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ZYGMUNT NIEDOJADŁO*, WOJCIECH GRUSZCZYŃSKI*

THE IMPACT OF THE ESTIMATION OF THE PARAMETERS VALUES ON THE ACCURACY OF PREDICTING THE IMPACTS OF MINING EXPLOITATION

WPŁYW OSZACOWANIA WARTOŚCI PARAMETRÓW MODELU NA DOKŁADNOŚĆ PROGNOZOWANIA WPŁYWÓW EKSPLOATACJI GÓRNICZEJ

The possibility of the assessment of the probability that the acceptable values of deformation indexes will be exceeded and the introduction of the coefficient of safety are more and more seriously considered in case of predicting the impacts of exploitation both on the surface and the shaft tube. This article presents the mentioned above issue focusing on the authors' project of the exploitation in the protective pillars of the shafts of the Legnica and Głogów Copper Area (LGOM). To assess the accuracy of the predicted deformation indexes Monte Carlo method (often used in statistics) was proposed as well as the strict and approximate analysis using the law of error propagation. The values of vertical strains were analysed, as the most important at the assessment of the state of deformation of the shaft tube. The comparison of the results obtained with different methods confirmed the correctness of the carried out calculations. The applied methods allowed the analysis of the impact of individual parameters of the model on the prediction of the accuracy of forecast. This article makes the continuation of the solutions of the article (Niedojadło & Gruszczyński, 2010) thus some contents essential for the continuity of understanding were repeated here.

Keywords: protective pillars, shaft, vertical strains, mean errors, predicting the impacts of exploitation, Monte Carlo method

Możliwość oceny prawdopodobieństwa przekroczenia dopuszczalnych wartości wskaźników deformacji oraz wprowadzenie współczynnika bezpieczeństwa coraz częściej są brane pod uwagę w przypadku prognozowania wpływów eksploatacji tak na powierzchnię jak i rurę szybową. W niniejszym artykule przedstawiono powyższe zagadnienie na przykładzie autorskiego projektu eksploatacji w filarach ochronnych szybów LGOM. Do oceny dokładności prognozowanych wskaźników deformacji zaproponowano wykorzystywaną w statystyce metodę Monte Carlo oraz analizę ścisłą i przybliżoną wykorzystującą prawo przenoszenia się błędów średnich. Analizowano wartości odkształceń pionowych, jako najistotniejsze przy ocenie stanu deformacji rury szybowej. Porównanie wyników uzyskanych różnymi metodami potwierdziły poprawność wykonanych obliczeń. Zastosowane metody pozwoliły na analizę wpływu poszczególnych

* AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, FACULTY OF MINING SURVEYING AND ENVIRONMENTAL ENGINEERING, AL. A. MICKIEWICZA 30, 30-059 KRAKOW, POLAND

parametrów modelu na dokładność prognozy. Niniejszy artykuł stanowi kontynuację rozważań zawartych w artykule (Niedojadło & Gruszczyński, 2010) stąd pewne treści niezbędne dla zachowania ciągłości rozumowania zostały powtórzone.

Słowa kluczowe: filary ochronne, szyb, odkształcenia pionowe, błąd średni, prognozowanie wpływów eksploatacji, metoda Monte Carlo

1. Introduction

Every expert dealing with the problem of predicting the impacts of mining exploitation on the surface and the rock mass, in the first phase of the speculations referring to a concrete mining region and object, asks a question which parameters of the accepted for calculations model correspond the real conditions. In principle it can be said that there is no correct answer to this question, in the meaning of deterministic values of these parameters. The authors of this article, with their experience of decades in this field, can state that almost never the hypotheses accepted „a priori" to predict the parameters strictly correspond the values marked „a posteriori", after exploitation and revealing the earlier predicted impacts. This results from the randomness of the phenomenon, incomplete compatibility of the model with its real course, wrong estimation of the theory parameters or deviations from the assumptions of the project of exploitation (depth, thickness of the layer, the way of filling the cavity, the shape and size of the exploitation field). This does not mean that the model is incorrect. One should simply use the model in an proper way.

In case of the Legnica and Głogów Copper Area, at the assessment of the impacts of planned exploitation in protective pillars of the shafts on the shaft tube, we know very little about the values of the parameters of theories for the inside of the rock mass, necessary for the calculations of the values important for the safety of shaft deformation indexes. The project of exploitation in the protective pillar of the shaft in tubing, functioning in the conditions of water threat, at the very beginning requires establishing the extreme values of deformation indexes, possible to occur during the planned exploitation. In the section of the tubing in the watered zone, one must not allow any damage to the shaft, compromising its integrity. Thus it is necessary to evaluate the risk of transgressing the limit value of the analyzed deformation index, established for the shaft.

For the calculations, often a „standard" set of parameters is accepted, which consequently does not give information on a real state of the deformations in the surface or the rock mass in the analyzed region. The article presents the methods of making calculations, enabling to determine the predicted values with the assessment of the probability of exceeding acceptable values. The author's project of the copper deposit exploitation in the protective pillars of LGOM shafts (Niedojadło, 2008a) was analyzed. The presented methods are, however, of general character and are possible to be used also in other projects of the exploitation of useful minerals.

2. General relationships and formulae of Knothe's theory in the area of predicting the rock mass deformations

The process of the rock mass deformation caused by the mining exploitation takes place in the conditions of significant complexity of the rock mass structure. The phenomena occurring within the rock mass are hard to be record by the measurements, especially in case of one-seam copper deposit. The measurements inside the rock mass are carried out only in the shafts and usually in the scale not allowing the full verification of the theoretical model and its parameters.

The state of the threat to the shaft tube with the deformations is defined by the following indicators:

– vertical displacement (subsidence) of the points in the rock mass for various horizons $w(x,z)$ [m],
– vertical specific strains ε_z [mm/m],
– horizontal displacements (inclination of the shaft in a horizontal plane) u [m].

In the S. Knothe's theory the distribution of the subsidence of points (Sroka, 1976; Piwowarski at al., 1995) in any horizon of the rock mass z, in the cross section perpendicular to a straight-line edge of the front of mining exploitation, of the half-planar shape is described by the formula (l):

$$w_k(x,z) = -\frac{a(z)\cdot g}{r(z)} \cdot \int_x^{\infty} \exp\left[-\pi \frac{\lambda^2}{r^2(z)}\right] d\lambda \tag{1}$$

where:

a — so-called exploitation coefficient,

r — the radius of dispersion of the main impacts [m], $r = \dfrac{H}{\mathrm{tg}\beta}$

$\mathrm{tg}\beta$ — the parameter of the dispersion of the impacts (tangent of the angle of the dispersion of main impacts).

z — vertical distance of the examined horizon from the mined seam [m],

g — thickness of the mined seam [mm];

x — distance of the examined point from the edge of the exploitation front [m],

$w_k(x,z)$ — final subsidence of the examined point in a given horizon z [mm].

