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Arch. Min. Sci., Vol. 59 (2014), No 1, p. 179–188

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2014-0013

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# INVESTIGATING THE SLOPE STABILITY BASED ON UNCERTAINTY BY USING FUZZY POSSIBILITY THEORY

# BADANIE STABILNOŚCI SKARPY W OPARCIU O ANALIZĘ NIEPEWNOŚCI Z WYKORZYSTANIEM TEORII PRAWDOPODOBIEŃSTWA I LOGIKI ROZMYTEJ

The main purpose of this paper is to investigate the slope stability condition by using fuzzy estimation method based on fuzzy possibility theory. Due to use of this theory, the inaccuracy, ambiguity and uncertainty in input parameters are considered and therefore, the calculated factor of safety (FOS) is highly reliable. In this research, first, the input parameters of slope stability analysis, based on statistical characteristics and grade of membership concept, as a fuzzy numbers are defined. Then the performance function of slope behavior is defined and by using the fuzzy parameters, the FOS is calculated. In next step, by using the several $\alpha - cut$, the calculated FOS is defined as a fuzzy form and subsequently, the slope stability condition based on fuzzy presentation of FOS is evaluated. The results show that, although based on deterministic analysis the studied slope is stable but based on fuzzy interpretation of FOS, the slope stability condition is scare. The fuzzy analysis of slope stability condition, by applying the uncertainty in calculating the FOS and defining the grade of membership for each unknown input parameters in model, a more realistic interpretation of slope stability condition is provided. In addition, the fuzzy presentation of the FOS, allowing more accurate judgments about slope stability condition.

**Keywords:** fuzzy possibility theory, uncertainty, factor of safety, slope stability

Celem pracy jest zbadanie warunku stabilności skarpy w oparciu o metody szacunkowe wykorzystujące elementy logiki rozmytej i teorii prawdopodobieństwa. Z uwagi na zastosowanie tych teorii w parametrach wejściowych uwzględniono niedokładność, niepewność i niejednoznaczność, dlatego też obliczone współczynniki bezpieczeństwa uznać można za wysoce wiarygodne. W pierwszym etapie pracy zdefiniowano parametry wejściowe do analizy stabilności skarpy w oparciu o charakterystyki statystyczne i przyjęte funkcje przynależności, i określone parametry rozmyte. Następnie zdefiniowano zachowanie zbocza skarpy w oparciu o parametry rozmyte i obliczono współczynnik bezpieczeństwa. W kolejnym kroku przy założeniu wielokrotnego wybierania przy założonym kącie $\alpha$ ($\alpha - cut$), obliczono współczynnik bezpieczeństwa w formie rozmytej i zbadano warunek stabilności skarpy oparty na współczynniku bezpieczeństwa w formie rozmytej. Wyniki wskazują, że pomimo iż zbocze badane w oparciu o analizy deterministyczne wydaje się stabilne, to współczynnik bezpieczeństwa w ujęciu rozmytym sugeruje, że

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warunek stabilności zbocza będzie spełniony w niewielkim stopniu. Analiza warunku stabilności zbocza wykorzystujące elementy logiki rozmytej uwzględniająca zagadnienie niepewności przy obliczaniu współczynnika bezpieczeństwa i przy określaniu stopnia funkcji przynależności dla poszczególnych parametrów wejściowych modelu pozwala bardziej realistycznie określić warunek bezpieczeństwa. Ponadto, przedstawienie warunku bezpieczeństwa w postaci rozmytej pozwala na bardziej dokładną ocenę warunków stabilności.

**Słowa kluczowe:** teoria możliwości, niepewność, współczynnik bezpieczeństwa, nachylenie

# 1. Introduction

The slope stability is one of the main subjects in engineering rock mechanics. To assess the slope stability, the factor of safety (FOS) is calculated. FOS is defined as a ratio of resisting force to unresisting force. FOS is a function of physical and mechanical properties of surrounding rock mass (and soil) of slope as well as geometrical characteristic of slope (Park et al., 2012). The quantitative process of obtaining data from ground (including soil and rock mass properties) is always uncertain in nature. Because the ground material is not uniform and even the analogous materials may also show different mechanical behavior (Zhian et al., 2012; Oreste & Soldano, 2012). The effect of data scattering and dissimilar results in measurement of the natural parameters in engineering problems was under attention since years. Previously, use of statistical methods was only scientific way to deal with such issues. If lack of time or the skills to use statistical methods was encountered, and/or in many cases that the level of accuracy result was not essential, by using of conventional averaging methods, for each parameters a suitable value was determined and thus, such problems were solved. In some cases, instead of using the averaging methods, the value which causes the critical results, was used in calculation (Daftaribesheli et al., 2011; Saboya et al., 2006). Therefore, the other value that could even lead to decreases the project costs was excluded in order to comply of safety.

