in the measurement result.

3.3. Synthesis of the additional measurement channel

The additional measurement channel is developed on the basis of a calculation-based measuring method, where we assume that the information on the state of the system of the main channel can be obtained by means of the theoretical model of its dynamics, whose input vector is obtained as a result of measurement. We presume that the vector of state of the dynamic system satisfies the following system of differential equations:

$$\frac{d^2\mathbf{E}(t)}{dt^2} = F1(t)\cdot\frac{d\mathbf{E}(t)}{dt} + F2(t)\cdot\mathbf{E}(t) + G1(t)\cdot\mathbf{U_0}(t) + G2(t)\cdot\mathbf{U}(t), \tag{8}$$

where: $\mathbf{E}(t)$ – the n-dimensional vector of the state of dynamic system; $\mathbf{U_0}(t)$ and $\mathbf{U}(t)$ – the r – dimensional vectors defining the linear and angular inertial effects, respectively; $F1(t)$ and $F2(t)$ – matrices of $n \times n$ type; $G1(t)$ and $G2(t)$ – matrices of $n \times r$ type.

Since, according to the concept of the proposed method, the additional measurement channel aims to determine the current values of the dynamic error which should be used for correcting the results from the main channel, it follows that the vector of state is identical with the model of the dynamic error presented in the previous section. Therefore, synthesis of the additional channel will be based on the differential (7), which can be reduced to the form:

$$\begin{aligned}\ddot{\alpha}(t) &= -\frac{b_1}{J_{y_1}+m_1\cdot l^2}\cdot\dot{\alpha}(t) - \frac{m_1\cdot g\cdot l}{J_{y_1}+m_1\cdot l^2}\cdot\alpha(t) + \frac{m_1\cdot l}{J_{y_1}+m_1\cdot l^2}\cdot\ddot{\eta}_0(t) - \frac{J_{y_1}+m_1\cdot l\cdot z}{J_{y_1}+m_1\cdot l^2}\cdot\ddot{\theta}(t),\\ \ddot{\beta}(t) &= -\frac{b_2}{J_{x_1}+m_1\cdot l^2}\cdot\dot{\beta}(t) - \frac{m_1\cdot g\cdot l}{J_{x_1}+m_1\cdot l^2}\cdot\beta(t) - \frac{m_1\cdot l}{J_{x_1}+m_1\cdot l^2}\cdot\ddot{\xi}_0(t) - \frac{J_{x_1}+m_1\cdot l\cdot z}{J_{x_1}+m_1\cdot l^2}\cdot\ddot{\psi}(t).\end{aligned} \tag{9}$$

In conformity with the vector-matrix differential (8) we can write the vector of the state of the dynamic system, *i.e.*:

$$\mathbf{E}(t) = \begin{vmatrix}\alpha(t)\\ \beta(t)\end{vmatrix} = \begin{vmatrix}\varepsilon_{\alpha_{de}}(t)\\ \varepsilon_{\beta_{de}}(t)\end{vmatrix}, \tag{10}$$

where $\varepsilon_{\alpha_{de}}(t)$ and $\varepsilon_{\beta_{de}}(t)$ are the dynamic errors along the measuring coordinates of the heel and trim, respectively.

Then, the matrix coefficients $F1$ and $F2$ from (8) will be defined by the following diagonal matrices:

$$F1 = \begin{vmatrix}-\frac{b_1}{J_{y_1}+m_1\cdot l^2} & 0\\ 0 & -\frac{b_2}{J_{x_1}+m_1\cdot l^2}\end{vmatrix}, \quad F2 = \begin{vmatrix}-\frac{m_1\cdot g\cdot l}{J_{y_1}+m_1\cdot l^2} & 0\\ 0 & -\frac{m_1\cdot g\cdot l}{J_{x_1}+m_1\cdot l^2}\end{vmatrix}. \tag{11}$$

The inertial interference effects enter the input of the dynamic system. They determine the emergence of a dynamic error along the two measuring coordinates. Within the mathematical models (8) and (9) those effects are divided into the linear and angular ones, whose vector-matrix form can be written as follows:

$$\mathbf{U_0}(t) = \begin{vmatrix} \ddot{\eta}_0(t) \\ \ddot{\xi}_0(t) \end{vmatrix}, \qquad \mathbf{U}(t) = \begin{vmatrix} \ddot{\theta}(t) \\ \ddot{\psi}(t) \end{vmatrix}, \tag{12}$$

