

## Two-dimensional CFD modeling of the heat and mass transfer process during sewage sludge drying in a solar dryer

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**Abstract** The paper presents key assumptions of the mathematical model which describes heat and mass transfer phenomena in a solar sewage drying process, as well as techniques used for solving this model with the Fluent computational fluid dynamics (CFD) software. Special attention was paid to implementation of boundary conditions on the sludge surface, which is a physical boundary between the gaseous phase – air, and solid phase – dried matter. Those conditions allow to model heat and mass transfer between the media during first and second drying stages. Selection of the computational geometry is also discussed – it is a fragment of the entire drying facility. Selected modelling results are presented in the final part of the paper.

**Keywords:** Heat and mass transfer; CFD; Sewage sludge drying

### Nomenclature

- $a, b, c$  – constants in the equation
- $a_w$  – water activity
- $c_p$  – specific heat of the hydrated sludge, J/(kgK)
- $c_{pw}$  – specific heat of water, J/(kgK)
- $c_{s.m.o.}$  – specific heat of the dry sludge, J/(kgK)

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$D_0$	–	constant equation
$D_{H_2O}$	–	diffusion coefficient of water vapor in the air, $m^2/s$
$D_w$	–	diffusion coefficient of water in sludge, $m^2/s$
$D_{weff}$	–	effective diffusion coefficient of water in sludge, $m^2/s$
$h_{sorp}$	–	enthalpy of bound water, $J/kg$
$J$	–	diffusive mass flux density, $kg/m^2s$
$k$	–	constant in the equations
$\dot{m}_{H_2O}$	–	mass flux density of water vapor, $kg/m^2s$
$q_{evap}$	–	evaporative heat flux, $W/m^2$
$q_\alpha$	–	convective heat flux, $W/m^2$
$r$	–	heat of vaporization of water, $J/kg$
$t$	–	sludge temperature, $^\circ C$
$T$	–	temperature, $K$
$u$	–	velocity, $m/s$
$X$	–	water content in the sludge, $kg H_2O/kg$ dry matter
$Y_{H_2O}$	–	the mass fraction of water

#### Greek symbols

$\lambda_p$	–	thermal conductivity of the air, $W/m^2K$
$\lambda_o$	–	thermal conductivity of sludge, $W/m^2K$
$\rho$	–	density, $kg/m^3$
$\rho_o$	–	density of sludge, $kg/m^3$

## 1 Introduction

Solar sewage sludge drying facilities represent a greenhouse (chamber) type, without separated solar energy collector system. All existing solar sludge driers have similar design. A greenhouse structure with transparent roof (polycarbonate, glass) is installed over a paved yard. Dehydrated sewage sludge is spread over the paved floor surface. Solar radiation reaches the dried matter layer directly, passing through the transparent roof, and delivers heat required to evaporate residual humidity. Water removed from the surface of the dried sludge is removed from the facility by the flow of ventilation air. Typically a ventilation system enforces air flow over the sludge surface and exchanges the air volume inside the facility (Fig. 1).

Drying processes where surface of the dried body is subjected to a perpendicular gas flow (as it happens in most solar drying facilities) are frequently encountered in some branches of industry. Nevertheless design of such process presents great difficulties, as conditions of heat and mass transfer between the humid surface and gaseous phase change considerably with the distance from the blower axis. Parameters variability applies to both hydrodynamic conditions which govern local heat transfer coefficient values, and driving force of the process determined by the local humid surface

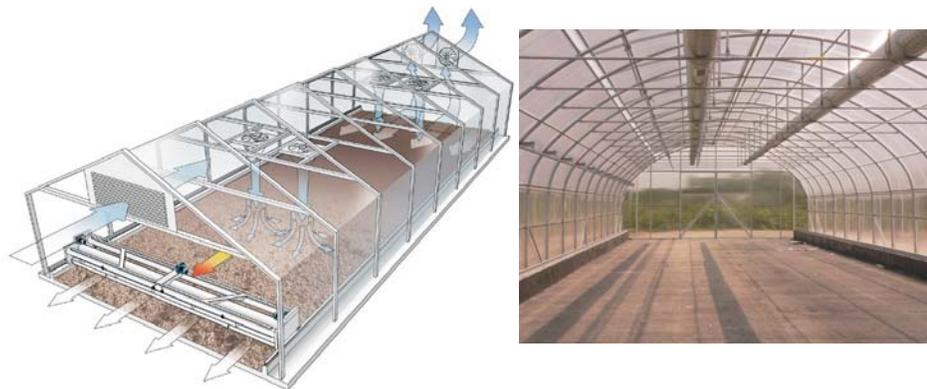


Figure 1. Solar sewage sludge dryer – examples: a) Huber AG technology [10],  
b) Research installation in Skarżysko-Kamienna.

