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Fluid flow consideration in fin-tube heat exchanger optimization

PIOTR WAIS*

Cracow University of Technology, Department of Thermal and Power Engineering, al. Jana Pawła II 37, 31-864 Kraków, Poland

Abstract The optimization of finned tube heat exchanger is presented focusing on different fluid velocities and the consideration of aerodynamic configuration of the fin. It is reasonable to expect an influence of fin profile on the fluid streamline direction. In the cross-flow heat exchanger, the air streams are not heated and cooled evenly. The fin and tube geometry affects the flow direction and influences temperature changes. The heat transfer conditions are modified by changing the distribution of fluid mass flow. The fin profile impact also depends on the air velocity value. Three-dimensional models are developed to find heat transfer characteristics between a finned tube and the air for different air velocities and fin shapes. Mass flow weighted average temperatures of air volume flow rate are calculated in the outlet section and compared for different fin/tube shapes in order to optimize heat transfer between the fin material and air during the air flow in the cross flow heat exchanger.

Keywords: Exchanger optimization; Fluid flow; Fin profile; CFD

Nomenclature

A_{fin}	–	fin surface area, m^2
a_{fin}	–	mesh element size on the fin surface that surrounds the local node, m^2
c_f	–	fluid (air) specific heat capacity, J/(kg K)
$d\,r$	–	derivative of radial coordinate
$f(r)$	–	fin profile function
$\dot{m}_f$	–	fluid (air) mass flow rate, kg/s
$\dot{m}_{f\,n}$	–	local fluid (air) mass flow at a node, kg/s

*E-mail address: wais@mech.pk.edu.pl

m_s – tube and fin mass (solid), kg
r – radial coordinate, m
r_a – radial coordinate of fin tip, m
r_b – radial coordinate of fin base, m
r_{ch} – radial coordinate of chamfer, m
p_f – fin pitch, m
p_t – tube pitch, m
$\dot{Q}$ – heat flow removed from fin and tube to the fluid (air), J/s
$T(r)$ – temperature on the fin surface (depends on r), °C
$T_{f\ n}$ – local fluid (air) temperature at a node, °C
T_{IN} – fluid (air) temperature in the inlet section, °C
T_S – surrounding temperature, °C
T_T – internal tube surface temperature, °C
T_{OUT} – average fluid (air) temperature in the outlet section, °C
V_s – volume of tube and fin material (solid), m^3
v_{IN} – fluid (air) velocity in the inlet section, m/s

Greek symbols

α – heat transfer coefficient, W/(m^{2}K)
β_1 – fin profile angle between tube and fin surface, deg
β_2 – fin profile angle between two fin surfaces after chamfer introduction, deg
ΔT = $T_{OUT} - T_{IN}$ – difference in fluid (air) temperature between outlet and inlet section, °C
$\Delta\varepsilon$ – optimization function change relative to the function value received for rectangular profile (a), %
δ – fin thickness, m
δ_f – max fin thickness, m
δ_t – tube thickness, m
ε – optimization function, (kg °C)/(s cm^3)
θ – temperature difference between a point on a fin surface (with coordinate r) and the surroundings, °C
θ_a – temperature difference between a point on a fin tip and the surroundings, °C
θ_b – temperature difference between a point on a fin base and the surroundings, °C
λ – thermal conductivity, W/(m K)
ξ – ratio between the heat removed from the tube/fin component to the tube/fin weight, J/(kg s)
ρ_s – material density for solid (tube and fin), kg/m^3

1 Introduction

Extended surfaces, in the form of longitudinal or radial fins are common in applications where the need exists to enhance heat transfer between the surface and adjacent fluid. Applications range from very large to small scale (tubes in heat exchangers, the temperature control of electronic components). Finned tube heat exchangers are used in different thermal systems for applications where heat energy is exchanged between different media. The analysis of heat transfer from radial finned surfaces involves solving second-order differential equations and is often a subject of research including also the variable heat transfer coefficient as a function of temperature or the fin geometrical dimensions.

Such equipment is often chosen from the sets of similar exchangers without real design work. Quick selection among the existing solutions can cause that the design does not fulfill the specification or does not consider the proper functionality. Cost optimization of the heat exchanger is an important target for the designers. Saving material and energy are common objectives for optimization. One of the important issues that should be defined during the design work is the optimization of the heat efficiency taking in consideration the cost of material.

There are different types of optimization for radial fin heat exchangers. The optimization can consider [1]:

- minimum weight for a specified heat flow,
- fin profile based on a set of specified conditions (for instance the dissipation from the fin faces and calculation of minimum volume as well as minimum profile area),
- placement of individual fins to form channels.

