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COMPARATIVE ANALYSIS OF SELECTED MODELS OF WATER CONING IN GAS RESERVOIRS

ANALIZA PORÓWNAWCZA WYBRANYCH MODELI POWSTAWANIA STOŻKÓW WODNYCH W ZŁOŻACH GAZOWYCH

Exploitation of natural gas fields with edge or underlying water is usually defined per analogy to the oil fields. The existing models do not correspond to reality as they do not describe relevant processes related with a turbulent gas flow near the well. The natural gas exploitation with productivity greater than critical may be advantageous in view of summaric depletion and rate of depletion. Article presents: the analysis of the selected critical rates models, determining the influence of specific parameters on the critical rate values, introducing new modified formula for critical rates, and comparative calculations for various configurations with the numerical model.

Keywords: water coning, critical gas rate

Problem eksploatacji złóż gazu ziemnego z wodą podścielającą lub okalającą jest określany zwykle na podstawie analogii ze złożami ropnymi. Istniejące modele nie odpowiadają rzeczywistości, ponieważ nie opisują istotnych procesów związanych z turbulentnym przepływem gazu w pobliżu odwiertu. Równocześnie eksploatacja gazu z wydajnością większą od krytycznej może być korzystna z punktu widzenia sumarycznego sczerpania złoża oraz szybkości jego sczerpania. W artykule przedstawiono: analizę wybranych modeli wydajności krytycznej, określenie wpływu poszczególnych parametrów na wartości wydajności krytycznych, wprowadzenie nowych zmodyfikowanych formuł określających wydajności krytyczne oraz przeprowadzenie obliczeń porównawczych dla różnych konfiguracji z wykorzystaniem modelu numerycznego.

Słowa kluczowe: powstawanie stożków wodnych, wydajność krytyczna gazu

Most important denotations

b – length of a section of a completion borehole, [m]

B_i – volumetric coefficient of i-th phase

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q_{cD} – dimensionless critical rate
Q – rate, [m^3/s]
Q_{cDP} – critical gas rate after Dupuit, [m^3_n/s]
Q_{cSC} – critical gas rate after Schols, [m^3_n/s]
h – thickness of layer, [m]
k – permeability, [mD]
k_a – permeability of aquifer, [mD]
k_h – horizontal permeability, [mD]
k_v – vertical permeability, [mD]
k_i – relative permeablity for i-th phase, [mD]
p – pressure, [Pa]
p_o – primary pressure, [Pa]
r_e – radius of well's impact, [m]
r_{eq} – equilibrium radius, [m]
r_w – radius of the well, [m]
S – skin effect
S_i – saturation with i-th phase
v – filtration rate after Darcy, [m/s]
μ_i – viscosity of i-th phase, [Pas]
ρ_i – density of i-th phase, [kg/m^3]

Indices

g – gaseous phase
o – oil phase
w – water phase

1. Introduction

The effect of water coning is typical of hydrocarbon completion wells, with a zone saturated with edge or underlying water. Originally, i.e. before the exploitation begins, the hydrocarbon and water interface usually constitutes a horizontal plane. When the exploitation begins, a lower pressure zone is formed around it causing relaxation and flux of fluids into the well. The disturbed pressure zone may reach the watered part of the field, causing water movement towards the well and deformation of the hydrocarbon and water interface. For vertical wells, such a deformation assumes the form of a cone.

The analyses of water coning effect in hydrocarbon fields are usually focused on the equilibrium between forces evoked by pressure difference and force of gravity. Forces produced by the pressure gradient tend to increase water level towards the well and are proportionate to the rate with which the well is exploited. The maximum output, for which the hydrocarbon is still produced waterfree is called critical rate and is most frequently modeled when solving water cone expansion problems.

2. Selected analytical and experimental models describing water coning at the gas-water flow

2.1. Dupuit model

Investigations of water coning carried out by Dupuit (1865) were performed at certain simplifying assumptions. The fluid flow in the reservoir was assumed maintain a distinct division line between gas and water, which signifies that capillary forces are neglected, and pushing of gas with water has a piston character. This is justified by considerable difference of density and viscosity of both those fluids. Then follows the analysis of radial fluid flowing to the borehole, which partly opens up the natural gas field with underlying water, and which produces gas at critical rate (fig. 1).

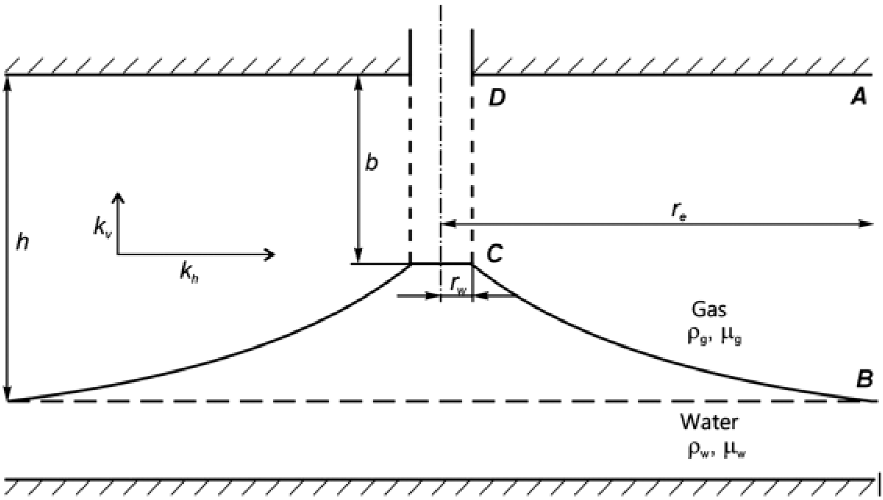

Fig. 1. Scheme of water cone in natural gas field

It was additionally assumed that the flow in the reservoir was steady, the reservoir was homogeneous, and fluid density and viscosity were constant. The assumption of constant viscosity and density is justified till the moment the drop of pressure in the near-zone area is small as compared to the average reservoir pressure. The detailed derivation of an equation for critical rate has been presented in Hagoort (1988). On this basis the following relation has been assumed:

$$Q_{cDP} = \frac{\pi g k_g k_{rg} (\rho_w - \rho_g)(h^2 - b^2)}{\mu_g B_g \ln\left(\frac{r_e}{r_w}\right)} \tag{1}$$

This holds true for an isotropic medium. As Polubarinova-Kochina (1962) suggests, for an anisotropic medium the dependence (1) can be multiplied by $1/\sqrt{k_v/k_h}$.