temperature and gaseous phase properties. Due to those factors traditional process description methods where heat and mass flows across phase border are determined upon heat transfer coefficients may not be reliably used in such situations. Therefore optimisation of process parameters may only be carried out on the existing industrial system. Introduction of new numerical methods creates new opportunities to describe and model the drying processes in a way previously impossible, providing an opportunity to optimise the process parameters still at the design stage.

This paper presents key assumptions for the mathematical model describing solar sludge drying process and its numerical implementation with Fluent techniques. The primary premises for that model were described in [1–4].

## 2 Mathematical model

The proposed model includes heat and mass transport:

- within the dried matter (sludge),
- in the ambient air,
- on the border of both media.

Due to the fact that thermodynamic characteristics of the sludge as well as drying conditions (intensity of solar radiation, temperature and humidity of

ventilation air) change over time, the thermal and flow processes occurring inside a dryer should be considered unsteady.

## 2.1 Mass and heat transport in the air

To model the process of heat and mass transfer in the air around the dried matter it is necessary to solve fluid mechanics equations for turbulent flow – continuity equation, momentum equation and substance transport equation. Those equations involve physical and chemical parameters of the described fluid. In the discussed problem this fluid is humid air treated as a mixture of oxygen, nitrogen and water vapour.

## 2.2 Mass transport in the dried matter

The unsteady diffusion equation was used to calculate water diffusion rate in the dried matter. To find the diffusion coefficient of the water in the sludge according to their temperature and water content, we used equation [7]:

$$D(X, T) = D_0 X^a \exp\left(-\frac{b}{T}\right). \quad (1)$$

## 2.3 Heat transport in the dried matter

Heat transport in the dried matter is determined by the Fourier's law. Thermodynamic properties of the dried sludge at different temperatures and water content required by this approach (specific heat and heat conductivity) can be obtained thanks to the studies [8] and [9]. The specific heat is calculated with equations:

$$c_p(X) = \frac{X}{1+X} c_{pw} + \frac{1}{1+X} c_{s.m.o.}, \quad (2)$$

where

$$c_{s.m.o.} = 1434 + 3.29t. \quad (3)$$

Relation between the heat conductivity of the sludge and its water content can be described with the equation [9]:

$$\lambda(X) = 0,5148e^{-0,0051X}. \quad (4)$$

### 3 Geometry of the computational area

Due to the fact that solving computational fluid dynamics (CFD) models requires high computing power, the geometry of the modelled object should be simplified as far as possible and physically justified. It may be assumed with reasonable accuracy that the modelled drying facility consists of a number of identical elements (modules) – from the thermal hydraulics point of view. A single module is understood as an area around a single blower (fan), including neighbouring sludge surface. Bearing that in mind it has been decided to model only a part of the drying facility's volume in the neighbourhood of a single ventilation grate (single fan).

The proposed model is two-dimensional. The modelled object is assumed to be axisymmetrical, the axis protruding from the centre of the blower, perpendicular to the sludge surface (Fig. 2). This approach considerably accelerates computational process, allowing to investigate higher number of cases, while maintaining unchanged accuracy level. The assumed geometry of computational area is presented in Fig. 2, and its dimensions are given in the Tab. 1.

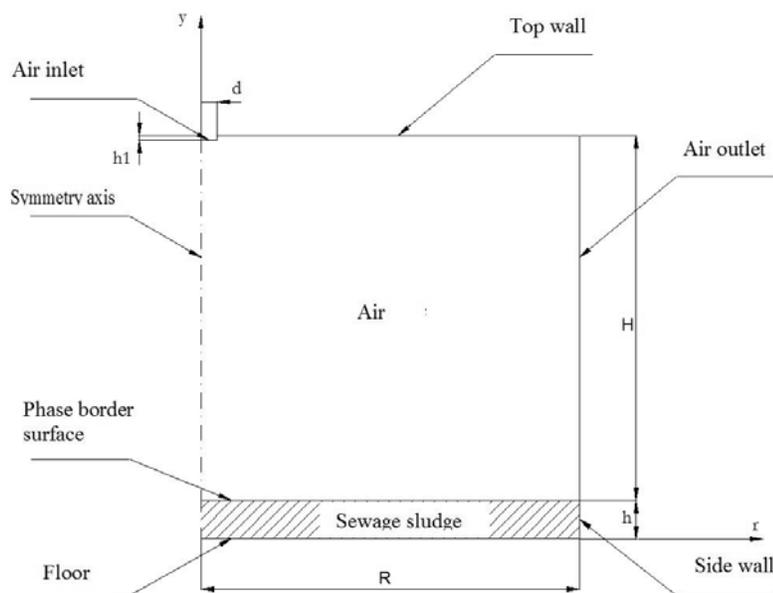


Figure 2. Geometry of the modelled area used in calculations.