Analytical investigations and search activities that allow to find the optimal profile of the fin are available under assumptions that simplify the problem of heat transfer. These basic assumptions are proposed by Murray (1938) and Gardner (1945), and are called Murray-Gardner assumptions [1]:

- heat flow in the fin and its temperatures remain constant with time,
- fin material is homogeneous, its thermal conductivity is the same in all directions, and it remains constant,
- convective heat transfer on the faces of the fin is constant and uniform over the entire surface of the fin,
- temperature of the medium surrounding the fin is uniform,
- fin thickness is small, compared with its height and length, so that temperature gradient across the fin thickness and heat transfer from the edges of the fin may be neglected,

- temperature at the base of the fin is uniform,
- there is no contact resistance where the base of the fin joins the prime surface,
- there are no heat sources within the fin itself,
- heat transferred through the tip of the fin is negligible compared to the heat leaving its lateral surface,
- heat transfer to or from the fin is proportional to the temperature excess between the fin and surrounding medium.

In general, the study of extended surface heat transfer compromises the movement of the heat within the fin by conduction and the process of heat exchange between the fin and the surroundings by convection.

The simple radial fin with a rectangular profile is sketched in Fig. 1:

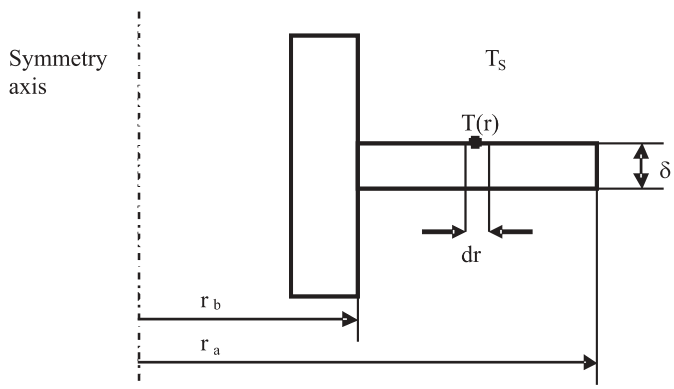

Figure 1. Radial fin of rectangular profile.

The heat flux in a parabolic fin is less sensitive to the variation of the tip temperature than in the case of rectangular and trapezoidal fin profiles. This can be seen after resolving the differential equations analytically.

For the ideal case, if the convection is considered in a fin heat exchanger and the surrounding temperature is equal to T_S, the temperature difference between any point on the fin surface and the surrounding temperature can be written as:

$$\theta = T(r) - T_S \ , \tag{1}$$

where $T(r)$ is the fin surface temperature that varies from the fin base to the fin tip.

The optimized profile of the symmetrical radial fin of least material can be found from the generalized differential equation [1]:

$$f(r)\frac{d^2\theta}{d\,r^2}+\frac{f(r)}{r}\frac{d\theta}{d\,r}+\frac{d\,f(r)}{d\,r}\frac{d\theta}{d\,r}-\frac{\alpha}{\lambda}\theta=0\,. \tag{2}$$

Assuming that the temperature excess changes linearly:

$$\theta=\theta_b\left(1-\frac{r-r_a}{r_a-r_b}\right)\,, \tag{3}$$

and resolving above equation with two differential conditions:

$$\frac{d\theta}{dr}=\frac{-\theta_b}{r_a-r_b}\,, \tag{4}$$

$$\frac{d^2\theta}{dr^2}=0\,, \tag{5}$$

the profile function is derived for the radial fin of least material [1]:

$$\frac{\lambda\,f(r)}{\alpha\,r_a^2}=\frac{1}{3}\left(\frac{r}{r_a}\right)^2-\frac{1}{2}\left(\frac{r}{r_a}\right)+\frac{1}{6}\left(\frac{r_a}{r}\right)\,. \tag{6}$$

However, due to the manufacturing problem, the profile described by Eq. (6) is not used.

The fin profile and its optimization issue is often the subject of research. Different authors eliminate some of Murray-Gardner assumptions in their investigations that makes the problem more complex.

In paper [2], Ullman and Kalman proceed an optimization for a single fin for a known fin mass. Four different cross-sections are analyzed: constant thickness, constant area of heat flow, triangular and parabolic shapes. They show that the fins with sharp edges and a sharper reduced thickness have lower efficiencies and higher quantities of heat dissipation per a fin mass. The fin that has the best performance is one of parabolic shape.

The efficiency of annular fin, when subjected to simultaneous heat and mass transfer mechanism, is analyzed by Sharqawy and Zubair [3]. Analytical solutions are obtained for the temperature distribution over the fin surface when the fin is fully wet.