2.2. Schols model

Schols (1972) determined the critical rate value using a physical Hele-Shaw model, made of two parallel transparent plates with glass granulate between them to represent a porous medium. This type of model reduced the water coning effect to two dimensions. It was observed that the size of the model should be properly selected, especially the distance between the plates, being a representation of permeability of the medium. According to previous studies by Aravin (1938) and Efros (1957), the distance should be multiple of cube root of horizontal length of the model. Thus selected distance should allow for using the Hele-Shaw model for simulating the symmetrical, radial flow of fluid in the porous medium. Schols used three models of various size in his experiments.

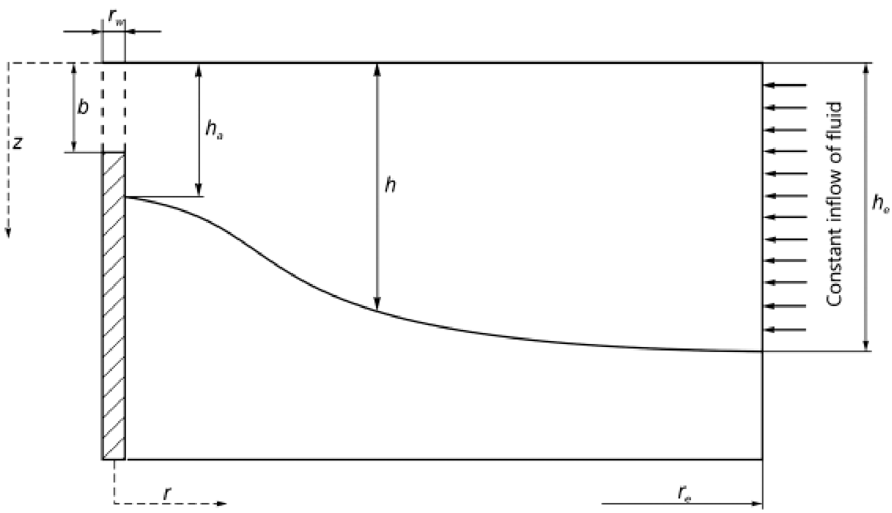

Fig. 2. Scheme of Hele-Shaw model used by Schols in his experiment

In all three models the critical rate was measured as a function of length of the well completing a productive layer and its thickness. On this basis the dimensionless critical rate q_{cD} could be determined in the form:

$$q_{cD} = \left[0.432 + \frac{\pi}{\ln\left(\frac{r_e}{r_w}\right)}\right]\left[1 - \left(\frac{b}{h_e}\right)^2\right]\left(\frac{h_e}{r_e}\right)^{2.14} \tag{2}$$

Applying additional assumptions and using a combination of equations of flow and continuity at boundary conditions resulting from the geometry of the physical model, the following formula for the critical rate was obtained:

$$Q_{cSC} = \frac{gk_o \Delta\rho (h_e^2 - b^2)}{\mu_o} \left[0.432 + \frac{\pi}{\ln\left(\frac{r_e}{r_w}\right)} \right] \left(\frac{h_e}{r_e}\right)^{0.14} \tag{3}$$

Introducing an element accounting for the reservoir anisotropy, Hagoort (1988) presented Schols correlation for critical gas rate in the following form:

$$Q_{cSC} = \frac{gk_g k_{rg} (\rho_w - \rho_g)(h^2 - b^2)}{\mu_g B_g} \left[0.432 + \frac{\pi}{\ln\left(\frac{r_e}{r_w}\right)} \right] \left(\frac{h}{r_e}\right)^{0.14} \left(\frac{k_v}{k_h}\right)^{-0.07} \tag{4}$$

That model is commonly used for gas engineering, mainly as disseminated in monographs and handbooks, e.g. Hagoort (1988). The actual properties of natural gas (change of density) have not been accounted for in this model.

3. Comparative analysis of selected analytical and experimental water coning models with a numerical model

The water coning at water and gas flow was simulated in this paper with the use of a 3D and three-phase reservoir simulator Eclipse by Schlumberger. For the numerical model of a reservoir (fig. 3) and reservoir parameters (tables 1 and 2) over 2800 numerical simulations were made.

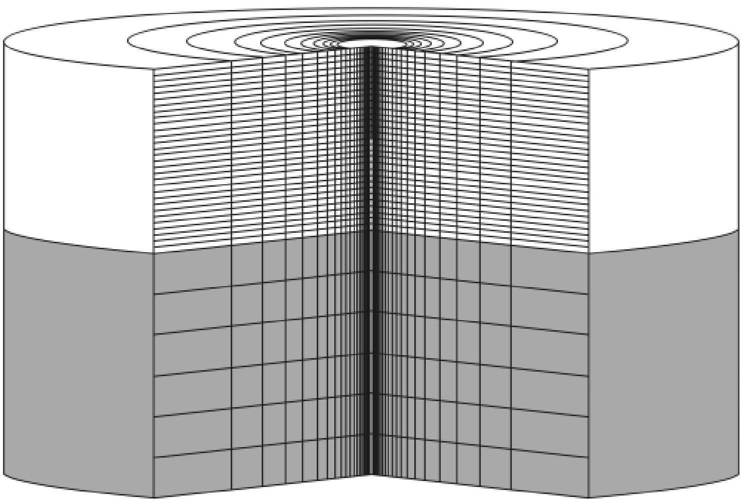

Fig. 3. Scheme of a mesh used for the numerical model

The simulation results were compared with the results of calculations performed with the use of selected analytical-experimental models. The influence of the following parameters on critical rates was analyzed:

- radius of well's impact,
- length of completion well to the thickness of productive layer ratio,
- difference between vertical and horizontal permeability,
- effective radius of well, on which the so-called mechanical skin effect has influence.