Vertical strain is defined in the following way:

$$\varepsilon_z(x,y,z) \overset{def}{=} \frac{\partial w(x,y,z)}{\partial z} \tag{2}$$

To define rock mass deformation, the important issue is the distribution of r parameter: $r = r(z)$, i.e. the radius of the dispersion of main impacts in the rock mass, described by the following formula:

$$r(z) = r(H)\cdot\left(\frac{z+z_o}{H+z_o}\right)^n \tag{3}$$

where:

H — depth of exploitation [m],

z_o — parameter depending on the values of the radius of impacts r_s (in the roof of the exploited seam).

The value of parameter z_o is found based on the following relationship (4):

$$z_o = \frac{\sqrt[n]{\frac{r_s}{r(H)}} \cdot H}{1 - \sqrt[n]{\frac{r_s}{r(H)}}} \tag{4}$$

where n — parameter of the rock mass, taking values of the range:

$$0,45 \leq n \leq 1,0 \tag{5}$$

Regarding (3), according to the definition of vertical strain (2) we obtain:

$$\varepsilon_z(\cdot, z) = \frac{\partial w(\cdot, z)}{\partial r(z)} \cdot \frac{\partial r(z)}{\partial z} \tag{6}$$

Distribution of vertical strains for a so-called infinite half-plane is described by the relationship:

$$\varepsilon_z(x, z) = -\frac{n}{z + z_o} a \cdot g \frac{x}{r(z)} \exp\left[-\pi \frac{x^2}{r^2(z)}\right] \tag{7}$$

With formula (7) one can calculate the distribution of vertical strains alongside any horizontal or vertical line in the rock mass, for the exploitation in the shape of half-plane.

3. The variability of parameters and coefficients of Knothe's theory in the conditions of LGOM

Accepting the ranges of the variability of the parameters related to the theory has a fundamental significance for the assessment of the accuracy of predicting the values of deformation indexes. Below the ranges of the variability of parameters (Sroka, 1974, 1976; Hejmanowski at al., 2004; Kwinta, 2009; Hejmanowski & Kwinta, 2009) are presented for LGOM conditions.

The angle of dispersion (range) of impacts β

Above 75% of determined values of parameter tgβ are in the range within 1.3÷1.8, and 88% within the range of 1.2÷1.9. The distribution of this parameter is close to the *rectangular (uniform)* distribution.

Exploitation coefficient a

For room-and-pillar systems applied in LGOM usually the following values of coefficient are accepted:

a = 0.5 for systems with caving (natural roof settlement),

a = 0.2 for systems with hydraulic backfill.

This parameter is, however, quite variable. Recently made verifications of parameters indicate that for caving systems this parameter ranges within:

$$0.45 < a < 0.8$$

and for hydraulic backfill:

$$0.2 < a < 0.3$$

The parameter of the rock mass – *n*

The value of parameter n is within the limits (Preusse, 1990):

$$0.45 < n < 0.70$$

The parameter of the dispersion of impacts in the roof of the seam r_s

The value of parameter r_s for the Polish copper ore deposits is (Biliński, 1989) *between 40 and above 150 m*. In expertises and opinions, the calculations are often carried out with the assumptions that $z_o = 0$ ($r_s = 0$).

The presented views on individual parameters and their values, show how difficult it is to accept correct values for the calculations of predicted deformation indexes. The ranges of possible values of parameters are in some cases very wide. Accepting the values of indexes predicted for mean values of the parameters of the model is not a correct procedure. The assumption that occurrence of each value of a parameter from the determined range is nearly equally probable was taken.

4. Preliminary assumptions of exploitation project, with the analysis of the distribution of vertical strains

In case of the exploitation in the protective pillars of the shafts, the most important deformation index is vertical strain.

In most LGOM shafts significant compressing vertical strains occur on the level of the floor of intensively dehydrated tertiary layers and the roof of the buntsandstein ($H \approx 370 \div 420$ m). On this level vertical strains are observed, which in some shafts achieve maximal value:

$$\varepsilon_{z\_odw} \approx -1.5 \text{ mm/m}$$

The total compressing vertical strains should not exceed values:

$$\varepsilon^{(-)}_{z\_gr} \approx -3.0 \text{ mm/m}$$

which means that during the exploitation in the shaft pillar the tubing in the threatened area cannot be subdued to additional compressing strains higher than:

$$\Delta\varepsilon_z = \varepsilon_{z\_dop} = \varepsilon_{z\_gr} - \varepsilon_{z\_odw} = -1.5 \text{ mm/m}$$

Taking into account a significant water threat in the LGOM shafts, in the carried out analyses and theoretical calculations of the predicted deformation indexes, one shoud also consider a random character of deformations.

Two aspects of this issue should be taken into account. The first refers to the irregularities in actually observed deformation indexes. The second question is the assessment of the accuracy of calculations of the predicted deformation indexes, at the presumed variability of input parameters.

The variability of the observed deformation indexes is often characterized by the so called variability coefficient M_D (D – analysed deformation index). The standard deviation of the deformation index e.g. ε_z is:

$$\sigma_{\varepsilon_z} = M_{\varepsilon_z} \cdot \varepsilon_z \tag{8}$$

The value M_{ε_z} was accepted according to the literature (Sroka, 1975; Popiołek, 1994):

$$M_{\varepsilon_z} = 0{,}2 \tag{9}$$

Thus a general formula is:

$$\varepsilon_{z\_\max} = (1 - M_{\varepsilon_z}) \cdot (\varepsilon_{z\_gr} - \varepsilon_{z\_odw}) \tag{10}$$

where: $\varepsilon_{z\_\max}$ — maximal value of compressing vertical strains caused by exploitation.

For the values presented in this chapter we obtain:

$$\varepsilon_{z\_\max} = (1 - 0{,}2) \cdot ((-3{,}0) - (-1{,}5))\,\text{mm/m} = -1{,}2\,\text{mm/m} \tag{11}$$

The scheme of exploitation should be designed in such a way that the *predicted* values of compressing vertical strains for mean values parameters, do not exceed the *limit* value of the Tertiary layers in the floor (–1.2 mm/m) decreased by the *estimated* error of the forecast. It was presumed a priori that the predicted *average* value of maximal vertical strain (for H ≈ 400 m) should equal:

$$\varepsilon_{z\_prog} \approx -1{,}0 \text{ mm/m}$$

The general scheme of the project of exploitation in the protective pillars of LGOM shafts (Niedojadło 2008a) is presented in Fig. 1.