Their matrix coefficients according to (9) have the following form:

$$G1 = \begin{vmatrix} -\dfrac{m_1 \cdot l}{J_{y_1} + m_1 \cdot l^2} & 0 \\ 0 & -\dfrac{m_1 \cdot l}{J_{x_1} + m_1 \cdot l^2} \end{vmatrix}, \quad G2 = \begin{vmatrix} -\dfrac{J_{y_1} + m_1 \cdot l \cdot z}{J_{y_1} + m_1 \cdot l^2} & 0 \\ 0 & -\dfrac{J_{x_1} + m_1 \cdot l \cdot z}{J_{x_1} + m_1 \cdot l^2} \end{vmatrix}. \tag{13}$$

To put the above written mathematical dependences into the consistent logic of the metrological procedure, a block diagram of the additional measurement channel has been worked out. It is shown in Fig. 4. The latter is based on the vector-matrix differential (8), where the structure of the main measurement channel and the mathematical models (9) determining the formation of the dynamic errors $\varepsilon_{\alpha_{de}}(t)$ and $\varepsilon_{\beta_{de}}(t)$ are taken into consideration.

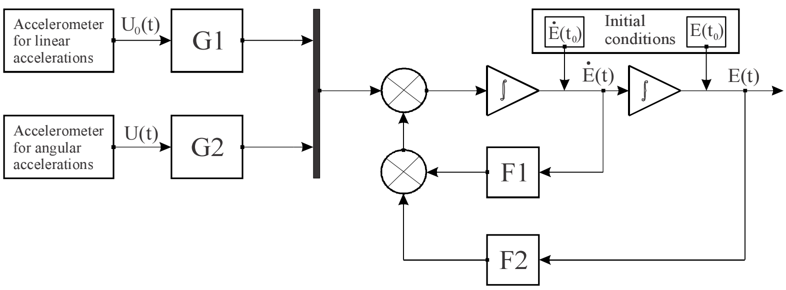

Fig. 4. A block diagram of the additional measurement channel.

3.4. A block diagram of the measuring system

A block diagram of the measuring system is shown in Fig. 5. As it can be seen, at the outputs of two absolute encoders the signals $\theta_r(t) = \theta(t) + \alpha(t)$ and $\psi_r(t) = \psi(t) + \beta(t)$ are obtained. Due to behaviour of the physical pendulum in the field of force they contain the dynamic errors $\alpha(t) = \varepsilon_{\alpha_{de}}(t)$ and $\beta(t) = \varepsilon_{\beta_{de}}(t)$. To eliminate the dynamic errors from the measurement result, the two output signals are used.

The correction signals $\alpha_s(t)$ and $\beta_s(t)$ are obtained as a result of operation of the additional measurement channel whose operational algorithm is formed as a measuring calculation-based analogue of the pendulum current motion. The calculation operations are performed in real time on the basis of the current measurement information about the interference effects which at the same time influence the real sensor. The external actions are measured by two *Micro-Electromechanical Systems* (MEMS) whose positions in the frame of the mechanical block are shown in Figs. 4 and 5. To measure the linear accelerations $\ddot{\eta}_0(t)$ and $\ddot{\xi}_0(t)$ and the angular

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accelerations $\ddot{\theta}(t)$ and $\ddot{\psi}(t)$, Digital Type Accelerometer AKE392B [15] and Digital Type Gyroscope TL632D [15] MEMS are used, respectively.