TABLE 1

Basic parameters of reservoir model

Reservoir geometry				
Radius of range of well's impact, [m]	750			
Radius of well, [m]	0.1			
Thickness of gas-bearing layer, [m]	30			
Thickness of water-bearing layer, [m[	60			
Number of mesh blocks towards the model radius	26			
Distribution of mesh blocks towards the model radius, [m]	0.15	0.76	0.95	1.18
	1.47	1.84	2.29	2.85
	3.55	4.42	5.50	6.85
	8.53	10.62	13.23	16.47
	20.51	25.54	31.81	39.61
	49.33	61.43	76.50	95.26
	118.37	150		
Number of mesh blocks towards axis Z	36			
Number of mesh blocks towards axis Z (gas-bearing layer)	30			
Number of mesh blocks towards axis Z (water-bearing layer)	6			
Depth of productive layer top, [m]	1500			
Basic properties of rocks and reservoir fluids				
Compressibility of rock matrix, [1/bar]	$1.45 \cdot 10^{-5}$			
Compressibility of reservoir water, [1/bar]	$4.0 \cdot 10^{-5}$			
Density of reservoir water in normal conditions, [kg/m^3]	1019			
Density of natural gas in normal conditions, [kg/m^3]	0.737			
Initial conditions				
Depth of gas/water interface, [m]	1530			
Primary pressure at gas/water interface, [bar]	200			
Primary capillary pressure at gas/water interface, [bar]	0			
Parameters of well				
Length of completion well section, [m]	15			
Skin effect	0			

TABLE 2

Reservoir parameters assumed for comparative analysis of models

Parameter	Value
Radius of range of well's impact, [m]	750
Radius of well, [m]	0,1
Permeability, [mD]	10
Thisckness of gas-bearing layer, [m]	30
Length of completion well section, [m]	15
Porosity, [%]	25
Depth of productive layer, [m]	1500
Density of reservoir water, [kg/m^3]	1019

The relative error was calculated for case:

$$E_w = \frac{Q_{cma} - Q_{cmn}}{Q_{cmn}} \cdot 100\% \tag{5}$$

Q_{cma} — critical rate calculated on the basis of analytical model,
Q_{cmn} — critical rate obtained with numerical simulation method.

The results of calculation of relative error were illustrated in figs. 4 to 11.

TABLE 3

Relative errors of calculation for specific Dupuit and Schols models of critical rate as compared to numerical model

Model	r_e			b/h			k_v/k_h			r_{ef}		
	50	100	200	50	100	200	50	100	200	50	100	200
Dupuit	55.7	55.3	76.1	126.3	122.9	129.0	158.1	169.3	167.4	74.2	83.6	93.9
	62.4			126.1			164.9			83.2		
	Mean weighted error – 121.7											
Schols	129.5	131.3	162.2	223.3	218.6	227.3	152.2	163.9	162.2	59.7	68.6	79.0
	141.0			223.1			159.4			69.1		
	Mean weighted error – 204.9											

Very big values of relative errors (over 100%) inspired author to look for their causes. One of them may be ignoring the influence of turbulent flow near the well.

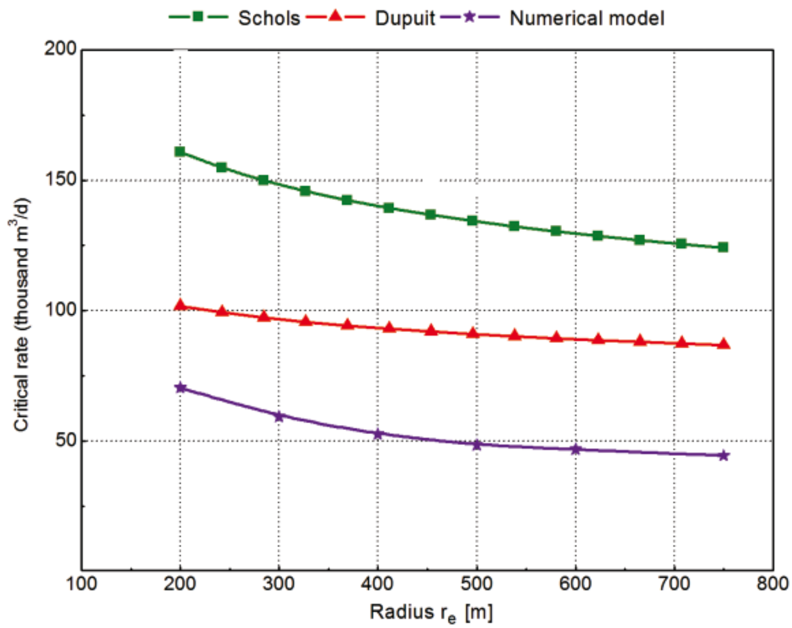

Fig. 4. Critical rate vs. radius of well's impact r_e for selected models and numerical model

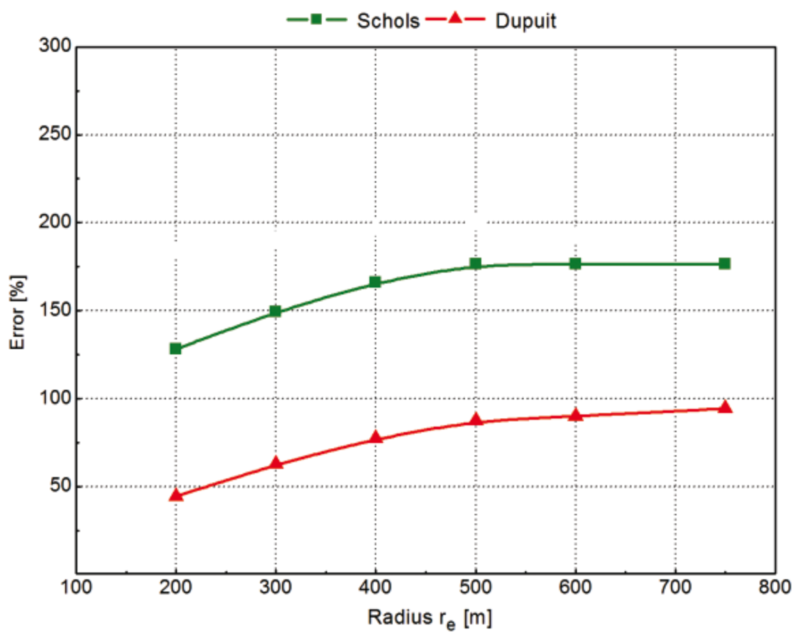

Fig. 5. Error of critical rate vs. radius of well's impact r_e for selected models as compared with numerical model