The analysis refers to the impact of exploitation of the external zone (field I and II as well as III and IV – Fig. 1). This exploitation will generate in the shaft-tube maximal values of vertical compressing strains, the border value should not exceed $\Delta\varepsilon_z = -1.5$ mm/m. The calculations were carried out for the mean depth of the floor of the Tertiary horizons $h \approx 400$ m. The stabilizing zone (field V), will make a square of side $\mathbf{2 \cdot d}$. The size of the zone mainly depends on the depth of the exploited deposit and on its thickness and the system of exploitation.

For each field the possibility that different values of parameters characterizing the exploitation (i.e. H, g, a) will occur was accepted. Taking into account state-of-the-art presented in chapter 3, in calculations with Monte Carlo method the following ranges of possible values of the parameters for the theory and coefficients were accepted:

$$1{,}3 \le \text{tg}\beta \le 1{,}8 \quad E(\text{tg}\beta) = 1{,}55$$

$$0{,}2 \le a \le 0{,}3 \quad E(a) = 0{,}25 \text{ (exploitation coefficient for hydraulic backfill)}$$

$$0{,}45 \le n \le 0{,}70 \quad E(n) = 0{,}575$$

$$50\,\text{m} \le r_s \le 150\,\text{m} \quad E(r_s) = 100\,\text{m}$$

$$\Delta H = 10\,\text{m}; \quad H_{avg} - 5\,\text{m} \le H_{avg} \le H_{avg} + 5\,\text{m} \quad E(\Delta H) = 0$$

$$\Delta g = 0{,}3\,\text{m}; \quad g_{avg} - 0{,}15\,\text{m} \le g_{avg} \le g_{avg} + 0{,}15\,\text{m} \quad E(\Delta g) = 0$$

where: $E(\cdot)$ — expected value.

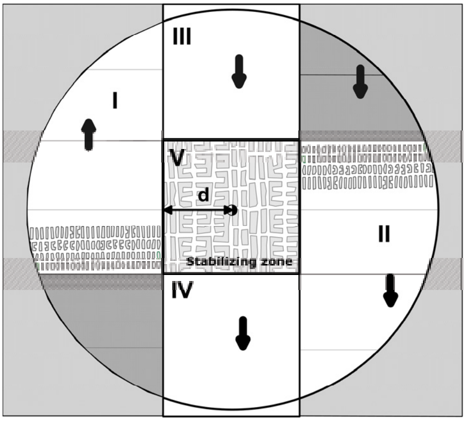

Fig. 1. The scheme of exploitation fields

We ask the question what is the probability that as the result of the exploitation, according to the proposed exploitation scheme (Fig. 1) the values of compressing vertical strains will not exceed the established limit value.

The solution, practically not applied so far at the analysis of the deformation of the shaft tube, is the algorithm of calculations, in which the ranges of the values of parameters are given instead of deterministic values. In every calculation series the set of parameters from the accepted ranges is randomly selected.

The method of the simulation of physical processes with a random selection of input parameters is known in statistics and belongs to so-called *Monte Carlo methods* (MCM). However, they have not been used so far in case of predicting mining damage in the conditions of LGOM.

The application of such algorithm gives possibility of multiple calculations of deformation indexes (Naworyta, 2004). As a result we obtain the set of values, having the character of the continuous random variable. Large enough number of repetitions of calculations and the application of definition and the relation of the probability calculus allows the assessment of the distribution of random variable and estimation of the statistics.

The main disadvantage of the above method is the fact that it is time-consuming. One should make dozens of repetitions of the calculations of the predicted values of deformation indexes, and then the set of these values should be subdued to statistic analysis. For complicated conditions of exploitation and considerable number of fields the time of such calculations can be relatively long. Because of the above, we consider that this method has limited utilitarian value. It was proposed mainly to be applied in research, to define the distribution and accuracy of the determination of the predicted values of deformation indexes. This is a time-consuming method, but gives the fullest description of the distribution.

The assumption was made that the parameter can get any value from the range with equal probability. This means that the parameters are characterized with *a rectangular (uniform) distribution*.

All the calculations were made introducing the interval range of parameters and coefficients as well as the “choice” of their values by programme generator of pseudo-random numbers. As a result of a full series of calculations $N = 100$ of result files were obtained with the values of the calculated deformation indexes, for the generated sets of parameters and coefficients, different every time.

Calculations were repeated 10 times, to assess the repeatability of the results in "samples" and descriptive statistics.

4.1. Deformations of the shaft tube

To verify the accepted assumptions for the method of the exploitation of the copper deposit in protective pillars of LGOM shafts, the calculations were made according to the presented above methods for the exploitation option of the following data:

$$H = 700 \text{ m}, \; a \cdot g = 1.0 \text{ m}$$

For the mentioned above conditions the predicted vertical strains in the floor of Tertiary horizons ($H \approx 400$ m) after the exploitation of fields I-IV (Fig. 1) for the expected values $E(\cdot)$ of parameters will be $E(\varepsilon_z^{(-)}) = -1{,}0$ mm/m if the distance of the edges of the stabilizing zone to the shaft is $\boldsymbol{d} = \mathbf{188}$ **m**.

Dispersion of these values is characterized by, among others, standard deviation. The applied calculation method gives the possibility of determining the values of standard deviation, and also defining the probability of exceeding the limit value ($\varepsilon_{z\_gr} \approx -1.2$ mm/m – equation 11).

The results of calculations were presented in the graphical form in the summary chart (Fig. 2), from which the distribution and the border minimal and maximal values of calculated strains can be found.

In table 1. the following values were put:

- ε_{z_avg} — mean value,
- $\sigma(\varepsilon)$ — standard deviation of the deformation index,
- $\varepsilon_{z\_\max}$ — maximal (absolute) value of vertical compressing strain in the sample,
- $\varepsilon_{z\_\min}$ — minimal value of vertical compressing strain in the sample,
- $\varepsilon_{z\_avg(10)}$ — mean value ε_{z_avg} from ten repetitions,
- $\sigma_{\_avg(10)}$ — mean value σ_{avg},
- $\varepsilon_{z\_avg} - \sigma$ — mean value ε_{z_avg} decreased by standard deviation,
- $\varepsilon_{z\_avg} + \sigma$ — mean value ε_{z_avg} increased by standard deviation.