Kundu and Das [4] determine optimum dimensions of plate fins for fin tube heat exchangers considering rectangular and hexagonal profile of fins. Maximum heat dissipation is obtained for a particular value of pitch length or fin thickness for a fixed fin volume.

Rocha *et al.* [5] analyze the heat transfer in one and two row tubes. They use plate heat exchangers in which fins are in the shape of circular or elliptical section. Two dimensional model allows to study and compare the performance of two configurations with experimental results (published by others). The elliptic fin heat exchangers have better overall performance than circular tubes.

Closed-form solution of 1-D heat conduction problem for a single straight fin and spine of a constant cross-section is obtained by Dul'kin and Garas'ko [6,7]. The local heat transfer coefficient is assumed to vary as a power function of temperature excess. They also determine the temperature difference between the fin tip and the ambient fluid.

Variable heat transfer coefficient is investigated by Mokheimer [8] for annular fins. He assumes the natural convection and analyses heat transfer for the heat transfer coefficient that is the function of local temperature. The results show that the application of constant heat transfer coefficient underestimates the fin efficiency.

Two dimensional heat transfer equation is solved analytically to obtain temperature distribution and heat transfer rate by Arslanturk [9] for non symmetric convective boundary conditions. In this work, the fin volume is fixed and the optimum geometry is searched to maximize the heat transfer rate for a given volume. The optimization variables are the fin thickness and ratio of outer radius to inner radius of a fin. The temperature distribution and the heat transfer rate is reached analytically. The same author, in paper [10], uses the Adomian decomposition method (ADM) to evaluate the efficiency of straight fins with temperature dependent thermal conductivity. He determines the temperature distribution within the fin and also notices that the thermal conductivity parameter has a strong influence over the fin efficiency. The received data are correlated for a wide range of thermogeometric fin parameters and the thermal conductivity parameter.

Malekzadeh *et al.* [11] optimize the shape of non-symmetric, convective radiative annular fins based on two nonlinear dimensional heat transfer analysis. The results received by means of the differential quadrature method are compared with those obtained from the finite difference method.

The homothropy analysis method (HAM) is used, by Khani *et al.* [12], to evaluate the analytical approximate solutions and efficiency of the nonlinear fin problem with temperature dependent thermal conductivity and heat transfer coefficient that can change along the surface especially in boiling, condensing or natural convection situations.

Aziz and Fang [13] determine the energy equations for one-dimensional steady conduction in the longitudinal fins of rectangular, trapezoidal, and concave parabolic profile. The temperature and the heat flux is specified at the base of the fin and the temperature distribution in the fins are provided for these conditions. They resolve the problem for a two-point boundary value with one thermal condition, given at the fin base and the other at the fin tip.

The thermal analysis and optimization of straight taper fins are presented by Kundu and Das [14]. The temperature profile is determined analytically for longitudinal, spine and annular fins. It is observed that the variable heat transfer coefficient has a strong influence over the fin efficiency. Laor and Kalman [15] present the theoretical-numerical analysis of longitudinal and annular fins and spines of different profiles. The optimum dimensions are presented for the temperature dependent heat transfer coefficient.

2 Problem description and fin heat exchanger models

Fin heat exchangers consist of tubes and fins. The main task of the fins is to increase the heat transfer rate between the heat exchanger surface and the surroundings. If the fin is positioned into an air stream, the flow applies a force from the fin tip surface in the direction of the oncoming flow (drag). The resistance of the body results in a pressure drop. The fin and tube surface orientation also affects the flow route and causes the variation of the air streamlines. Described phenomena modify the conditions of the heat exchange between the plate and the fluid having the effect on the heat transfer. The rate of heat transfer does not depend only on wall surface dimensions, heat transfer coefficient and the temperature difference between the fluid that surrounds the plate and the plate surface temperature. The air velocity and the fin shape are also essential because the fin profile influences the flow direction. For heat exchangers, built with many fins and designed for real industry, it is important to pay attention and calculate the heat transfer considering the fluid flow and flow paths.

The main objective of this research is to determine numerically the performance of a given heat exchanger for three different fin profiles, with emphasis on the flow rates. Different flow velocities are assumed in the inlet section for heat transfer estimations. The potential air flow effect on the heat transfer is calculated and the forced convection heat transfer is modeled by means of Ansys Workbench commercial program. The optimized dimension of the fin is defined as the fin profile for which the ratio between the effective heat amount removed from the tube/fin component to the tube/fin weight reaches the maximum. The heat flux depends on the temperature difference between the local plate/tube and local air temperatures. This means that the temperatures vary along the cross section of the air stream and along the fluid flow direction.

