Reduction of dynamic error in measurements of transient fluid temperature

MAGDALENA JAREMKIEWICZ[*]

Cracow University of Technology, Department of Thermal Power
Engineering, al. Jana Pawła II 37, 31-864 Kraków, Poland

Abstract Under steady-state conditions when fluid temperature is constant, temperature measurement can be accomplished with high degree of accuracy owing to the absence of damping and time lag. However, when fluid temperature varies rapidly, for example, during start-up, appreciable differences occur between the actual and measured fluid temperature. These differences occur because it takes time for heat to transfer through the heavy thermometer pocket to the thermocouple. In this paper, a method for determinig transient fluid temperature based on the first-order thermometer model is presented. Fluid temperature is determined using a thermometer, which is suddenly immersed into boiling water. Next, the time constant is defined as a function of fluid velocity for four sheated thermocouples with different diameters. To demonstrate the applicability of the presented method to actual data where air velocity varies, the temperature of air is estimated based on measurements carried out by three thermocouples with different outer diameters. Lastly, the time constant is presented as a function of fluid velocity and outer diameter of thermocouple.

Keywords: Temperature measurement; Transient conditions; First-order model; Time constant; Uncertainty analysis

Nomenclature

a	–	constant, 1/s
A_t	–	outer surface area of the thermocouple, m^2
b	–	constant, (m s)$^{-1/2}$
c	–	average specific heat of the thermocouple, J/(kg K)

---

[*]E-mail address: mjaremkiewicz@pk.edu.pl

d_t	–	outer diameter of thermometer, mm
$f(t)$	–	temperature indicated by the thermometer, °C
m_t	–	thermocouple mass, kg
s	–	complex variable
Δt	–	time step, s
T_0	–	initial thermometer temperature, °C
T_f	–	fluid temperature, °C
$T(s)$	–	Laplace transform of the thermometer temperature
$T_f(s)$	–	Laplace transform of the fluid temperature
u	–	unit-step response of the thermometer
w	–	fluid velocity, m/s

Greek symbols

α_t	–	heat transfer coefficient on the outer surface of the thermocouple, $W/(m^2 K)$
τ	–	time constant of the thermometer in the first order model, s

1   Introduction

Most of the studies on temperature measurements concentrate on steady-state measurements of the fluid temperature [1–9]. Only the unit-step response of thermometers is considered to estimate the dynamic error of the temperature measurement. Little attention is paid to measurements of transient fluid temperature, despite the great practical significance of the problem [10–12]. It is very difficult to measure the transient temperature of steam or flue gases in thermal power stations. Massive housings and low heat transfer coefficients cause the measured temperature to differ significantly from the actual temperature of the fluid. Some particularly heavy thermometers have time constants of three minutes or more, thus requiring about 15 min for a single measurement. On the other hand, some thermometer designs include more than one time constant in order to describe the transient response of a temperature sensor immersed in a thermowell. Measuring the temperature of a medium in a controlled process may require having two or three time constants that characterise the transient thermometer response.

   The problem of dynamic errors in temperature measurements of a superheated steam becomes particularly important when superheated steam temperature control systems use injection coolers (spray attemperators). Due to the large inertia of the thermometer, measurement of the transient temperature of the fluid can be inaccurate, thus adversely affecting the automatic control of the superheated steam system. A similar problem is

encountered in flue gas temperature measurements, since the thermometer time constant and time delay are large.

In this paper, a method of determining the transient temperature of the flowing fluid on the basis of thermometer temperature time changes is presented. In this method, the thermometer is considered to be the first-order inertia device. A local polynomial approximation based on nine points is used for the approximation of temperature changes. This assures that the first derivative of this function, which represents how the thermometer temperature changes with time, will be very accurate. The temperature time history when the fluid temperature increases step-wise is also determined using the proposed method.

2   Mathematical models of thermometers

Typically, thermometers are modelled as elements with lumped thermal capacity. Such a model assumes that temperature of a thermometer is only a function of time and neglects temperature differences that occur within the thermometer itself. Temperature changes of the thermometer in time $T(t)$ can be described by an ordinary first-order differential equation (i.e. first-order thermometer model)

$$\tau \frac{dT(t)}{dt} + T(t) = T_f(t) \, , \tag{1}$$

where $\tau = m_t c/(\alpha_t A_t)$ and the initial condition is:

$$T(0) = T_0 = 0 \, . \tag{2}$$

For structurally complex thermometers that measure the temperature of a fluid under high pressure, the accuracy of the first order model (1) is inadequate [13].