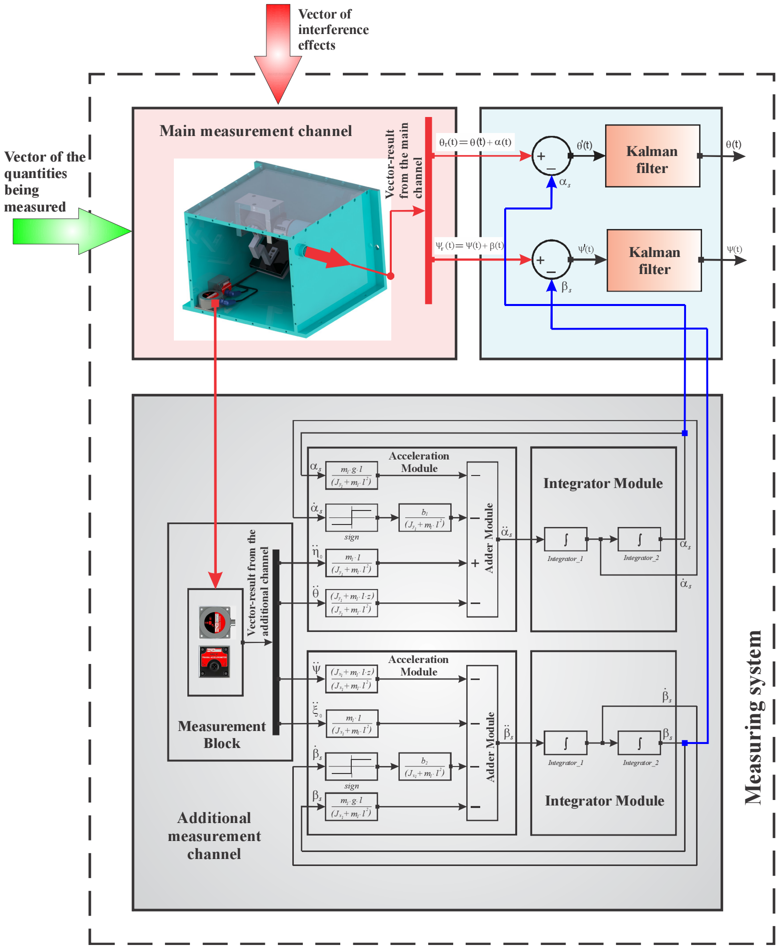

Fig. 5. A block diagram of the measuring system.

The calculation block consists of two main calculation modules – Acceleration Module and Integrator Module. The organization of operation of these two modules is presented in detail in Fig. 5. Due to the presence of interference sources of random characteristics, additional secondary processes of unpredictable behavior, as well as the possibility for the emergence of errors resulting from the transformation processes, additional errors which may considerably reduce the measurement accuracy could appear in the system. Consequently, to reduce the influence of the above listed effects, a module based on the Kalman method is included in the structure of the measuring system. The algorithm of that module is formed so as to define an optimal estimate of the measured quantity according to the minimum mean square error criterion. It is based on the actual model of the dynamics of the moving object. A detailed description of the module will be a subject of another work.

4. Experiments and results

The experiments have been carried out in order to check the actual characteristics of the proposed method. A stand with equipment reproducing the ship's motions along its six degrees of freedom has been developed so that the experiments can be performed under conditions that are maximally close to the real ones. The mechanical module of the equipment is a stand simulator based on the six-degree-of-freedom Stewart platform, which provides the required sensitivity and maneuverability of the operating platform. To increase the positioning accuracy, linear resistive sensors, whose error is ± 0,05%, are additionally mounted in the actuators. This enables to provide high positioning accuracy of the operating platform, where the error of angular positioning does not exceed 20″.

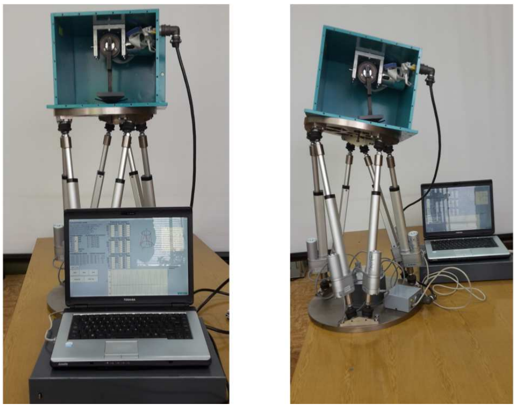

Fig. 6. Photos from the experiments.

The stand equipment has referential qualities which are provided by a specifically developed output instrument which is calibrated by means of references for length. It enables not only to examine the accuracy characteristics but also to calibrate instruments and systems measuring the space-temporal position of ships. Some photos illustrating the stand equipment and the experimental process are shown in Fig. 6.

Figures 7, 8, 9 and 10 present in a graphic form the results from the experiments referring to the prototype of the measuring system. Distribution of the boundaries of the systematic error in relation to the angular frequency with which the operating platform performs angular fluctuations upon examining two measurement channels is shown in Figs. 7 and 8.

Figure 7 shows the results from examining the accuracy of the measurement channel for the roll and heel movements, whereas Fig. 8 – the ones for the pitch and trim movements. Both figures present the boundaries of changing the systematic error ($\widetilde{\varepsilon}_{l\,max}$ and $\widetilde{\varepsilon}_{l\,min}$) and the examination has been performed when the module with the Kalman filter was disconnected. The figures show an obvious trend of increasing the systematic error value in the direction of increasing the frequency of fluctuations of the operating platform.