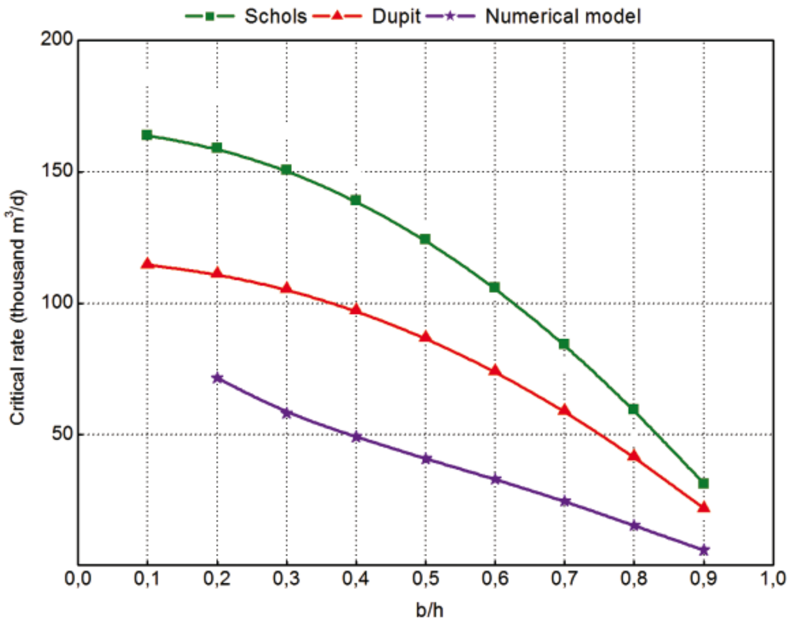

Fig. 6. Critical rate vs. *b*/*h* ratio for specific models and numerical model

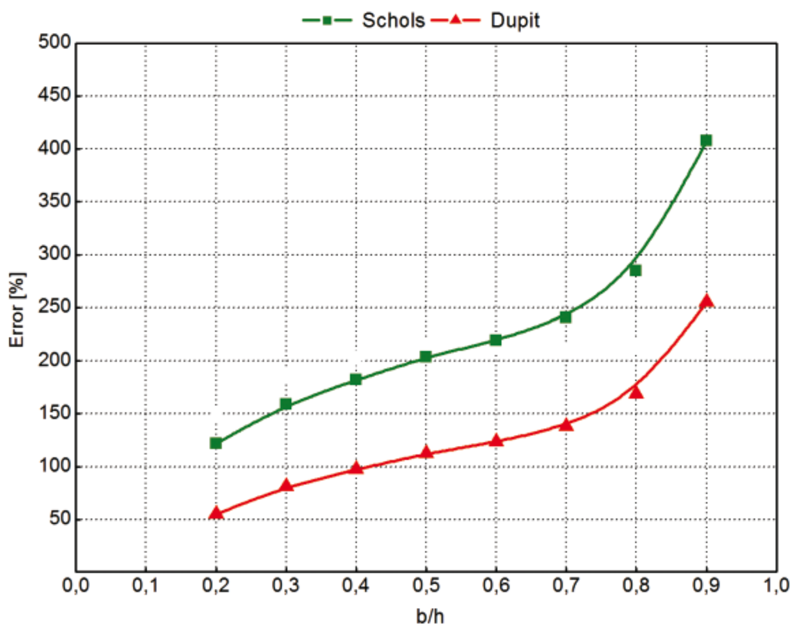

Fig. 7. Error of critical rte vs. *b*/*h* ratio for selected models as compared to numerical model

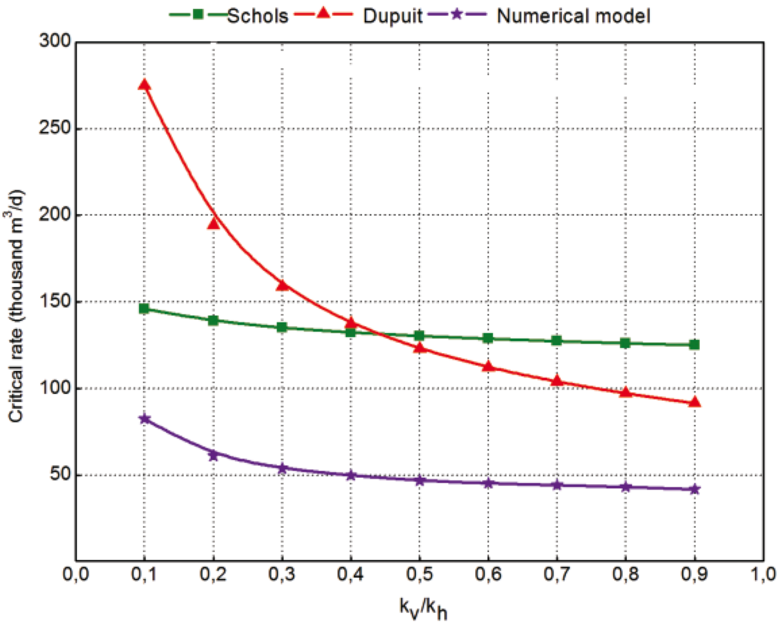

Fig. 8. Critical rate vs. vertical permeability to horizontal permeability ratio for specific models and numerical model

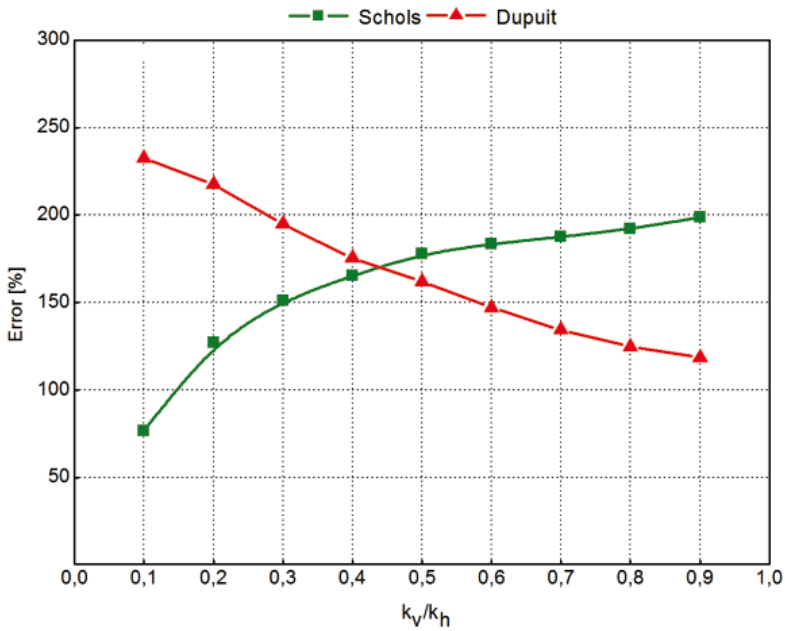

Fig. 9. Error of critical rate vs. vertical permeability to horizontal permeability ratio for specific models as compared with numerical model