The initial problem (1)–(2) was solved using the Laplace transformation. The operator transmittance $G(s)$ then assumes the following form:

$$G(s) = \frac{T(s)}{T_f(s)} = \frac{1}{\tau s + 1} \, . \tag{3}$$

For the step increase of the fluid temperature from $T_0 = 0$ °C to the constant value T_f, the Laplace transform of the fluid temperature assumes the form $T_f(s) = T_f/s$ and the transmittance formula can be simplified to:

$$\frac{T(s)}{T_f} = \frac{1}{s(\tau s + 1)} \, . \tag{4}$$

After writing Eq. (4) in the form:

$$\frac{T(s)}{T_f} = \frac{1}{s} - \frac{1}{\left(s + \frac{1}{\tau}\right)} \tag{5}$$

it is easy to find the inverse Laplace transformation and determine the thermometer temperature as a function of time:

$$u(t) = \frac{T(t) - T_0}{T_f - T_0} = 1 - \exp\left(-\frac{t}{\tau}\right) . \tag{6}$$

In this study, the time constant τ in Eq. (6) will be estimated from experimental data. The fluid temperature can then be determined on the basis of measurement histories from the thermometer temperature $T(t)$ and known time constant τ. $T(t)$ and its the first-order time derivative can be smoothed using the formulas [3]:

$$\begin{aligned}
T(t) \approx \frac{1}{693}\big[&-63f(t - 4\Delta t) + 42f(t - 3\Delta t) + 117f(t - 2\Delta t) + \\
&+162f(t - \Delta t) + 177f(t) + 162f(t + \Delta t) + \\
&+117f(t + 2\Delta t) + 42f(t + 3\Delta t) - 63f(t + 4\Delta t)\big],
\end{aligned} \tag{7}$$

$$\begin{aligned}
T'(t) = \frac{dT(t)}{dt} \approx \frac{1}{1188\Delta t}\big[&86f(t - 4\Delta t) - 142f(t - 3\Delta t) - \\
&-193f(t - 2\Delta t) - 126f(t - \Delta t) + \\
&+126f(t + \Delta t) + 193f(t + 2\Delta t) + \\
&+142f(t + 3\Delta t) - 86f(t + 4\Delta t)\big],
\end{aligned} \tag{8}$$

where $f(t)$ denotes the temperature indicated by the thermometer, and Δt is a time step. This smoothing eliminates, at least in part, the influence of random errors in the thermometer temperature measurements $T(t)$ on the determined fluid temperature $T_f(t)$. If temperature measurement histories are not overly noisy, the first-order derivative can be approximated by the central difference formula

$$T'(t) \approx \frac{f(t + \Delta t) - f(t - \Delta t)}{2\Delta t} . \tag{9}$$

Equation (1) can also be used for determining fluid temperature $T_f(t)$ when the time constant of the thermocouple τ is a function of fluid velocity w. After substituting the time constant $\tau(w)$ into Eq. (1), we can determine fluid temperature $T_f(t)$ for different fluid velocities using the proposed method.

3 Experimental determination of time constants

The method of least squares is used to determine the time constant τ in Eq. (6). The value for the time constant is found by minimising the function S:

$$S = \sum_{i=1}^{N} [u_m(t_i) - u(t_i)]^2 = \min , \qquad (10)$$

where $u(t_i)$ is the approximating function given by Eq. (6), and N denotes the number of measurements $(t_i, u_m(t_i))$. Here, the sum of the squares of the deviations of the measured values $u_m(t_i)$ from the fitted values $u(t_i)$ is minimised. Once the time constant τ has been determined, it can be substituted into Eq. (10) to find the value for S_{min}.

The uncertainties in the calculated time constant τ are estimated using the mean square error [14–16]:

$$S_N = \sqrt{\frac{S_{\min}}{N - m}} , \qquad (11)$$

where m is the number of time constants (i.e. $m = 1$ for Eq. (6)). Based on the calculated mean square error S_N, which is an approximation of the standard deviation, the uncertainties in the determined time constants can be calculated using the formulas given in the TableCurve 2D software [16].