Table 1. Dimensions of the computational geometry used in calculations.

Dimensions	Value [mm]
R	3000
H	2920–3120
h	100–300
d	240
h1	20

Geometrical parameters of the modelled area correspond with the research facility constructed at a sewage treatment plant in the City of Skarżysko-Kamienna. At this facility ventilation air inlets are located 3.2 m above the floor, with linear spacing 6 m [14]. Assumed cylindrical computational area has a radius of 3 m. Cross-section of the air discharge is 0.045 m<sup>2</sup> (radius 0.12 m).

The assumed computational geometry does not exactly match the real system, as an additional wall limiting the area from the top was introduced. This results from a necessity to fully define the modelled area. The author of this study has carried out supplementary calculations which analysed the influence of location (level) of this wall on drying parameters and found that it had no impact at all [1].

The assumed computational area was divided into two zones:

- air,
- sewage sludge,

separated with a “phase border surface”. Assumed boundary conditions on all external borders of the area are specified in Tab. 2. Boundary conditions at the phase border surface are not shown here – due to their specific features they are discussed in the following section of the paper.

## 4 Modelling air-sludge border

The Fluent CFD package does not provide standardised boundary condition type which would enable defining simultaneous mass and heat transfer processes occurring at a physical border between the solid (sludge) and gaseous (humid air) phases. In order to implement appropriate conditions it is required to incorporate appropriate procedures in the form of so-called user-

Table 2. Boundary conditions.

Boundary	Condition type	Value
Air inlet	Velocity inlet	8.8 m/s, temperature and humidity as per measured data
Symmetry axis	Axis	–
Air outlet	Pressure outlet	–
Floor	Wall	Temperature as per measured data
Top wall	Wall	–
Side wall	Wall	–

defined functions (UDF). A UDF is a user-created programme which may be dynamically linked to the Fluent. UDFs may be written in C or C++ programming languages. Each UDF must contain DEFINE macros and other functions defined in Fluent software, which allow to access data generated by the basic software package. UDFs are used to adapt the boundary conditions, model variable material properties, add new source components in transport equations or calculations initialisation. Such functions allow to extend models provided in the basic Fluent software package.

As modelled zones exist on both sides of the “phase border surface” boundary (air on one side, sludge on the other), Fluent software automatically creates two boundaries – one for each zone (Fig. 3). This is key information for correct specification of heat and mass transfer conditions between the zones. In order to implement the discussed conditions at the border between solid and gaseous phases, a following sequence of actions accomplished by six UDFs was assumed:

1. Mass fraction of water vapour in the air adjacent to the dried matter surface (C0 cells from the “sewage sludge” zone) is calculated from the temperature of the heated matter surface. This is achieved with the UDF 1, using the relation below [1]:

$$Y_{H_2O}(T, X) = (a_1T^6 - a_2T^5 + a_3T^4 - a_4T^3 + a_5T^2 - a_6T + a_7)a_w . \quad (5)$$

The coefficients from a1 to a7 are the coefficients of the polynomial which determines the mass fraction of water vapor in air (at saturation) as a function of temperature in the range 260–335 K.