In statistical methods a few variables under the study are assumed to be random in nature. However, the validity of this hypothesis for many parameters is questionable. In addition, the statistical methods are not applied to qualitative data (Zhang, 2011; Das, 2012). Furthermore, understanding of the probability distribution function of random variables requires a large number of almost hardly-achievable data. Therefore to solve the problem, a distribution function is assumed with unclear validity. Thus, if the collected sets of data are defective and inadequate, the probability theory and statistical methods uses the simplifying assumptions.

Finally in 1965 with invention of "fuzzy set theory" such weaknesses and bugs are overcome. According to L. A. Zadeh, in dealing with vague and insufficient information, the use of "grade of membership" concept is the key issue. The concept of fuzzy sets is a contractual theory for creation of a conceptual model that in many cases is similar to traditional sets (Pradhan, 2010; Vujić et al., 2011; Giasi et al., 2003). However, the fuzzy sets are more general than traditional sets, and applicable in most area, especially in areas such as classification, and data analysis. This model in fact is a way of dealing with problems that the source of ambiguity is the clear criteria for grade of membership.

In this paper, the fuzzy program estimated method is used to calculation the FOS of rock – soil slope. This method with determining the grade of membership for each value in each set of input parameters, the uncertainty in the input parameters is considered in calculation, and also enables the fuzzy interpretation of FOS. In addition, calculates the eigenvalue and variance of

FOS. Thus the "reliability" of slope stability is estimated, that in fact determines the probability of slope failure during the operation.

## 2. The concept of FOS based on possibility analysis

Due to type and properties of uncertainty in input parameters of FOS calculation, using of only probabilistic methods are not adequate (Chenga et al., 2012). However, it is better the possibility theory based on fuzzy theory as an effective method for uncertainty condition to be used. The FOS is defined as function of several certain and uncertain parameters by equation 1.

$$FS = f(c,\varphi,\gamma,\psi,\rho,H,u,...) \tag{1}$$

where $FS$ is FOS, $c$ is cohesion, $\varphi$ is internal friction angle, $\gamma$ is density, $\psi$ is face angle, $\rho$ is failure plane angle, $H$ is slope height, and $u$ is water pressure.

In theoretical point of view, the slope failure occurs when the FOS is smaller or equal than the unit. Therefore if $F$ represents the failure, then:

$$F = \left[FS \leq 1\right] \tag{2}$$

If $f(FS_i)$ is the probability density function (PDF) of FOS $FS_i$ along the failure plane $S_i$, the fuzzy possibility of slope failure along $S_i$ with each $FS$ and grade of membership in $A$ set is defined by equation 3.

$$P\left[F\middle|S_i\right] = \int_{-\infty}^{1} \alpha_A\left(FS_i\right)dp\left(FS_i\right) \tag{3}$$

where $\alpha_A(FS_i)$ is probability distribution function of $FS_i$, $dp(FS_i) = f(FS_i)d(FS_i)$, and $P[F|S_i]$ fuzzy possibility condition of failure in plane $S_i$, that may occur in the future.

For each $N$ possible failure plane with random probability $P[S_i]$, $i = 1,2,...,N$ by using general probability theory, fuzzy possibility failure of slope ($P_{ff}$) is calculated by equation 4.

$$P_{ff} = P[F] = \sum_{i=1}^{N} P[F|S_i]P[S_i] \tag{4}$$

With considering $\sum_{i=1}^{N} P[S_i] = 1$ along failure planes, the critical failure plane (a plane that has the least FOS, $FS = DS_{\min}$) is determined. If such plane is presented by $S_c$, and the possibility of this event is $P[S_c]$, equation 4 can be rewritten as equation 5:

$$P_{ff} = P[F|S_c]P[S_c] + \sum_{i=1}^{N} P[F|S_i]P[S_i] \tag{5}$$

Equation 5 represents a failure probability system. Obviously, a high probability degree is not reason of a high possibility degree. However, an improbable event is impossible too. Therefore probability method is the upper limit for possibility method.