The heat is transferred from the tube/fins to the air that flows over the fin. All results are calculated considering the air flow and its streamline deviations caused by the plate and tube configuration. The fluid flow is the variability that is often neglected in the fin optimization process. The literature includes large numbers of publications dealing with convective heat transfer for different surface geometry, fluid flow type, fluid composition, and thermal boundary conditions but without the fluid flow introduction.

The subject of this paper is a heat exchanger equipped with radial fins. A single row of tubes with fins that forms the heat exchanger is taken into consideration, including the fluid flow. A cross flow heat exchanger is shown schematically in Fig. 2 and its characteristic dimensions are written in Tab. 1.

Table 1. Heat exchanger characteristic dimensions.

	Fin and tube pitches
p_f [mm]	2.6
p_t [mm]	80

Three different radial fin profiles are used for the fin shape optimization. Due to the fact that the symmetry occurs in the air flow among the tubes and fins, the heat transfer area is limited to the quarter of one segment. The segment consists of one tube and fin sections. The fin and tube design, applied for profile modifications, is illustrated in Fig. 3. The shapes of fins, used for the simulations, are shown in Fig. 4(a)–(c). All profiles have the

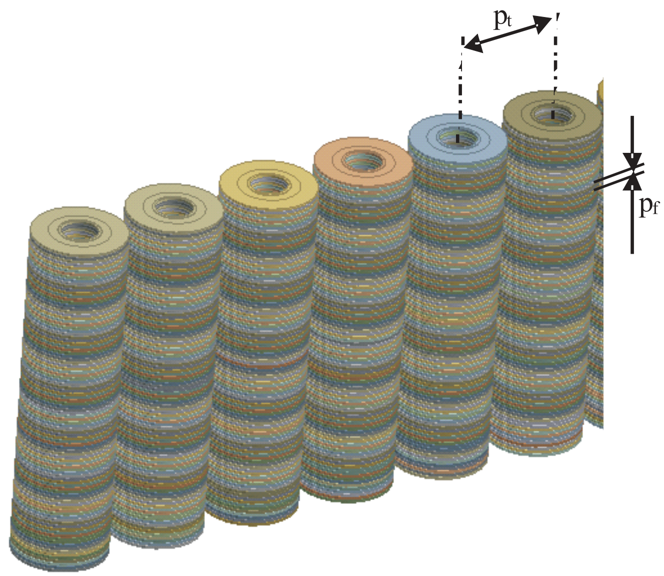

Figure 2. One row heat exchanger.

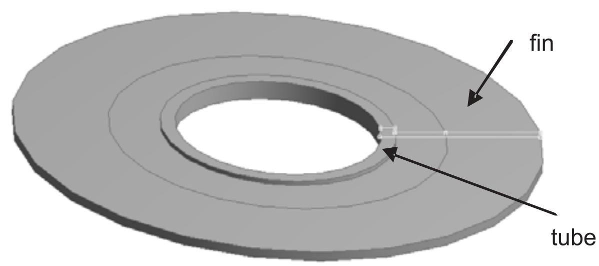

Figure 3. Fin and tube shape.

same radius r_a and thickness δ_f at the fin base. The coordinate dimension r is measured from the tube axis. The dimensions of all fins are presented in Tab. 2.

Each fin operates as a part of the heat exchanger presented in Fig. 2 and the initial thermal conditions are the same for all fins:

- air inlet temperature T_{IN},
- temperature on the internal tube surface T_T.