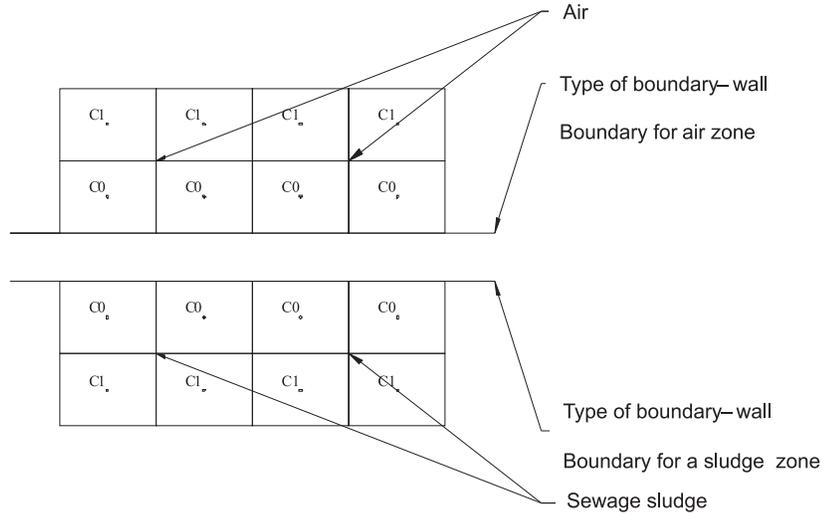


Figure 3. “Phase border surface” boundary in the modelled geometry, together with adjacent computational cells – schematic drawing.

$$\begin{aligned}
 a_1 &= 2.8256626131\text{E-}13, & a_2 &= 4.5278092896\text{E-}10, \\
 a_3 &= 3.037947077935\text{E-}7, & a_4 &= 1.090990317813\text{E-}4, \\
 a_5 &= 0.0220937883041836, & a_6 &= 2.39022255985961, \\
 a_7 &= 107,851727062049.
 \end{aligned}$$

The relation between the water activity for the sludge and the sludge temperature and humidity can be expressed as [6]:

$$a_w = \frac{e^A}{1 + e^A}, \quad (6)$$

where

$$A = \frac{\ln\left(\frac{X}{K_1 + K_2 T}\right)}{K_3 + K_4 T}. \quad (7)$$

Distribution of the water vapour mass fraction over the surface constitutes a boundary condition for the water vapour transfer equation in the “air” zone [11]:

$$\frac{\partial}{\partial t} (\rho Y_{H_2O}) + \frac{\partial}{\partial x_i} (\rho u_i Y_{H_2O}) = - \frac{\partial}{\partial x_i} J_{H_2O}, \quad i = 1, 2, 3, \quad (8)$$

where  $k_1 = 0.5424$ ;  $k_2 = -0.0015$ ;  $k_3 = -0.9232$ ;  $k_4 = -0.004$ .

2. Diffusion coefficient for the water vapour in the entire “air” zone is determined according to the air temperature at its assumed pressure (UDF 2 with Eq. (1)).
3. Then the mass flow of water which leaves the surface of the dried matter is determined (according to gradient of the water vapour mass fraction in C0 cells of the “air” zone) [12]:

$$\dot{m}_{H_2O} = -\rho D_{H_2O}(T) \left. \frac{\partial Y_{H_2O}}{\partial y} \right|_{y=0+}. \quad (9)$$

It becomes a boundary condition for the humidity distribution in the dried matter [5]:

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial x_i} \left( D_{weff}(X, T) \frac{\partial X}{\partial x_i} \right). \quad (10)$$

This task is accomplished by the UDF 3. Additionally at this step water mass flow values are recorded in user-defined memory (UDM 1) for use during later stages of the solution process.

4. UDF 4 is used to determine the heat flux reaching the dried matter surface (“wall” type boundary, belonging to the “sewage sludge” zone), which is driven by:

- Convection – calculated according to the temperature gradient in C0 cells of the “air” zone and thermal conductivity of the air [13]:

$$q_\alpha = -\lambda_p \left. \frac{\partial T}{\partial y} \right|_{y=0+}. \quad (11)$$

- Solar radiation – heat flux variable in time, assumed according to the measurement results, input through the UDF 6.
- Heat of evaporation for the mass flow specified in the UDF 3:

$$q_{evap} = \dot{m}_{H_2O} [r(T) + h_{sorp}(X, T)]. \quad (12)$$

The calculated heat flux value constitutes a boundary condition for the equation which describes temperature distribution within the sludge [13].

$$\rho_o c_o(X, T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i} \left( \lambda_o(X) \frac{\partial T}{\partial x_i} \right). \quad (13)$$

- UDF 5 assigns the sludge surface temperature (“wall” type boundary, belonging to the “sewage sludge” zone) to the adjacent air (“wall” type boundary, belonging to the “air” zone). Those temperature values constitute boundary condition for the air energy equation.

## 5 Results

In terms of the flow phenomena the investigated system is not a complex one. Air is flowing out of the blower perpendicularly to the dried matter surface. As the distance from the blower increases, the air flow diffuses and upon hitting the dried surface it spreads radially over this surface.

Distribution of air velocity values within the modelled geometry obtained by calculations is presented in the Fig. 4.