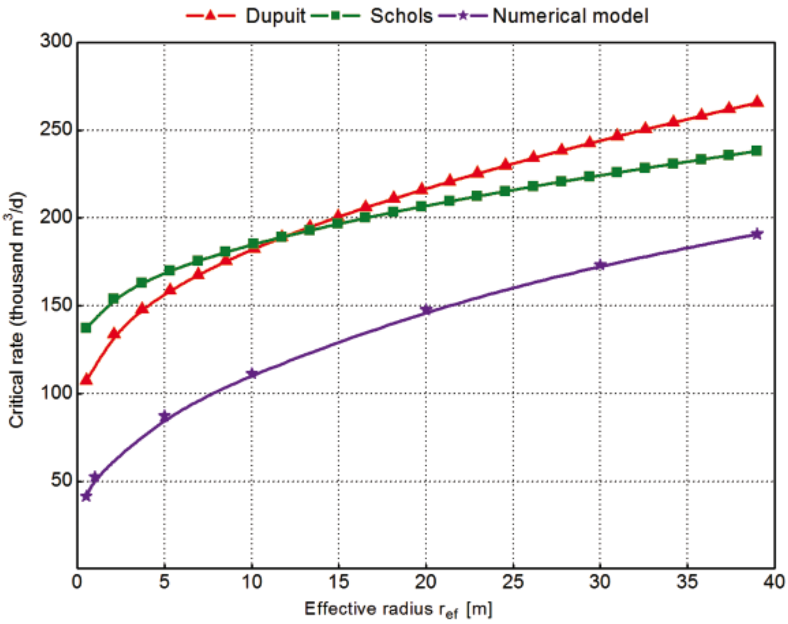

Fig. 10. Critical rate vs. effective radius of well for selected models and numerical model

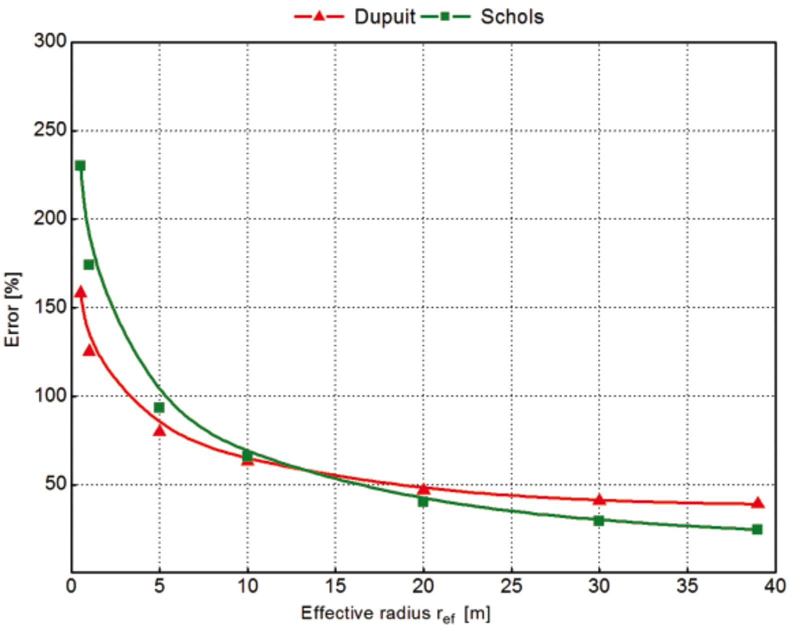

Fig. 11. Error of critical rate vs. effective radius of well for selected models as compared to numerical model

4. Influence of turbulent gas flow in the near-well zone on critical rate

Model of turbulent flow near well

It has been assumed that for a small flow of fluid in a porous medium the relation between its filtration rate and pressure gradient is determined by Darcy law:

$$v = -\frac{k}{\mu}\frac{dp}{dx} \tag{6}$$

However, there are known irregularities from this ideal model, e.g.: when skin effect or multiphase flows occur. If the flow rate in the porous medium is increased, Darcy law fully describes this effect. Forchheimer (1901) made an effort to solve this problem by proposing the following equation:

$$\frac{dp}{dx} = \frac{\mu}{k}v + \beta\rho v^2 \tag{7}$$

where β is the so-called inertia coefficient.

Equation (6) with the turbulence effect assumes the following form:

$$v = -\delta\frac{k}{\mu}\frac{dp}{dx} \tag{8}$$

where: $\delta = \dfrac{1}{1+\beta\dfrac{\rho k v}{\mu}}$

The inertia coefficient β may depend on a number of factors. According to Scheidegger (1974), who introduced the analytical model for it, this coefficient depended mainly on porosity, permeability and the so-called waviness of pore channels. On the other hand, Geertsma (1974), Pascal & Quillian (1980) and Jones (1987) proved in their experiments that the type of reservoir rock is the most important factor responsible for its value. Laboratory experiments were also a basis of introducing empirical correlations by Evans, Hudson and Greenlee (1987) as well as Lombard and Longeron (1999). In their opinion liquid saturation was the most influential on coefficient β. Finally, Frederic and Graves (1994) presented three empirical correlations for its determining for a broad range of permeability values.

Equation of gas flow to well in semi steady state

A full equation of gas flow from the reservoir to an imperfect hydrodynamic well can be written in the form:

$$\Delta p^2 = \frac{q_n \mu p z T_n}{\pi k h T_n}\left[P_D(t_D) + S + D_t q_n\right] \tag{9}$$

$$D_t = \frac{\beta \rho_n k}{2\pi h \mu_w r_w} \tag{10}$$

The semi-steady state of flow, lasting for most of the exploitation time is most frequent for flows taking place during exploitation of gas fields. The value of dimensionless function of pressure in a semi steady state at known difference of pressures between average pressure in the reservoir and bottom pressure in the borehole $\Delta p = p_z - p_w$ is expressed by the following formula:

$$P_D(t_D) = \ln \frac{r_e}{r_w} - \frac{3}{4} \tag{11}$$

Substituting dependence (11) to equation (9) we get:

$$\Delta p^2 = p_z^2 - p_w^2 = \frac{q_n \mu z p_n T}{\pi k h T_n} \left(\ln \frac{r_e}{r_w} - \frac{3}{4} + S + D_t q \right) \tag{12}$$

Equation (12) can be used for calculating gas inflow from the reservoir to the well. The gas properties μ and z were averaged in the equation. In practice it usually is the reservoir pressure which is known, and for which those gas properties are determined.