The obtained mean value of vertical strains is $\varepsilon_{z\_avg(10)} = -0.989$ mm/m ≈ -1.0 mm/m, which *corresponds exactly to the presumed values*. Mean standard deviation in the sample was:

$$\sigma_{avg(10)} = 0.152 \text{ mm/m}$$

Assuming the correctness of the model and assessment of the ranges of parameters and coefficients, the analysis of the obtained calculation results can be done. The analysis showed that vertical strain will range between:

$$-1{,}48 \text{ mm/m} \leq \varepsilon_z^{(-)} \leq -0{,}64 \text{ mm/m}$$

Border values occur sporadically, basically in 1000 results only single such values were found. However, being aware of possible values of the results of calculations is very valuable.

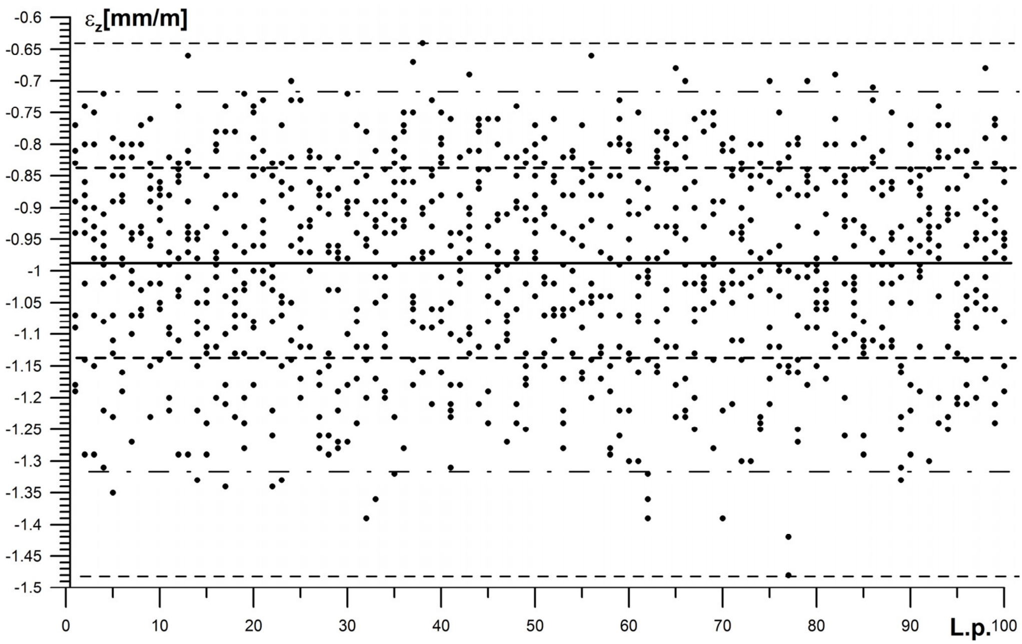

Fig. 2. Chart of the results of simulation calculations

TABLE 1

The results of the calculations of mean and extreme values of vertical strains

Nr	1	2	3	4	5	6	7	8	9	10
ε_{z_avg}	–0.986	–0.980	–1.003	–0.990	–0.997	–0.987	–0.965	–0.996	–0.988	–0.997
$\sigma(\varepsilon)$	0.147	0.156	0.155	0.152	0.146	0.161	0.138	0.158	0.156	0.147
$\varepsilon_{z\_\max}$	–1.34	–1.48	–1.36	–1.42	–1.3	–1.39	–1.35	–1.36	–1.39	–1.33
$\varepsilon_{z\_\min}$	–0.7	–0.73	–0.66	–0.66	–0.69	–0.69	–0.68	–0.64	–0.7	–0.73
$\varepsilon_{z\_avg(10)}$	–0.989									
$\sigma_{\_avg(10)}$	0.152									
$\varepsilon_{z\_avg} - \sigma$	–0.837									
$\varepsilon_{z\_avg} + \sigma$	–1.141									

As a result of calculations with the proposed algorithm the set of the values of deformation indexes is obtained, characterized by the normal distribution, confirmed by the test W Shapiro-Wilk on the significance level 0.05, of the following parameters:

$$N(-1{,}0\,;0{,}15)$$

Now the question about the probability that the predicted value of compressing vertical strain will not exceed the acceptable value ($\varepsilon_z^{(-)} \approx -1{,}2$ mm/m) can be answered.

For:

$$x = -1{,}2; \quad \mu = -1{,}0; \quad \sigma = 0{,}152$$

we obtain:

$$z^* = \frac{x-\mu}{\sigma} = \frac{-1{,}2-(-1{,}0)}{0{,}152} = -1{,}315$$

$$P(-1{,}2 \le x \le -1{,}0) = 0{,}40575$$

$$P(|x| \le |-1{,}2\,\text{mm/m}|) = 0{,}40575 + 0{,}5 \approx 0{,}91$$

where: z^* — variable standardised in the tables of the normal distribution.

In this case the probability of risk (α_{gr}) is 0.09. Often for important engineering objects, for which significant threats were defined, it is accepted that α_{gr} = 0,10 (10%). The obtained value α_{gr} for the acceptable value (–1,2 mm/m) corresponds the value above.

4.2. Distribution of vertical strains alongside the shaft tube

Apart from "point" analysis of the state of deformation, it is also necessary to assess the distribution of deformations and determine the predicted values of maximal vertical strains alongside the whole shaft tube.

Calculations were made for identical ranges of parameters, the results of which were presented on a summary chart (Fig. 3). They show that in the bottom part of the shaft the ranges of the values of strains are much bigger.

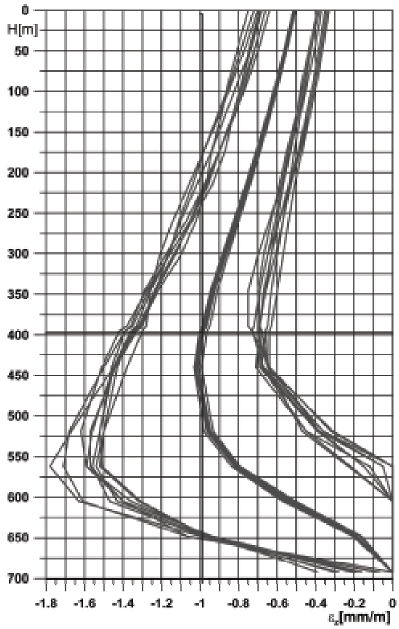

Fig. 3. The course of vertical strains alongside the shaft tube (mean and extreme maximal and minimal values)

The value of standard deviation s, also in the bottom part of the shaft grows to the value $\sigma \approx 0.35$ mm/m at the value $\varepsilon_z = -0.57$ mm/m, i.e. makes more than 60% of this value.

The above values show how difficult is to predict the state of deformations in the shaft-tube. Even small changes of parameters significantly change the predicted values of *maximal* deformations and the place of their occurrence. These values often determine the scale of the undertaken security measures or the place of cutting the shaft lining.