## 3. Fuzzy program estimated method

Possibility theory is a powerful tool for the analysis of uncertainty parameters. If the distribution function of effective parameters is in appropriate form, the main features of random variables are completely describable (Akgun et al., 2012; Tian & Zhuge, 2008). However in practice, may not specify the form of distribution function. In this condition, the description of the approximation of random variables is required. Therefore the uncertain parameters must be described as a distance by estimation of the upper and lower limits (Binaghi et al., 1998).

The conventional number fuzzy in rock mechanics is triangular fuzzy numbers (with three parameters *a*, *b* and *c*) and trapezoidal fuzzy numbers (with four parameters *a*, *b*, *c*).

To investigate the possibility theory, *M* multiple uncertain parameters, with mean vector $X = [X_1, X_2, ..., X_M]$ and expected value $E(X) = [E(X_1), E(X_2), ..., E(X_M)]^T$ as a diagonal matrix $D_{M \times M}$ of random variables of uncertain parameters are assumed (Chenga et al., 2012). In other word, the uncertain parameters are defined by mean and standard deviation.

For each uncertain parameter, an individual fuzzy set is created which relates to define a suitable reference set and also determine an appropriate membership function. For trapezoidal fuzzy variable *X*, parameters *a*, *b*, *c* and *d* are defined as:

$$\begin{aligned} a = E[X] - K_1\sigma[X];\ b = E[X] - K_2\sigma[X] \\ c = E[X] + K_2\sigma[X];\ d = E[X] + K_1\sigma[X] \end{aligned} \tag{6}$$

where $K_1$ and $K_2$ depending on available data and accuracy needed of results, vary from 0.5 to 3.

For multivariate problems with *M* uncertain parameters, it is suggested that the evaluation points of function be selected based on $\alpha - cut$ concept. Therefore, the selected points to calculate the function in parametric space are obtained by equation 7 and 8.

$$X_{\alpha i}^{+} = c + V_{\alpha i} \tag{7}$$

$$X_{\alpha i}^{-} = b + V_{\alpha i} \tag{8}$$

where *c* and *b* are parameters of fuzzy numbers for $\alpha = 1$, $V_{\alpha i}$ is value to obtain the related parameter with corresponding point of grade of membership, is high or low, $X_{\alpha i}^{+}$ and $X_{\alpha i}^{-}$ are the coordinate vectors of *M* uncertain parameters, and $\alpha i$ is the $\alpha - cut$ value to calculate the range of uncertain values.

For function $W = g(X)$, the sum of function value in each $\alpha - cut$ is represented the correlation between variables and calculated by equation 9.

$$W_{\alpha i}^{r} = P + g^{r}(X_{\alpha i}^{+}) + P - g^{r}(X_{\alpha i}^{-});\ i = 1, 2, ..., N \qquad r = 1, 2 \tag{9}$$

where $P_+$ and $P_-$ are weight factors that are calculated by equation 10.

$$P_{\pm} = 1 \pm \sum_{i=1}^{M-1} \left[ \sum_{j=i+1}^{M} \frac{P_{ij}}{\sqrt{\prod_{j=i+1}^{M} \left[ 1 + \left( \frac{\beta(l)_i}{2} \right)^2 \right]}} \right] \tag{10}$$

where $P_{ij}$ is the correlation coefficient between fuzzy variables $X_i$ and $X_j$, $\beta(l)_i$ is unbiased coefficient of fuzzy variable $X_i$.

In addition $r$ th time of function is calculated according to equation 11.

$$E[W^r] = \frac{\sum_{i=1}^{N} \alpha i.W_{\alpha i}^r}{N} \tag{11}$$

The $\alpha i$ must be more than zero. Based on definition, $\alpha = D$ is representing the reference set $X$. With determination of the upper and lower limits of FOS for each $\alpha - cut$, the FOS is defined as a fuzzy set.