Equation (12) can be written in a simpler form by introducing coefficients "*a*" and "*b*" defined as below:

$$a = \frac{\mu z p_n T}{\pi k h T_n} \left(\ln \frac{r_e}{r_w} - \frac{3}{4} + S \right) \tag{13}$$

$$b = \frac{\mu z p_n T}{\pi k h T_n} D_t \tag{14}$$

Then equation (12) can be written as:

$$\Delta p^2 = a q_n + b q_n^2 \tag{15}$$

The obtained equation (15) is known as a two-element formula. Despite the fact that coefficients "*a*" and "*b*" in this formula have effective forms, their values are usually defined on the basis of hydrodynamic tests. Determining coefficients "*a*" and "*b*" from hydrodynamic tests stems from the fact that reservoir significantly changes its physical properties over the entire production period, causing that theoretical formulae fail to fully describe the changing reality.

Modification of equations describing critical rates by accounting for the turbulence factor

As the critical rate is calculated for one-phase flow, then the following equation can be written for the semi-steady state:

$$p_1^2 - p_2^2 = a q_{ND} + b q_{ND}^2 \tag{16}$$

and ignoring the turbulence element

$$p_1^2 - p_2^2 = aq_D \tag{17}$$

in line with Darcy law for a gas deposit,
where:

q_D — rate related to flow according to Darcy law,
q_{ND} — rate related to turbulent flow.

As the critical rates are calculated with equations which do not account for turbulence, then by assuming a similar gradient of pressure for turbulent flow and Darcy flow the left sides of equations (16) and (17) can be equalized, analogous as for their right sides.

Accordingly, we may write:

$$aq_{cND} + bq_{cND}^2 = aq_{cD} \tag{18}$$

hence

$$q_{cND} = \frac{1}{2}\left(\sqrt{\left(\frac{a}{b}\right)^2 + 4\left(\frac{a}{b}\right)q_{cD}} - \frac{a}{b}\right) \tag{19}$$

And so for the modified Dupuit model the new expression will assume the following form:

$$Q_{cDPND} = \frac{1}{2}\left(\sqrt{\left(\frac{a}{b}\right)^2 + 4\left(\frac{a}{b}\right)\frac{\pi g k_g k_{rg}(\rho_w - \rho_g)(h^2 - b^2)}{\mu_g B_g \ln\left(\frac{r_e}{r_w}\right)}} - \frac{a}{b}\right) \tag{20}$$

For Schols model the new form of the equation for critical rate takes the form:

$$Q_{cSCND} = \frac{1}{2}\left(\sqrt{\left(\frac{a}{b}\right)^2 + 4\left(\frac{a}{b}\right)\frac{g k_g k_{rg}(\rho_w - \rho_g)(h^2 - b^2)}{\mu_g B_g}\left[0.432 + \frac{\pi}{\ln\left(\frac{r_e}{r_w}\right)}\right]\left(\frac{h}{r_e}\right)^{0.14}\left(\frac{k_v}{k_h}\right)^{-0.07}} - \frac{a}{b}\right) \tag{21}$$

By introducing this modification the value of critical rate is significantly decreased, which has been visualized in figs. 12, 14, 16 and 18.

Relative error calculated for modified equations is much lower as compared to errors of original equations, see figs. 13, 15, 17 and 19.

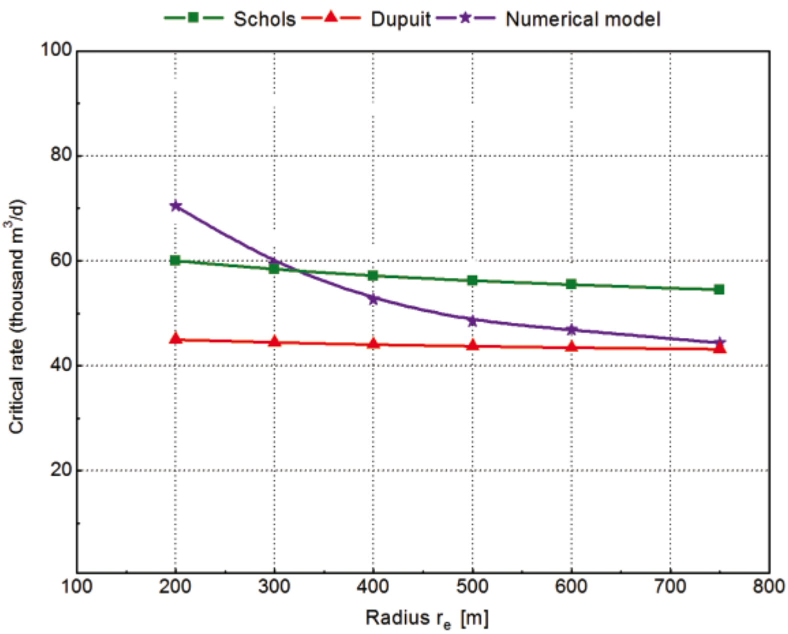

Fig. 12. Critical rate vs. radius of well's impact r_e for selected modified models and numerical model

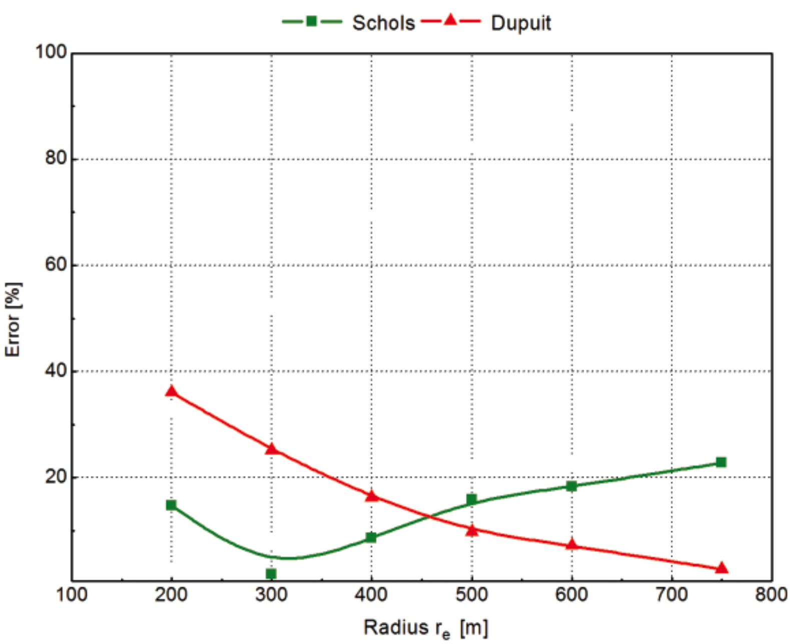

Fig. 13. Error of critical rate vs. radius of well's impact r_e for selected modified models as compared to numerical model