5. The analysis of the prediction accuracy based on the law of error propagation

The presented in the first part of the article assessment of the accuracy of predicting vertical strains in the shafts of LGOM, applying Monte Carlo method, has an experimental character. At the determination of the subsequent „samples" of the values of strains with „drawn" parameters strict formulae of Budryk-Knothe's theory were applied as presented in chapter 2. As it has already been stated, this method is relatively time-consuming and difficult in utilitarian application.

In the further part of this publication the attempt was made to do the analysis of the accuracy of predicting vertical strains with another method, based on the same formulae (1÷7). The calculations were made for the same geological and mining conditions based on often applied in the analysis of measurement results law of error propagation (LEP). RMS error is in this case equivalent to the standard deviation, analysed in previous chapters.

Because of the degree of the complication of the obtained functions for a larger number of fields, the calculations of mean error of vertical strain ε_z was made for the exploitation of one half-plane-shaped field. The edge of the half-plane was 188 metres far away from the axis of the shaft, like in the case of the example analyzed in chapter 4.2. The same set of parameters was accepted, as well as the same range of the intervals of parameters, and their (uniform) distribution.

The law of error propagation is commonly known and applied in engineering calculations (and not only) rule allowing the estimation of the values of the RMS error of the function. To achieve this is necessary to know RMS errors of the arguments of this function and their expected values.

Based on the introduction of the law of the propagation of errors (Hausbrandt, 1970) one can notice the following limitations of its application (resulting from the assumptions accepted at the introduction):

1. Independent variables' errors (arguments of the function, for which the RMS errors will be estimated) should be *independent*.
2. Mean values of the error of each independent variable should be approximately zero.
3. The values of RMS errors of independent variables should be *small*. Deriving of the law of error propagation is based on the development of the function (for which RMS is determined) in the Taylor series, skipping the elements in second and higher powers. What values of RMS errors can be regarded as small depends on the shape of the studied function and the desired accuracy estimation. It should be emphasized that this feature in practice means that the values RMS errors obtained from the application of the law of error propagation are only *estimation*.

For the considered function the law of the propagation of errors takes the form:

$$m_{\varepsilon_z}^2 = \left(\frac{\partial \varepsilon_z}{\partial a} m_a\right)^2 + \left(\frac{\partial \varepsilon_z}{\partial tg\beta} m_{tg\beta}\right)^2 + \left(\frac{\partial \varepsilon_z}{\partial n} m_n\right)^2 + \left(\frac{\partial \varepsilon_z}{\partial r_s} m_{r_s}\right)^2 + \left(\frac{\partial \varepsilon_z}{\partial g} m_g\right)^2 + \left(\frac{\partial \varepsilon_z}{\partial H} m_H\right)^2 \tag{12}$$

where:

$\varepsilon_z = f(a, \mathrm{tg}\beta, n, r_s, g, H, x, z)$,

m_a, $m_{\mathrm{tg}\beta}$, m_n, m_{rs}, m_g, m_H — RMS errors of independent variables (parameters of the model),

m_{ε_z} — RMS error of vertical strain,

x, z — coordinates of the analysed point.

For the calculations we accepted the values of RMS errors of independent variables calculated based on their distribution, thus for uniform distribution, according to the formula:

$$\sigma^2 = \frac{(b-a)^2}{12}$$
$$\sigma \cong \pm 0.29(b-a) \tag{13}$$

where (chapter 4.1):

b — upper limitation of the range, from which the values of the parameter are randomly drawn,

a — lower limitation of the range, from which the values of the parameter are randomly drawn,

σ — standard deviation of the distribution of parameter, here associated RMS error of this parameter.

RMS errors' values of subsequent parameters calculated based on this formula are presented in the table below.

TABLE 2

Parameters of the model and their errors accepted for the calculations of RMS error of vertical strains with the application of LEP

Parameter	**RMS error**
a	0.03
tgβ	0.14
n	0.07
r_s	29 m
H	3 m
g	0.09 m

Assuming formula (7) for the exploitation in the shape of the half-plane for value ε_z, after the differentiation the formula for RMS error of vertical strain was obtained. The resulting formula will not be presented here, because of its large size and uclarity. Better to present are the values RMS errors of vertical strain resulting from this formula. They were presented in figure 4 against the errors estimated with the application of Monte Carlo method for the same conditions.

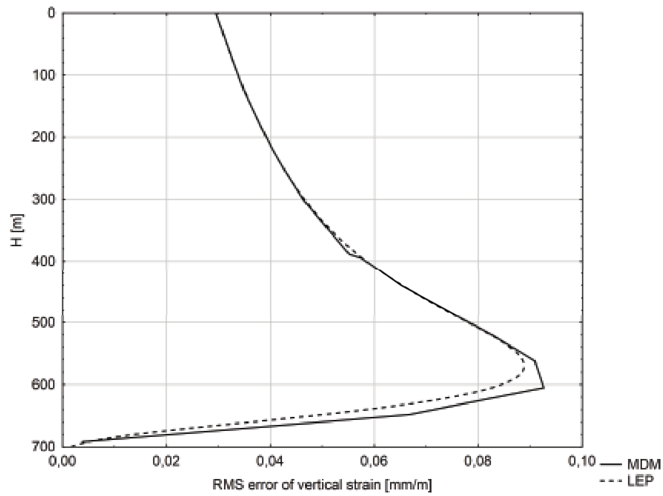

Fig. 4. The graph of the RMS error of predicted values ε_z in the depth function

As it can be noticed, the calculations carried out with two methods give similar results and this way mutually confirm one another (assuming the correctness of the applied premises). It should be underlined that the applied calculation methods have different origin, which is important for the assessment of the reliability of the obtained results.

The greatest discrepancy between the estimated values of RMS error ε_z from both methods take values below 0.02 mm/m, which makes about 25% RMS error of vertical strain in this point (H = 648 m) and about 18% of maximal value of RMS error. In most cases of the discrepancies between RMS errors obtained from both methods, are significantly lower.

The obtained similarity indicate practically sufficient accuracy estimation RMS error of vertical strain at the application of both methods.