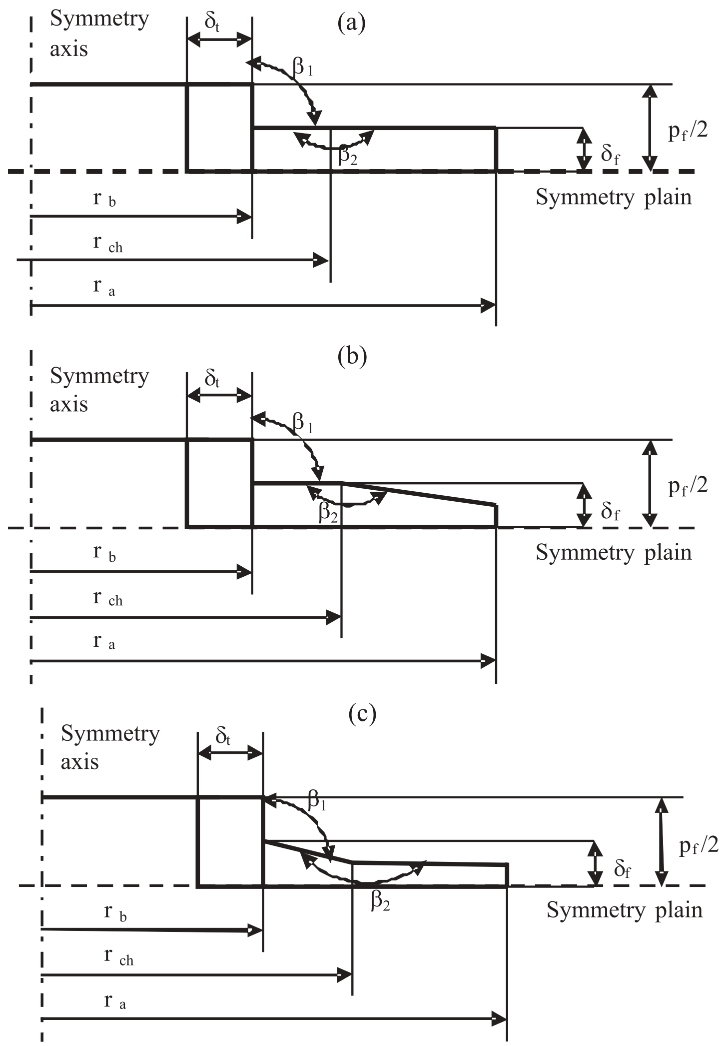

Figure 4. Fin profiles (a), (b), (c).

Inlet velocity v_{IN} varies from 0.5 m/s to 1.5 m/s.

Based on the described physical boundary conditions, the 3-D model is built. The boundary conditions are the same for all models: no slip walls, interface area between fluid and fin/tube surfaces, and symmetry for other surfaces (excluding inlet and outlet). The model sketch, including also an air volume attached to the fin and tube segment, is demonstrated in Fig. 5,

Table 2. Fin and tube dimensions

	Fin version		
	(a)	(b)	(c)
r_a [mm]	36	36	36
r_{ch} [mm]	–	23	23
r_b [mm]	16	16	16
δ_f [mm]	0.65	0.65	0.65
δ_t [mm]	1.3	1.3	1.3
β_1 [deg]	90	90	92
β_2 [deg]	180	178	182

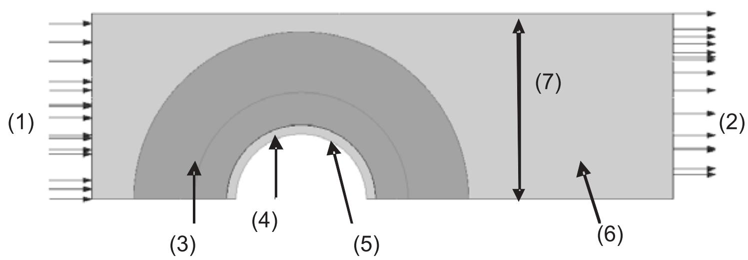

Figure 5. Model used for CFD simulation.

where:
(1) – inlet area with constant temperature $T_{IN} = 10\,^{\circ}$C and unanimous air velocity normal to the section,
(2) – outlet area,
(3) – fin made of steel,
(4) – tube made of steel,
(5) – inner tube surface with constant temperature $T_T = 90\,^{\circ}$C,
(6) – air volume,
(7) – model width equal to $\mathrm{p}_t/2$.

3 Results and discussion

Numerical analyses are carried out to examine finned tube heat exchanger. The solutions are obtained by means of ANSYS commercial program. Iterative convergence is monitored and ensured by the root mean square residuals. The numerical experiments are performed with different RMS residual values. The target value of the local imbalance of each conservative control volume equation, used for models, is 0.0001. All simulations run till the convergence criteria are met. The tube material is kept fixed as well as the heat exchanger fin and tube pitches (spacing). No changes are done to the inlet and outlet temperature and pressure values. The model allows to consider the heat transfer in three directions. This is an advantage, comparing to other optimization method, where the temperature profile is two-dimensional. For each velocity, the shape of the fin and tube is modified to calculate heat transfer for different conditions, reduce the total mass that refers to the cost of the whole heat exchanger.