## 4. Interpretation of FOS fuzzy set

After defining the fuzzy set of FOS, description of the slope stability is a requirement. Figure 1 is shown the slope stability condition based on fuzzy set interpretation. In many cases, it is necessary a deterministic value for FOS to be provided. To obtain a deterministic value from FOS fuzzy set, the $\alpha - cuts$ less than 0.5 should not be considered (Hosseini, 2012). A method to attain the deterministic value of FOS from fuzzy set is averaging the upper and lower limits of FOS in rang of $0.5 < \alpha < 1$ as shown in figure 2. Alternatively, it is selected the lower limit of FOS corresponding with $\alpha = 0.7$.

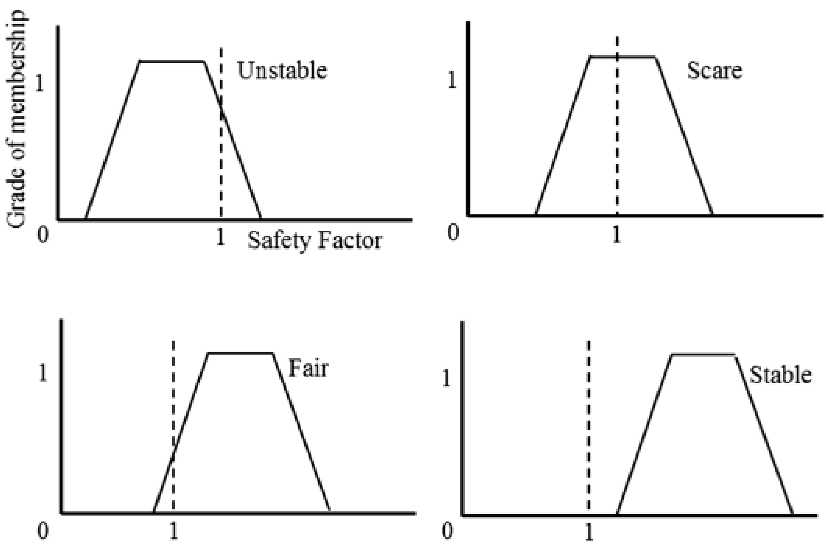


Fig. 1. Description of slope stability by using interpretation of FOS fuzzy set

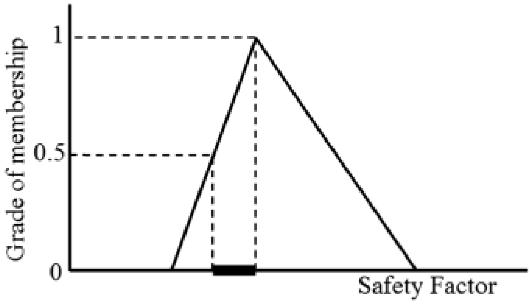


Fig. 2. Extraction of a deterministic value from FOS fuzzy set

## 5. Fuzzy analysis of slope stability

To illustrate the use of fuzzy analysis in slope stability investigation, the potential of plane failure in the slope is studied (Hosseini, 2012). For this purpose, a hypothetical slope is considered with the slope height of 45 m, the slope face of 65 degree, and the angle of failure plane of 40 degree as show in figure 3. The length of failure plane in cross-section is 70 m, whereas the slope stability for 1 m of slope width (per unit width) is evaluated, the area of failure plane is considered as 70 $m^2$. The average density of contain material of slope is 2.7 $t/m^3$, and hence the weight of sliding block is calculated as 1983.19 $t/m^2$. In failure plane, cohesion ($c$) is 10 $t/m^2$ and internal friction angle ($\varphi$) is 30 degree.

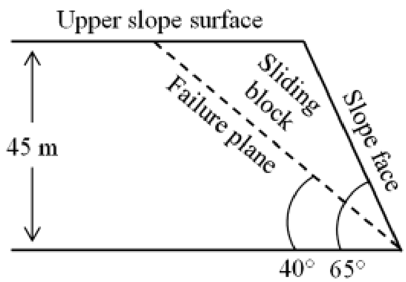


Fig. 3. The cross-section of slope

In probabilistic analysis of slope stability, cohesion strength ($c$) and internal friction angle ($\varphi$) are considered as random variables and density is assumed as constant value.