The law of error propagation allows simple estimation of the impact of the errors of individual independent variables on the value of the function error (vertical strain). The value of the square of the RMS error of the function is calculated as the sum of squares of the contributions to the error of this function resulting from subsequent variables (parameters of the model). Below only the formulae for parameter a were presented, but the impact of other parameters can be calculated in the same way. RMS error of vertical strain resulting from the RMS error of parameter a can be described by the formula:

$$m_{\varepsilon_z}^a = \pm \left| m_a \frac{\partial \varepsilon_z}{\partial a} \right| \tag{14}$$

Values of these errors do not sum up to the values of RMS error vertical strain, but sum up in squares (to the square of RMS error). Thus it can be presumed that the impact of a given parameter on the value of the square of RMS error of the function can be calculated according to the formula:

$$w_a = \left(\frac{m^a_{\varepsilon_z}}{m_{\varepsilon_z}} \right)^2 \cdot 100\% \tag{15}$$

where: w_a — the impact of parameter a on the value of the square of RMS error ε_z.

The calculated this way impacts of subsequent parameters express the percentage of the participation of errors in the parameters of the model in the calculated squared RMS error of ε_z. The graphs of these impacts are presented in figure 5.

One can notice that the impacts of errors in defining the thickness and depth of exploitation are negligible, however the significant impact of the error of defining r_s expires about 100-150 metres above the exploitation. The impact of the error of the definition of parameter n occurs in the area between 50 and 150 metres above the level of exploitation and from 300 metres above the level of exploitation to the surface.

In the broadest range the significant ones are the errors of parameters a and tgβ. The impact of the error of parameter a is significant more or less at 150$^{\text{th}}$ metre above the exploitation level up to the surface, while in case of parameter tgβ in the area between 50 and 550 metres above the exploitation level.

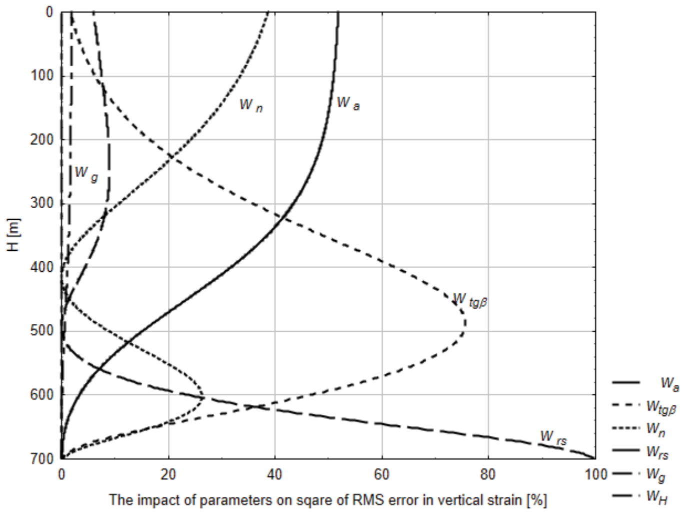

Fig. 5. The graph of the impact of RMS errors parameters of the model on RMS error of the vertical strain function (equation 15)

Knowing these impacts and applying Monte Carlo method for a specified level, one can neglect drawing the parameters that are not significant for RMS error of vertical strain, taking their values equalling their expected values.

The law of error propagation does not allow determining the distribution of the values of function, and only allows the estimation of the values of its RMS error, however, with Monte Carlo method it was shown that at the application of the parameters of the uniform distribution model in drawing, the values of vertical strain has a distribution close to normal. This result complies with central limit theorem, it is not surprising, but worth noticing.

The estimation of the shape of the distribution of the function values is of great significance for probabilistic interpretation of the obtained results. Failing to know the probability density function could lead to the overestimation of danger resulting from the planned exploitation. The application of the Chebyshev inequality, Gauss inequality or Camp-Meidell inequality, gives wider coverage intervals for the same probability than those found in the tables of the normal distribution.

6. Numeric approximation of the law of error propagation

The main disadvantage of the application of the law of the propagation of errors in its exact form is the necessity to define the formulae for the partial derivatives of the function for which the RMS error is calculated. In a particular case such as the exploitation of the half-plane shape, the formulae are possible to define, but for more complicated shapes of exploitation the formulae for derivatives gets much more complicated.

The solution of the problems connected with the formulae of partial derivatives is the application of the approximations in the place of accurate values of derivatives. The approximations are calculated as central differences:

$$\frac{\partial f(x,y)}{\partial x} = \frac{f(x+h,y)-f(x-h,y)}{2h} \tag{16}$$

where:

x, y — coordinates of the point, in which the derivative is calculated,

h — some small value.

In a particular case of the approximation of the values of derivatives for the formula of vertical strain, in the place of function *f* there is vertical strain, and in the place of arguments there are model parameters. If the derivative is to be used in the approximation of values obtained from the law of error propagation, the derivatives will be calculated in the point for which all the model parameters have the values equal to the expected value. This means that e.g. for parameter *a* the appropriate approximation is the following:

$$\begin{gathered}\frac{\partial\varepsilon_z\left(E(a),E(tg\beta),E(r_s),E(n),E(g),E(H),x,z\right)}{\partial a}= \\ =\frac{\varepsilon_z\left(E(a)+h_a,E(tg\beta),E(r_s),E(n),E(g),E(H),x,z\right)-\varepsilon_z\left(E(a)-h_a,E(tg\beta),E(r_s),E(n),E(g),E(H),x,z\right)}{2h_a}\end{gathered} \tag{17}$$

In practice, the application of this formula means that making calculations for mean values of all the parameters, except the parameter after which the derivative is calculated, thus, if adequate care is taken, the task is relatively simple with the application of the program for the predicting of such a deformation index.

The application of this solution allowsvoiding the main problem connected with applying the law of the propagation of errors in its exact form, i.e. deriving formulae for partial derivatives. This solution, however, brings two important problems, which must be considered. Firstly, what width of the range of the values of the parameter should be taken to calculate the derivative value (which is value *h*). Secondly, central differences allow only the calculation of the approximation of the values of derivatives produce inaccuracy in the calculations of the values of mean errors of vertical strain, regarding the law of the propagation of errors (in its accurate form). Both problems described here can be treated jointly (as the cause and result), which leads to the following questions:

1. For which width of the interval (values *h*) of approximation values RMS error ε_z is the most accurate?
2. Do obtained values m_{ε_z} significantly (from practical point of view) depend on the selection of values *h*?

In practice, solving the mentioned above questions, thus the definition of the optimal width *h* required calculations for different values *h* and selecting such a value for which the approximation errors are the smallest. The selected value for one case (values of parameters, shape of exploitation) does not have to be optimal for other cases, thus it is important to assess the impact of *h* selection to the accuracy of the approximations. Values m_{ε_z} obtained in the simulation with Monte Carlo method and the law of error propagation are similar, but there are certain differences between them. The purpose of the approximation should rather be the results of MCM than the law of error propagation. Comparison with the results based on the law of error propagation will present the accuracy of the approximation of the values of derivatives.