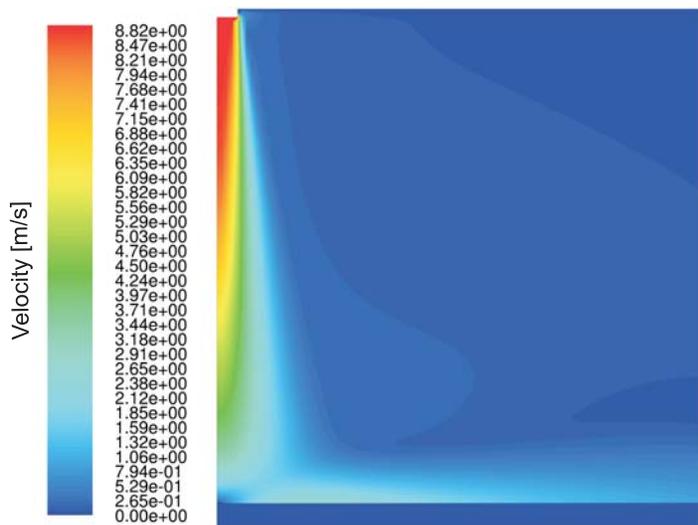


Figure 4. Air velocity distribution in the modelled geometry.

Distributions of temperature and humidity for the 13.00 hours of the modelled day (June conditions) are presented in Figs. 4 and 5, respectively. The first of those figures shows cooling of the ventilation air as it flows away from the blower axis, as well as sludge surface heating.

The other figure allows to see increase of relative humidity of air as it flows over the dried matter surface. Following charts present calculated drying rate (Fig. 7), dried sludge surface temperature (Fig. 8) and its humidity

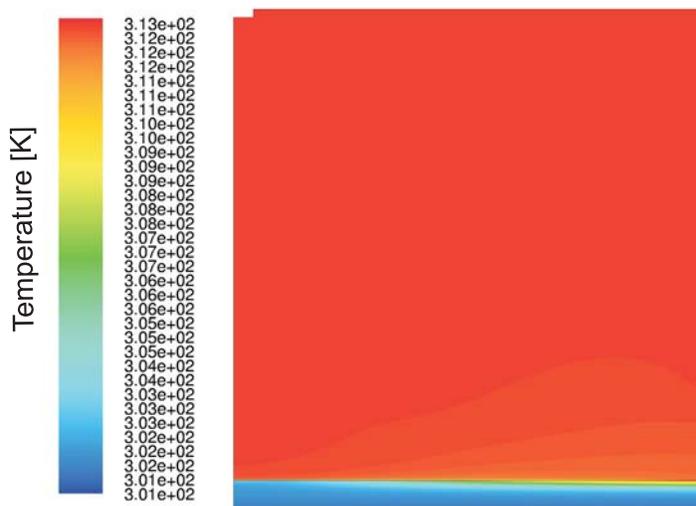


Figure 5. Distribution of air and sludge temperatures for the modelled day, 13:00 hrs.

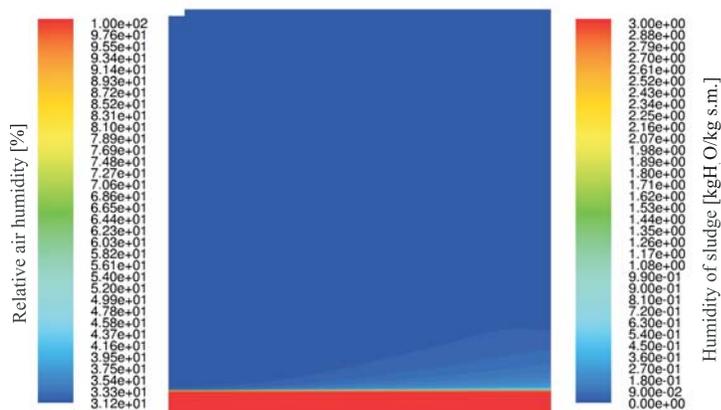


Figure 6. Distribution of humidity in the air and dried sludge for the modelled day; 13:00 hours.

(Fig. 9) as a function of distance from the blower axis for different times of the modelled day. Figure 7 shows that maximal drying rate values occur at the blower's axis (except for the overdrying periods).