The heat removed from the fin and tube to the air can be expressed as:

$$\dot{Q} = \dot{m}_f \, c_f \, (T_{OUT} - T_{IN}) \tag{7}$$

and the tube/fin mass can be written as:

$$m_s = \rho_s \, V_s \, , \tag{8}$$

so that the ratio ξ is equal:

$$\xi = \frac{\dot{Q}}{m_s} \, , \tag{9}$$

$$\xi = \frac{\dot{m}_f \, c_f \, (T_{OUT} - T_{IN})}{\rho_s \, V_s} \, . \tag{10}$$

If the values of c_f, ρ_s do not change during the air flow, then the optimization problem can be resolved by finding the maximum value of the optimization function:

$$\varepsilon = \frac{\dot{m}_f (T_{OUT} - T_{IN})}{V_s} = \frac{\dot{m}_f \Delta T}{V_s} \to \max \, . \tag{11}$$

The temperature difference is found numerically and the solid volume is calculated for different fin profile shapes. The air temperature value is also computed numerically in the outlet section and the average air temperature is evaluated according to the formula:

$$T_{OUT} = \frac{\sum (\dot{m}_{f\,n} \, T_{f\,n})}{\dot{m}_f} \, . \tag{12}$$

The values of the optimization function are found for different models and given in Tab. 3:

Table 3. Optimization function for different profiles and air velocity.

	Air velocity $v_{IN} = 0.50$ m/s			Air velocity $v_{IN} = 0.75$ m/s		
	Fin profile			Fin profile		
	(a)	(b)	(c)	(a)	(b)	(c)
ΔT [°C]	32.9	41.8	39.0	31.3	32.9	30.5
V_s [cm^3]	1.19	0.89	0.83	1.19	0.89	0.83
m_f [10^{-5}kg/s]	3.08	3.08	3.08	4.62	4.62	4.62
ε [10^{-5}(kg °C)/s cm^3)]	85.1	144.6	144.1	121.2	170.9	169.1
$\Delta\varepsilon$ [%]	0.0	69.9	69.3	0.0	41.0	39.5

	Air velocity $v_{IN} = 1.00$ m/s			Air velocity $v_{IN} = 1.50$ m/s		
	Fin profile			Fin profile		
	(a)	(b)	(c)	(a)	(b)	(c)
ΔT [°C]	27.5	22.7	26.9	20.2	13.7	17.0
V_s [cm^3]	1.19	0.89	0.83	1.19	0.89	0.83
m_f [10^{-5}kg/s]	6.16	6.16	6.16	9.24	9.24	9.24
ε [10^{-5}(kg °C)/s cm^3)]	142.3	156.6	198.5	156.4	142.0	188.1
$\Delta\varepsilon$ [%]	0.0	10.1	39.5	0.0	-9.2	20.2

The results illustrate how the fin dimensions influence the defined optimization function for different air rates. The best results, among three profiles, are received for profile (c) especially for higher inlet air velocities. In that case, value ε is higher than for profile (a) and (b).

The same conclusion may also be reached with the flow analysis. Evaluating the streamlines for all models, the influence of fin shape on mass flow distribution is seen. The examples are demonstrated in Fig. 6, Fig. 7 for fin profile (a) and in Fig. 8, Fig. 9 for fin profile (c) for different air velocities. The air, that comes through the heat exchanger, flows over the fin. The fin temperature varies and where the fin temperature is higher (closest the tube, higher the temperature on the fin surface), the air has a possibility to absorb more thermal energy from the heat exchanger. For higher air velocities, the profile (b) obstacles the flow to reach more profitable area. The temperature distribution is shown for profile (c) and velocity $v_{IN} = 0.5$ m/s, as an example, in Fig. 10.

To confirm the observation, the outlet area is divided into three sections: Outlet 1, Outlet 2 and Outlet 3, as explained in Fig. 11. Then, the mass

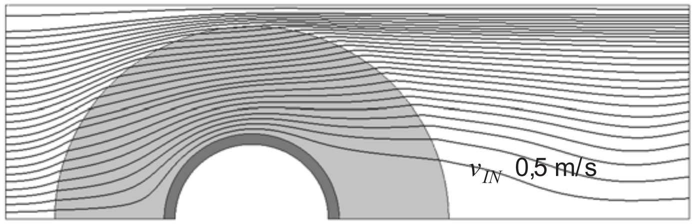

Figure 6. Air streamlines for fin profile (a) and v_{IN} 0.5 m/s.

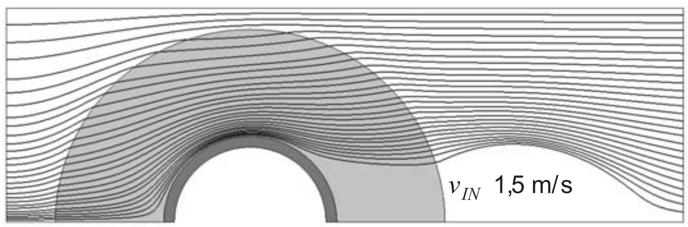

Figure 7. Air streamlines for fin profile (a) and v_{IN} 1.5 m/s.