Calculations were done for 4 *h* widths. The values *h* was determined in the proportion to the width of the ranges of their variability given in chapter 5.1. The general formula to calculate the values of parameters h for subsequent options is the following:

$$h_x = k\frac{x_{\max} - x_{\min}}{2} \tag{18}$$

where:

- x — parameter,
- $x_{\max}, x_{\min}$ — respectively: maximal and minimal value of parameter (upper and lower limitation of the range of its variability),
- k — coefficient.

For subsequent options – the following *k* values were accepted respectively 1/3, 1/2, 2/3 and 1. In table 3 the RMS errors of numeric approximations of the law of error propagation (further on called the LEP-N) for subsequent values *k*. Calculated RMS error was referred both to the results of Monte Carlo simulation, and the law of error propagation. Calculations were made for the previously described case of the exploitation of the first field in the protective pillar of the shaft (chapter 4).

TABLE 3

The values of RMS errors of the approximations with the application of the LEP-N

k	**RMS referring to MCM [mm/m]**	**RMS referring to LEP [mm/m]**
1/3	0.0039	0.0078
1/2	0.0059	0.0028
2/3	0.0044	0.0024
1	0.0029	0.0037

As one can notice, the RMS error of approximations of the LEP-N referring to the MCM is the smallest for the widest analyzed range ($k = 1$), i.e. for the calculations based on extreme values of parameters. Relatively good adjustment was obtained for all the calculation options, which proves small, from the practical point of view sensitivity of the accuracy of approximation on the width of range $2h$. This is particularly important in the use of the proposed approximation for the cases, where the MCM was not carried out, thus results of the LEP-N will not be verified.

To establish the wider context RMS errors were calculated in such a way that the reference method was MCM, and the tested one was the LEP (in its accurate form). The value of this error is 0.0028 mm/m. Taking into account the above, the results of the LEP-N can be regarded very good. In figure 6 the graphs m_{ε_z} of determined MCM and with the LEP-N for $k = 1$, were put together.

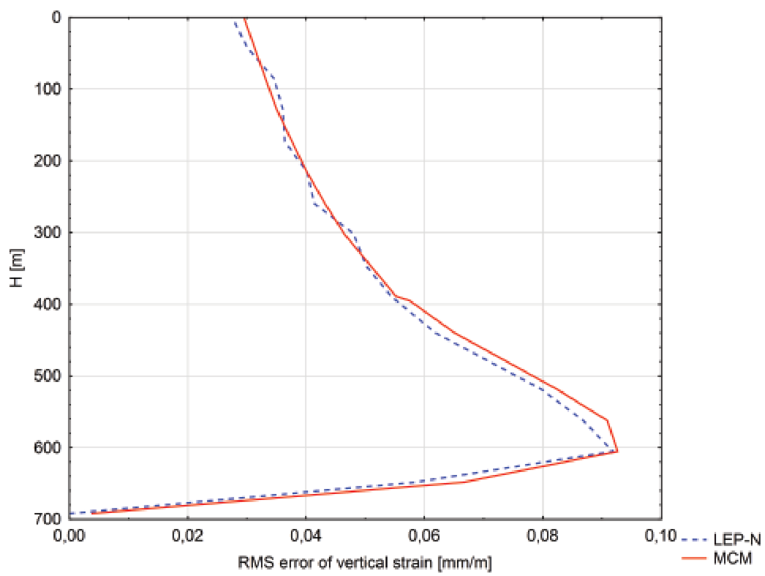

Fig. 6. Comparison of the graphs of RMS error ε_z estimated with MCM and LEP-N

For more exact illustration, showing which parameters generate approximation errors in figure 7 were put in the graphs of the contributions (RMS errors of vertical strains, resulting from the RMS error of the given parameter) for the LEP and the LEP-N ($k = 1$). As one can notice,

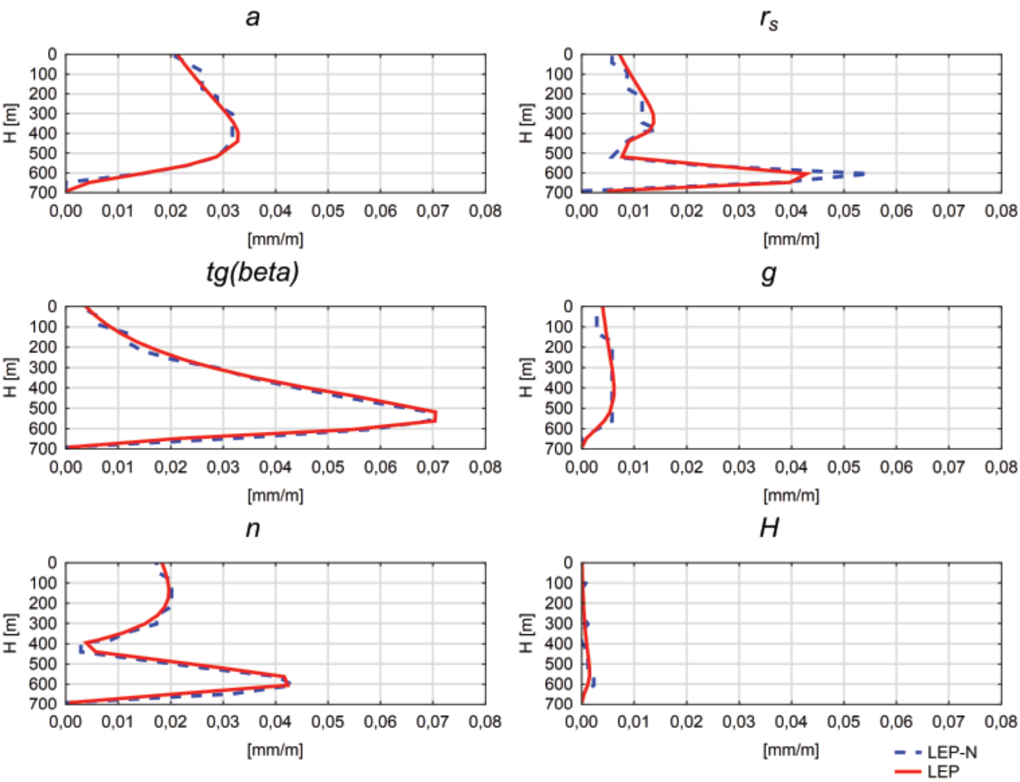

Fig. 7. RMS errors of the prediction of the vertical strain resulting from individual parameters of the model

the compliance of the graphs is very good, the existing small discrepancies partially result from the accuracy of the calculations of vertical strains with the applied program (i.e. 0.01 mm/m), and partially from other factors i.e. relatively large non-linearity of the function values in the relation to e.g. r_s parameter.