The boundaries which the error of the examined measuring system can reach at a 95% confidence level are shown in Figs. 9 and 10. As it can be seen, the measuring system is stable enough with respect to its dynamic accuracy even if the Kalman algorithm is not connected. In this case the maximum values of the dynamic error do not exceed 0,16°.

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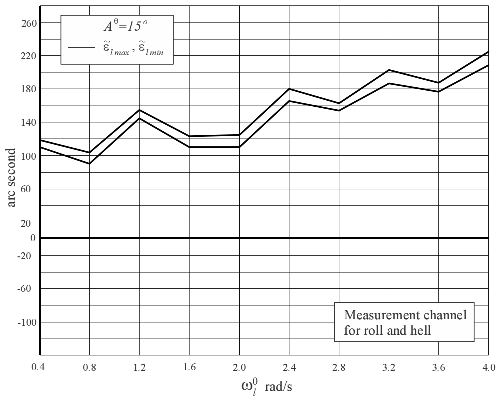

Fig. 7. Values of the systematic error, obtained upon measuring the roll and heel.

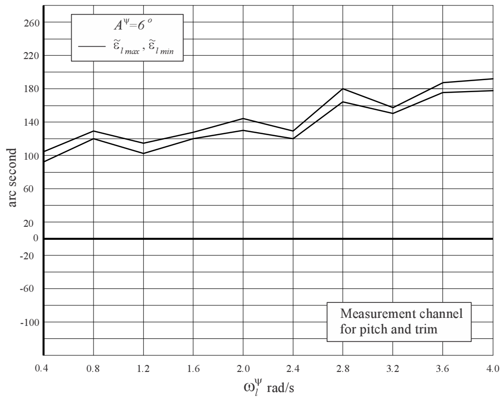

Fig. 8. Values of the systematic error, obtained upon measuring the pitch and trim.

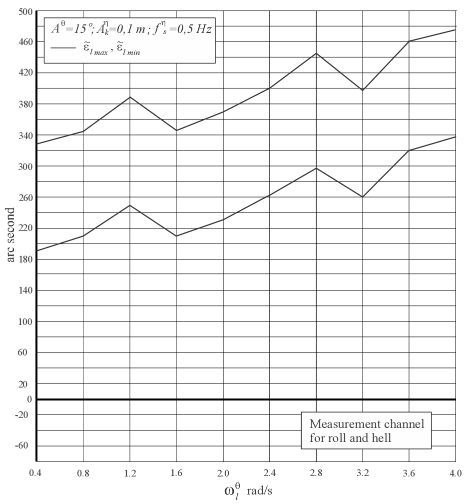

Fig. 9. Boundaries of the confident intervals of the random component for the channel of the roll and heel.

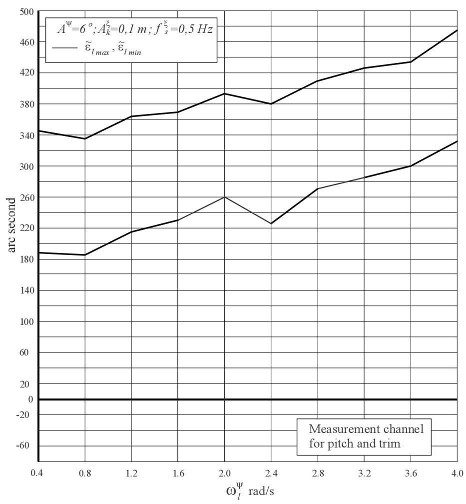

Fig. 10. Boundaries of the confident intervals of the random component for the channel of the pitch and trim.

5. Conclusions

The results of the experiments confirm high accuracy of the developed measuring system under conditions of dynamic actions close to the real ones. When the system operates in the mode where the Kalman filter is disconnected, its accuracy approaches the accuracy of the best modern measuring instruments and systems. As a result of operation of the module based on the Kalman filter the system accuracy considerably increases under the conditions of dynamic actions without using expensive stabilization systems.

The above conclusion demonstrates the qualities and perspectives of the proposed method which makes possible development of measuring systems for determining the parameters of moving objects that operate with a high accuracy under dynamic conditions. An example of this is the measuring system being considered in this paper, as well as the system based on the method and presented in [16].

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