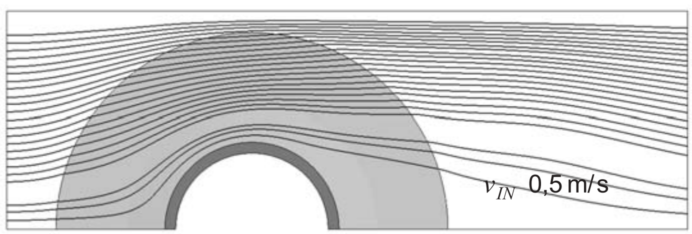

Figure 8. Air streamlines for fin profile (c) and v_{IN} 0.5 m/s.

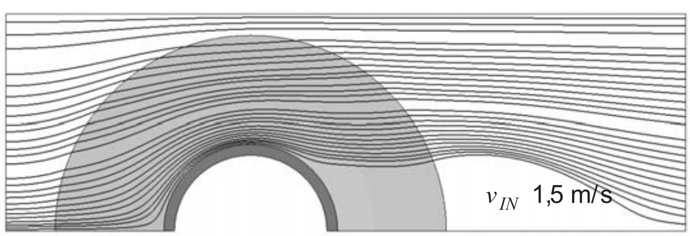

Figure 9. Air streamlines for fin profile (c) and v_{IN} 1.5 m/s.

Figure 10. Temperature on fin surface and flowing air for profile (c), velocity $v_{IN} = 0.5$ m/s.

flow through these specified surfaces are calculated for each of three different fin models. The results for different fin profiles are presented in Tab. 4, in the reference to the whole mass flow in outlet section (total mass flow in the outlet section is 100%).

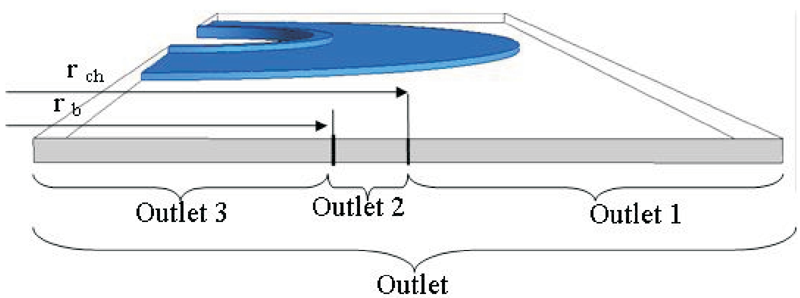

Figure 11. Outlet area partition.

The results of mass flow, collected in Tab. 4, confirm that the heat transfer between the fluid and the heat exchanger is sensitive to different fin profiles. The fin profiles do not cause only the variation of the fin surface temperature in the direction r, but also affect the flow and cause the variation of the air streamlines.

Table 4. Mass flow in two sections of outlet area.

	Air velocity $v_{IN} = 0.50$ m/s			Air velocity $v_{IN} = 0.75$ m/s		
	Fin profile			Fin profile		
	(a)	(b)	(c)	(a)	(b)	(c)
Outlet 1 [%]	68.7	48.4	55.3	68.6	58.0	62.7
Outlet 2 [%]	14.3	17.4	25.9	14.2	17.4	15.9
Outlet 3 [%]	17.0	34.2	18.7	17.3	24.6	21.4

	Air velocity $v_{IN} = 1.00$ m/s			Air velocity $v_{IN} = 1.50$ m/s		
	Fin profile			Fin profile		
	(a)	(b)	(c)	(a)	(b)	(c)
Outlet 1 [%]	73.2	62.2	61.9	69.0	66.3	59.6
Outlet 2 [%]	12.7	17.5	15.9	14.2	19.6	16.9
Outlet 3 [%]	14.1	20.4	22.2	16.8	14.2	23.6

4 Conclusions

The subject, investigated in the paper, is inspired by the increasing need for optimization in engineering applications, aiming to rationalize use of the available energy.

The heat exchange optimization parameter is defined as the amount of dissipated heat to the heat exchanger weight for a one raw heat exchanger. The three different models are built to optimize the heat transfer process and calculate the temperature distribution in the air outlet of the fin heat exchanger. The relationship between the difference in the air temperature ΔT, and the fin profiles for various air inlet velocties is presented in Tab. 3. Fin geometry affects the heat transfer phenomenon between the plate itself and the air. It is seen that the flow streams vary and change the flow direction depending on fin profile modification that impact on the fin surface temperature. For lower air velocities, profiles (b) and (c) give similar results. When the air inlet rate increases, better results are received for profile (c). All obtained results show that the optimum fin design should be calculated and chosen assuming also the flow parameters.