Modelling results indicate that the lowest sludge surface temperature value occurs at the blower axis (Fig. 8). The sludge surface humidity is inversely proportional to the drying rate. The lowest values of this param-

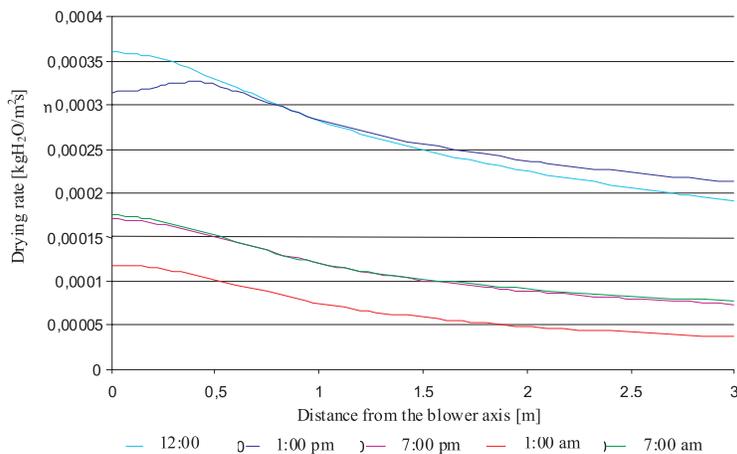


Figure 7. Sludge drying rate as a function of the distance from the blower axis for various times of the modelled day.

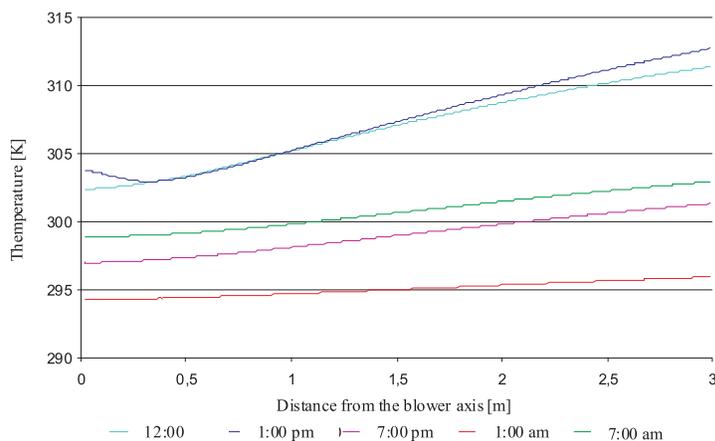


Figure 8. Sludge surface temperature as a function of the distance from the blower axis for various times of the modelled day.

eter are recorded at the blower axis for the periods when the drying rate is highest (Fig. 9).

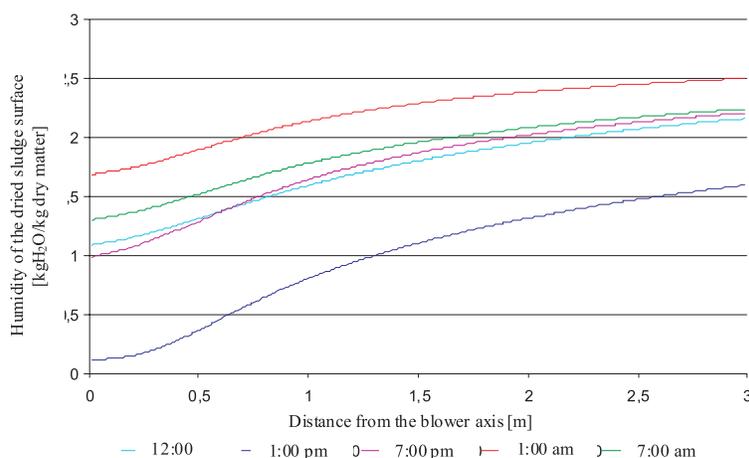


Figure 9. Humidity of the dried sludge surface as a function of distance from the blower axis for various times of the modelled day.

## 6 Summary

The unsteady-state simulation of the simultaneous heat and mass transfer process during sewage sludge drying is not straightforward. Many difficult practical problems are caused by the coupling of several transport equations, the existence of a solid phase and a fluid phase, the species and thermal balances and equilibria at the interface of these phases. Special techniques, using Fluent's UDF and UDM, have to be applied to model the simultaneous heat and mass transfer across two phases during first and second drying stages. Very fine grid near the wall must be used in order to accurately model the fluid all the way to the wall and calculate the convection fluxes.

The selected results of modeling of drying process were examined in this work. Calculated drying rate, dried sludge surface temperature and its humidity as a function of distance from the blower axis were presented. These results indicate the usefulness of the presented approach in the course of calculations of solar drying of sludge.

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