Limit equilibrium method is used to calculate the FOS. In this method, regardless of the types and forms of slope failure, the FOS is calculated as the ratio of the resistance forces to the unstable forces (Johari et al., 2013), according to equation 12.

$$F = \frac{cA + W\cos\psi\tan\varphi}{W\sin\psi} \tag{12}$$

where $W$ is weight of sliding block, $A$ is area of failure plane, $\psi$ is angle of failure plane with respect to horizontal, $c$ and $\varphi$ are cohesion strength and internal friction angle in failure plane, respectively.

## 6. Results and discussion

Considering the geometrical, physical and geomechanical properties of slope and failure plane, the performance function of FOS is defined as equation 13.

$$F = 0.05491c + 1.192 \tan\varphi \tag{13}$$

Based on deterministic analysis, with replacing the values of cohesion and internal friction angle in equation 13, the calculation result of deterministic FOS is 1.2372. Since the critical-FOS is considered 1.2, therefore the slope is stable.

In the first step of fuzzy analysis, the statistical parameters of random variables are essentially determined and the result is presented in table 1.

TABLE 1

Random variables parameters

| **Parameters** | ***c*** | **tan*φ*** |
|---|---|---|
| Mean | 10 t/m$^2$ | 0.5774° |
| Coefficient of variation (%) | 20 % | 10 % |
| Standard deviation | 2 t/m$^2$ | 0.05774° |

With considering the value of table 1 for each input parameter and by using of statistical distribution, the fuzzy numbers of $c$ and $\tan\varphi$ are plotted as shown in figure 4 and 5, respectively.

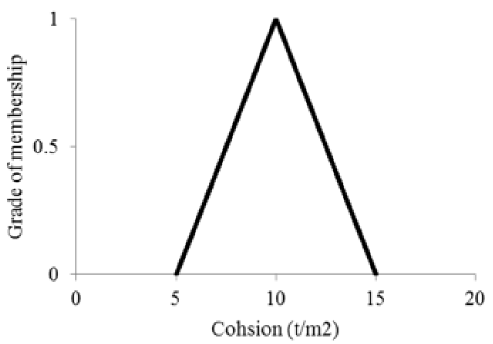


Fig. 4. The fuzzy number of $c$

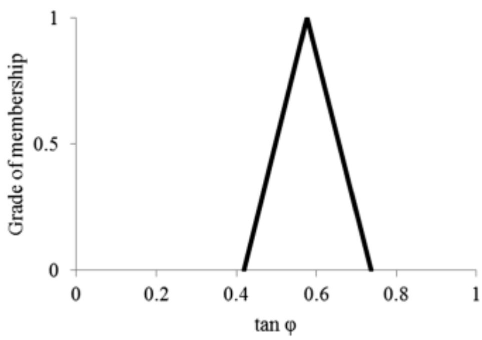


Fig. 5. The fuzzy number of $\tan\varphi$

After defining the fuzzy sets for input parameters, with creating the $\alpha - cuts$, the upper and lower limits for each cut are determined, as shown in table 2.

TABLE 2

The values of $\alpha - cuts$ and upper and lower limits of $c$ and $\tan\varphi$

| $\alpha - cuts$ | $c$ | | $\tan\varphi$ | |
|---|---|---|---|---|
| | **Lower limit** | **Upper limit** | **Lower limit** | **Upper limit** |
| 0.0 | 5.00 | 15.00 | 0.4182 | 0.7366 |
| 0.1 | 5.50 | 14.50 | 0.4341 | 0.7207 |
| 0.2 | 6.00 | 14.00 | 0.4500 | 0.7048 |
| 0.3 | 6.50 | 13.50 | 0.4660 | 0.6888 |
| 0.4 | 7.00 | 13.00 | 0.4819 | 0.6729 |
| 0.5 | 7.50 | 12.50 | 0.4978 | 0.6570 |
| 0.6 | 8.00 | 12.00 | 0.5137 | 0.6411 |
| 0.7 | 8.50 | 11.50 | 0.5296 | 0.6252 |
| 0.8 | 9.00 | 11.00 | 0.5456 | 0.6092 |
| 0.9 | 9.50 | 10.50 | 0.5615 | 0.5933 |
| 1.0 | 10.00 | 10.00 | 0.5774 | 0.5774 |

Next, the FOS based on the performance function (equation 13) for each $\alpha - cut$ is calculated. With considering the combination of upper and lower limits of cohesion and tangent of internal friction angle, four FOS are obtained for each $\alpha - cut$. The calculated FOS for $\alpha = 0.5$ is shown in table 3 as an example.