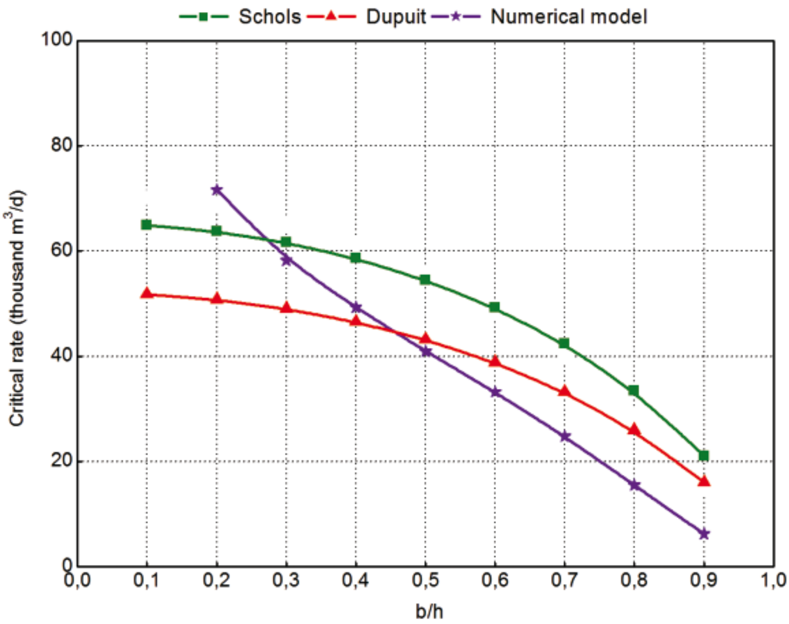

Fig. 14. Critical rate vs. *b*/*h* ratio for specific modified models and numerical model

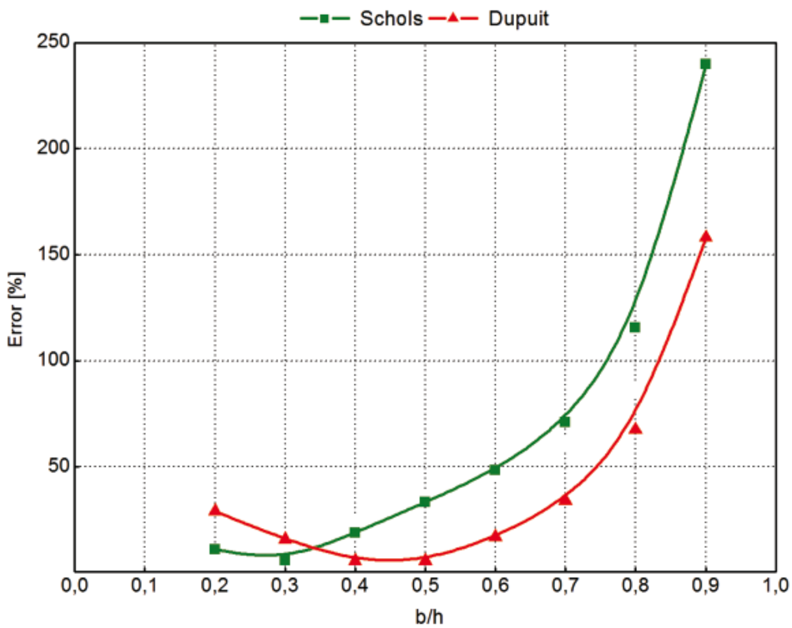

Fig. 15. Error of critical rate vs. *b*/*h* rate for selected modified models as compared to numerical model

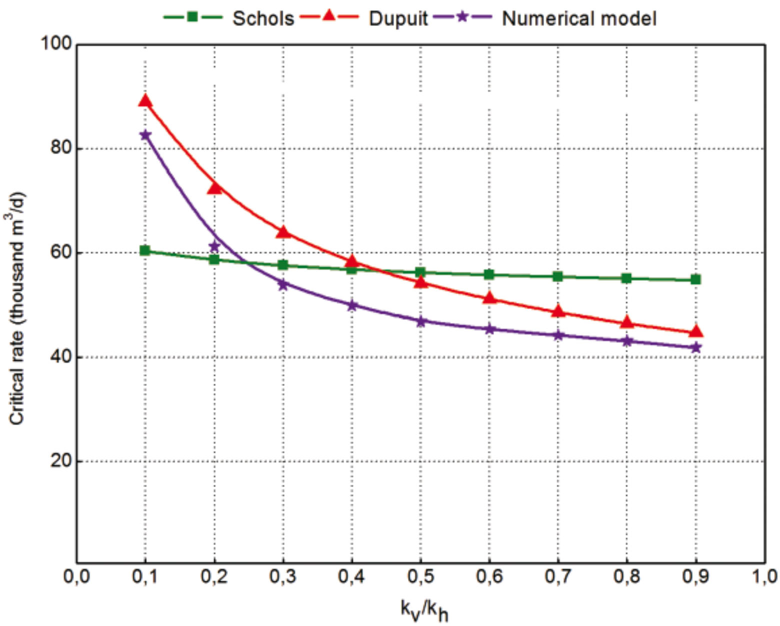

Fig. 16. Critical rate vs. vertical permeability to horizontal permeability ratio for specific modified models and numerical model