To check the solution error, the grid refinements are done and additional simulations are run for fin geometry (a) and $v_{in} = 1$ m/s. It can be noticed that for certain number of grid elements (higher than 1 100 000) the variation of temperature T_{OUT} is diminutive, Fig. 12. The difference between solutions, obtained for coarser and refined grids, are shown in Tab. 5. All simulations, described in the paper, are run for models with grids between 1 200 000 and 1 500 000 elements.

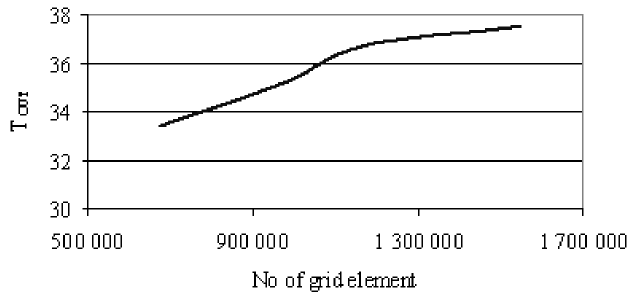

Figure 12. T_{OUT} and grid element relationship.

Table 5. ΔT for different number of grid elements.

	Air velocity $v_{IN} = 1.0$ m/s			
	Fin profile (a)			
	grid 1	grid 2	grid 3	grid 4
No of grid elements	674000	982000	1162000	1548000
T_{OUT}	33.4	35.3	36.7	37.5

The main objective of this work is to determine the performance of the heat transfer process in a given heat exchanger for different fin profiles, considering the fluid flow as a variability often neglected for the fin optimization.

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References

[1] Kraus A., Aziz A., Welty J.: *Extended Surface Heat Transfer*. Willey-Interscience Publication, 2001.

[2] Ullmann A., Kalman H.: *Efficiency and optimized dimensions of annular fins of different cross-section shapes*. Int. J. Heat Mass Transfer, **32**(1989), 6, 1105–1110.

[3] Sharqawy M.H., Zubair S.M.: *Efficiency and optimization of an annular fin with combined heat and mass transfer — An analytical solution*. Int. Journal of Refrigeration, **30**(2007), 751–757.

[4] Kundu B., Das P.K.: *Optimum dimensions of plate fins for fin-tube heat exchangers*. Int. J. Heat and Fluid Flow, **18**(1997), 530–537.

[5] Rocha L.A.O., Saboya F.E.M.: *A comparative study of elliptical and circular sections in one and two-row tubes and plate fin heat exchangers*. Int. J. Heat and Fluid Flow, **18**(1997), 247–252.

[6] Dul'kin I.N., Garas'ko G.I.: *Analytical solutions of 1-D heat conduction problem for a single fin with temperature dependent heat transfer coefficient — I. Closed-form inverse solution*. Int. Journal of Heat and Mass Transfer, **45**(2002), 1895–1903.

[7] Dul'kin I.N., Garas'ko G.I.: *Analytical solutions of 1-D heat conduction problem for a single fin with temperature dependent heat transfer coefficient — II. Recurrent direct solution*. Int. Journal of Heat and Mass Transfer, **45**(2002), 1895–1903.

[8] Mokheimer E.M.A.: *Performance of annular fins with different profiles subject to variable heat transfer coefficient*. Int. Journal of Heat and Mass Transfer, **45**(2002), 3631–3642.

[9] Arslanturk C.: *Performance analysis and optimization of a thermally non-symmetric annular fin*. Int. Comm. Heat Mass Transfer, **31**(2004), 8, 1143–1153.

[10] Arslanturk C.: *A decomposition method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity*. Int. Comm. Heat and Mass Transfer, **32**(2005), 831–841.

[11] Malekzadeh P., Rahideh H., Setoodeh A.R.: *Optimization of non-symmetric convective-radiative annular fins by differential quadrature method*. Energy Conversion and Management **48**(2007), 1671–1677.

[12] Khani F., Raji M.A., Nejad H.H.: *Analytical solutions and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient*. Communications in Nonlinear Science and Numerical Simulation, **14**(2009), 3327–3338.

[13] Aziz A., Fang T.: *Alternative solutions for longitudinal fins of rectangular, trapezoidal, and concave parabolic profiles*. Energy Conversion Management, 2010, doi:10.1016/j.enconman.2010.03.012

[14] Kundu B., Das P.K.: *Performance analysis and optimization of straight taper fins with variable heat transfer coefficient*. Int. Journal of Heat and Mass Transfer **45**(2002), 4739–4751.

[15] Laor K., Kalman H.: *Performance and optimum dimensions of different cooling fins with a temperature-dependent heat transfer coefficient*. Int. J. Heat Mass Transfer, **39**(1996), 9, 1993–2003.