TABLE 3

The calculated FOS for $\alpha = 0.5$

| **No.** | **c** | | **$\tan\varphi$** | | **FOS** |
|---|---|---|---|---|---|
| 1 | Upper limit | 12.50 | Upper limit | 0.6570 | 1.4695 |
| 2 | Upper limit | 12.50 | Lower limit | 0.4978 | 1.2798 |
| 3 | Lower limit | 7.50 | Upper limit | 0.6570 | 1.1950 |
| 4 | Lower limit | 7.50 | Lower limit | 0.4978 | 1.0052 |

As can be seen in table 3, upper and lower limits of FOS are calculated as 1.4695 and 1.0052, respectively. However, the upper and lower limits of FOS for all $\alpha - cut$ are given in table 4, and accordingly the fuzzy number of FOS is also plotted as shown in figure 6.

TABLE 4

The upper and lower limits of FOS for all $\alpha - cut$

| $\alpha - cut$ | **FOS** | |
|---|---|---|
| | **Lower limit** | **Upper limit** |
| **1** | **2** | **3** |
| 0.0 | 0.7730 | 1.7017 |
| 0.1 | 0.8195 | 1.6552 |
| 0.2 | 0.8659 | 1.6088 |
| 0.3 | 0.9123 | 1.5624 |
| 0.4 | 0.9588 | 1.5160 |

| 1 | 2 | 3 |
|---|---|---|
| 0.5 | 1.0052 | 1.4695 |
| 0.6 | 1.0516 | 1.4231 |
| 0.7 | 1.0981 | 1.3767 |
| 0.8 | 1.1445 | 1.3302 |
| 0.9 | 1.1909 | 1.2838 |
| 1.0 | 1.2374 | 1.2374 |

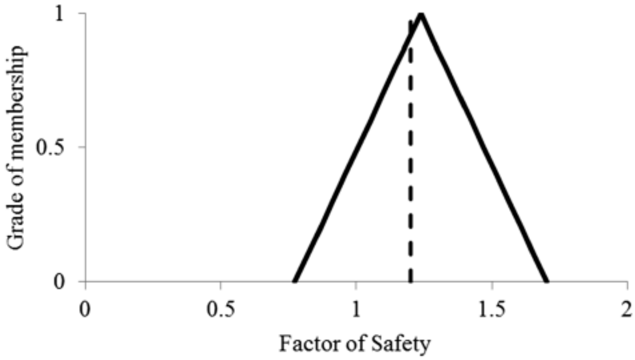


Fig. 6. The fuzzy number of FOS

By comparison of figure 6 and figure 1, it is deduced that with considering the critical-FOS = 1.2, the slope stability is in dangerous state. However based on deterministic analysis, the FOS is calculated 1.2372 that indicates the stable state of slope stability.

## 7. Conclusions

In the analysis of slope stability by using deterministic method, the uncertainty in input parameters is not considered. While, fuzzy possibility theory due to defining the fuzzy ranges for each parameter, the undesired performance probability of uncertainties is decreased. The results show that the uncertainty in input parameters of slope stability problem can significantly effects on the calculation results. However by considering the upper and lower limits in fuzzy definition of each parameter, the uncertainty is reduced. Therefore the evaluation of slope stability by using of fuzzy possibility theory, the realistic concept of slope stability is provided. In addition, the fuzzy interpretation of calculated FOS increases the reliability of engineering design and prevents the occurrence of uncontrolled failure. Although based on deterministic analysis the slope is stable, but based on fuzzy analysis and fuzzy interpretation of calculated FOS, the slope stability is in dangerous state. Therefore fuzzy analysis has a significant role in risk reduction and by considering the uncertainty resources, allows reliable judgments with respect to slope stability condition to be provided.

## Acknowledgment

This research paper is a part of research program entitled "Large-Scale Slope Stability Analysis based on the Combination Study of Effective Parameters under Uncertainty Conditions", which has been supported financially by Islamic Azad University, South Tehran Branch.

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*Received: 12 March 2013*