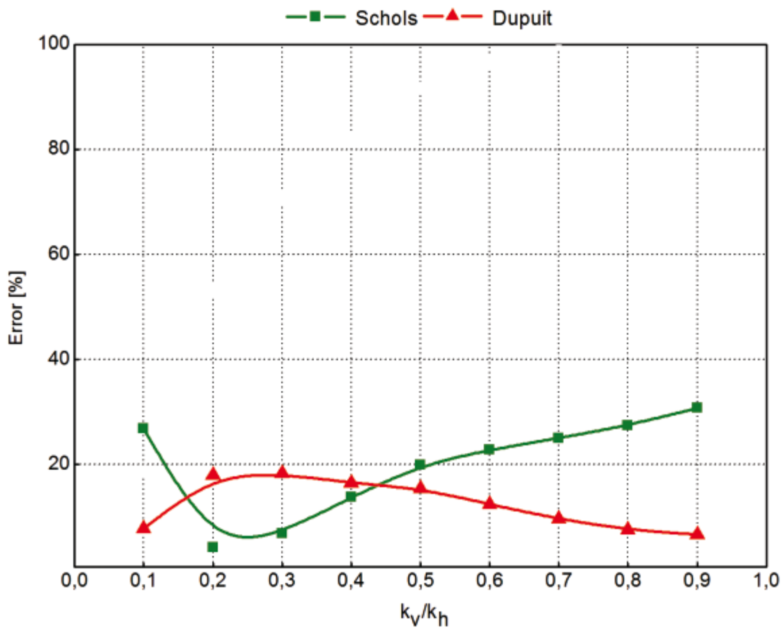

Fig. 17. Error of critical rate vs. vertical permeability to horizontal permeability ratio for selected models modified as compared to numerical model

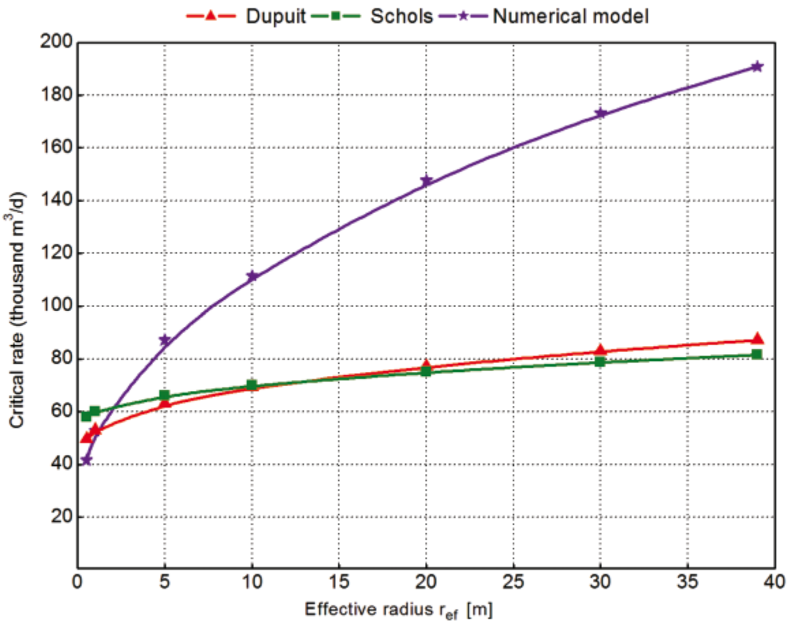

Fig. 18. Critical rate vs. effective radius of well for selected modified models and numerical model

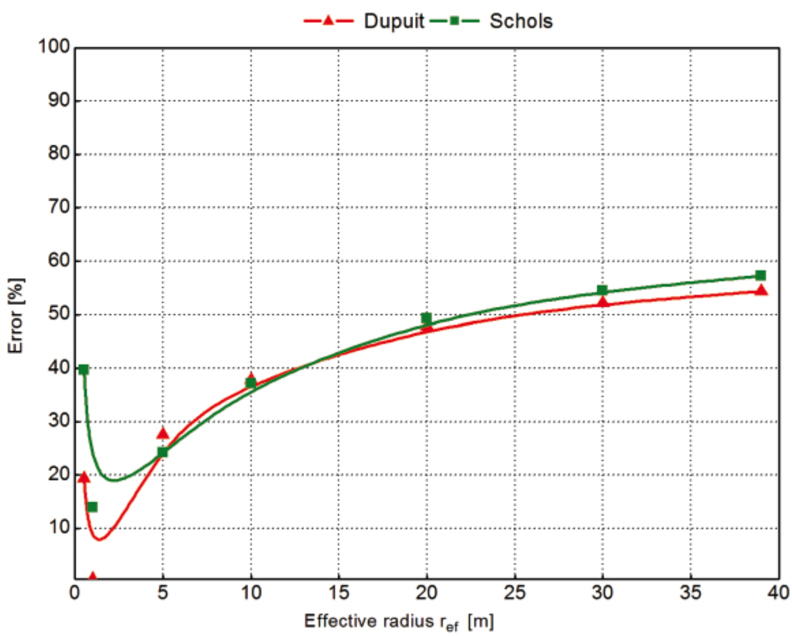

Fig. 19. Error of critical rate vs. effective radius of well for selected models modified with respect to numerical model

TABLE 4

Realtive error of calculation of specific modified Dupuit and Schols models of critical rate as compared to the results of numerical model

Model	r_e			b/h			k_v/k_h			r_{ef}		
	50	100	200	50	100	200	50	100	200	50	100	200
Dupuit	10.2	20.8	16.2	57.1	44.0	41.5	40.9	21.9	12.5	36.9	37.1	34.2
	15.7			47.5			25.1			36.1		
	Mean weighted error – 44.4											
Schols	25.2	9.9	13.7	80.6	75.7	67.9	41.9	26.6	19.8	43.9	42.8	39.4
	16.3			74.7			29.4			42.0		
	Mean weighted error – 67.9											

5. Conclusions

1. The existing literature on water coning mainly refers to the steady state cases. Presented solutions are derived for critical rates, time water enters the well and two-phase exploitation.

2. Most of existing correlations are adaptations of models of water coning in oil deposits. Relative errors of such correlations exceed 100%. None of those correlations accounts for non-linear filtration processes N-D ("Non Darcy Flow").

3. The use of the numerical model for evaluating the quality of critical rate approximation allowed the correlation to be verified. The analysis presented in this paper shows the necessity of refuting most of the correlations which do not account for turbulence N-D.

4. The proposed modified method of calculating critical rate N-D can be used for assessing this parameter with moderate error of 40% for modified Dupuit correlation. The analysis reveals that the critical rate is most influenced by reservoir permeability "kh". Drastically limited critical rate is related with the increasing of completion parameter (b/h) and existence of mechanical damage to the well (S).

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