The proposed method of the approximation of the values of the LEP has the features (both advantages and disadvantages) of the LEP, as well as MCM. From the point of view of the disadvantages of calculations for the LEP-N, last longer than the LEP (already after calculating derivatives). At the same time the results provided by the LEP-N are limited compared to the results provided by MCM. The LEP-N, like the classical law of error propagation provides information only about the RMS error of the given function and does not inform us on the distribution of the random variable.

From the point of view of the advantages for the LEP-N the calculations are much faster than for MCM, and contrary to the classical law of error propagation, the numeric approximation of derivatives can be applied also for complicated shapes of exploitation. Owing to the calculations made by MCM, distribution, thus probabilistic interpretation of the results is also known. This means that also this potential disadvantage of the application of the LEP-N does not have practical significance. From practical point of view it can be presumed that the approximation of MCM by the LEP-N is accurate enough to be attributed the same probabilistic interpretation.

Taking into account all the described features of the LEP-N, its application for practically occurring complex cases seems simpler, faster and not much less accurate than the other described method. Thus the LEP-N is the most practical method of all the described.

To illustrate the results the calculation m_{ε_z} as made applying the LEP-N ($k = 1$) for the state after exploitation of 4 fields in the protective pillar of the shaft (chapter 4). Fig. 8 presents the set of graphs with the results of calculations made by the LEP-N and for MCM for this case.

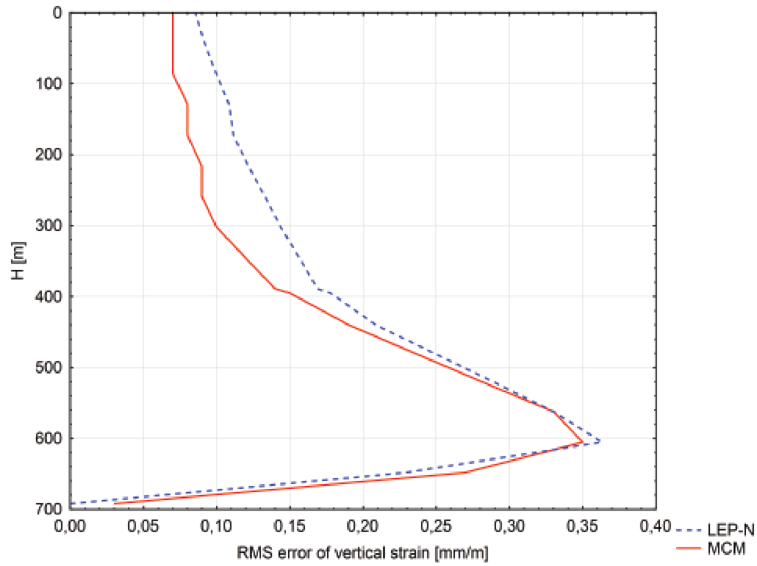

Fig. 8. RMS errors of the prediction of vertical strain in the shaft after exploiting 4 fields of the pillar estimated with MCM and the LEP-N

The obtained compliance is sufficient from the practical point of view. RMS error of the LEP-N at the assumption of MCM as reference method has the value of 0.03 mm/m, which makes almost 10% of maximal value m_{ε_z} calculated with MCM for this case. Maximal discrepancy between the results of the calculations with both methods does not exceed 0.05 mm/m, which makes almost 15% of maximal values m_{ε_z} for the results obtained from MCM.

7. Conclusions

In the research dealing with the prediction of the impact of the exploitation of the copper deposit in the protective pillars of the shafts in LGOM, it was necessary to apply a new calculation method, based on Knothe's theory, giving the possibility of the assessment the probability of risk exceeding acceptable values of deformation indexes, and introducing proper coefficient of safety. The proposed calculation algorithm based on the applied in statistic Monte Carlo method, has not been applied before in the case of predicting deformations of the rock mass and the shaft tube. Proper computer applications were also made, supporting the calculation process and statistic analyses.

The analysis of the accuracy of vertical strains calculations, based on the same equations and assumptions, based on the law of error propagation confirmed the correctness of the method selection and meaning of the application of this method in predicting the impacts of mining exploitation.

The presented method can be applied not only in the estimation of the values of deformation indexes, but also in different issues. The disadvantage of the presented theoretical analysis (the law of error propagation) is significant complication of the derived formula, at any shapes of exploitation. The advantage is the short time of making the calculations once the formula is derived. On the other hand, the application of Monte Carlo method is often too much time consuming, however allows the estimation not only of the values of root mean square error of the function, but also the distribution of values for more complicated shapes of exploitation. This method is relatively simple to program, while practical requirements of the accuracy of the estimation of errors of vertical strains allows the limitation of the number of iterations.

The proposed method, preliminarily called the LEP-N based on the law of error propagation at the application of central differences for the approximation of the values of derivatives seems the most practical of all the presented in the article. It combines the features of both methods, however from the practical point of view the balance of advantages and disadvantages compared to the other methods seems positive. This method guarantees the simplest way of processing the results and in practice the shortest time of obtaining them at small loss of accuracy compared to Monte Carlo method. To use the LEP-N it is necessary that the number of calculations equals the double number of significant model parameters. The full version of presented algorithm requires 12 calculations (there are 6 model parameters: a, tgβ, n, r_s, g, H). Rejection of the parameters that do not have a significant impact on worsening the accuracy of the assessment of the RMS error (i.e. g and H) only 8 calculations are required. It is a small number compared to the number of calculations necessary in Monte Carlo method. Moreover, the calculations are done for fixed values of parameters, thus the problem of the quality of random numbers generator disappears, while the problem of the selection of the width of the intervals of parameters taken to calculate finite differences appears. The results of the presented calculations indicate, however that the accuracy of approximation with the use of the LEP-N is relatively resistant to the selection of the width of these intervals, which is very important for practical purposes. The carried out calculations indicate that the best results are obtained for relatively wide range of parameters used to calculate central differences ($k = 1$).

For the forecast and estimation of the accuracy of the calculated values of vertical strains the relationships based on Knothe's theory were applied. In case of the intention to apply another model to carry out the operations as the calculations described in this article, it is necessary to know the range of all the parameters significant for the calculations in this model. This can cause certain difficulties, however the authors of the publication encourage doing such calculations for the research, because this way the assessment of the accuracy of the models used for the predictions would be made, and not, as usually it takes place, for the approximation of deformation indexes for the exploitation already carried out. This would allow their comparison, and possibly also the choice of the most appropriate model. One should emphasise that in case of the forecasts, unlike the approximation, not necessarily more "flexible" models will turn out to be more accurate than the models of a smaller number of parameters.

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