4 Determining the fluid temperature on the basis of time changes in the thermometer temperature

A sheathed thermocouple with outer diameter 1.5 mm at the ambient temperature was suddenly immersed into hot water at saturation temperature; the results are presented in Fig. 1. The thermometer temperature data was collected using the Hottinger-Baldwin Messtechnik data acquisition system. The measured temperature changes were approximated using Eq. (6), and the time constant τ in Eq. (6) was determined using the TableCurve 2D code [16]. The estimated value of the time constant and the uncertainty at the 95% confidence level is $\tau = 1.54 \pm 0.09$ s.

First, the transient fluid temperature $T_f(t)$ was calculated using Eq. (1) together with Eqs. (7) and (8). Then, the raw temperature data was used. The first-order time derivative dT/dt in Eq. (1) was also calculated using the central difference quotient of Eq. (9). The results shown in Fig. 1 indicate

that the central difference approximation of the time derivative in Eq. (1)
leads to less accurate results, since it is more sensitive to random errors in
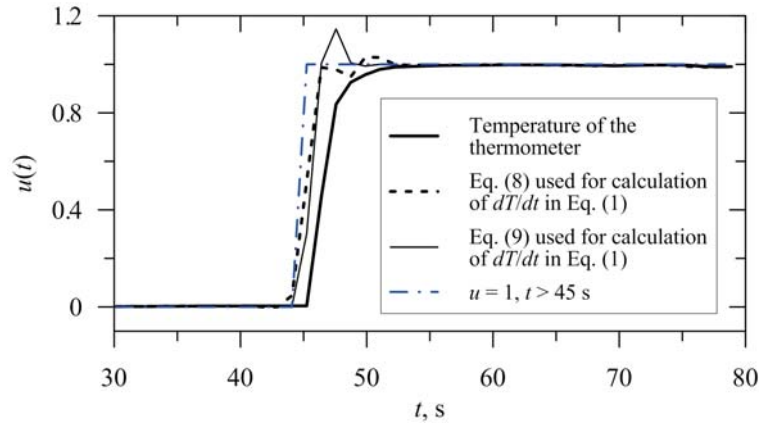the experimental data.

Figure 1. Fluid and thermometer temperature changes determined from the first-order
Eq. (1) for the sheathed thermocouple with outer diameter 1.5 mm.

5   Time constant as a function of fluid velocity and outer diameter of thermocouple

The thermocouple time constant τ for various air velocities w were deter-
mined in an open benchtop wind tunnel (Fig. 2). The WT4401-S benchtop
wind tunnel is designed to give uniform flow rate over a 100 mm × 100 mm
cross section [17]. The variations of the thermocouple time constants τ with
the fluid velocity for the sheathed thermocouples with the outer diameter of
0.5 mm, 1.0 mm, 1.5 mm and 3.0 mm are shown in Fig. 3. The experimen-
tal data collected for the four thermocouple diameters of 0.5 mm, 1.0 mm,
1.5 mm, and 3.0 mm, as presented in Fig. 3, were approximated by the least
squares method. The following function was thus obtained:

$$\tau = \frac{1}{a + b\sqrt{w}} \; , \tag{12}$$

where τ is expressed in s, and w in m/s.

The best estimates for the constants a and b, with uncertainty at the
95% confidence level, for the thermocouples with the following outer diam-
eters are:

Figure 2. Benchtop wind tunnel used for determining the thermocouple time constant.

- $d_t = 0.5$ mm
  $a = 0.004337 \pm 0.000622$ 1/s, $b = 0.022239 \pm 0.001103$ (m s)$^{-1/2}$;

- $d_t = 1.0$ mm
  $a = 0.020974 \pm 0.006372$ 1/s, $b = 0.103870 \pm 0.011240$ (m s)$^{-1/2}$;

- $d_t = 1.5$ mm
  $a = 0.040425 \pm 0.003301$ 1/s, $b = 0.056850 \pm 0.004479$ (m s)$^{-1/2}$;

- $d_t = 3.0$ mm
  $a = 0.128220 \pm 0.035716$ 1/s, $b = 0.220641 \pm 0.051122$ (m s)$^{-1/2}$.

The time constant of the thermocouple $\tau = m_t c / (\alpha_t A_t)$ depends strongly on the heat transfer coefficient α_t on the outer thermometer surface, which in turn is a function of the air velocity [18]. When the velocity and temperature of air stream change in time, the velocity-dependent time constant in Eq. (12) can be used in Eq. (1) to estimate the air temperature based on the temperature readings from the sheathed thermocouples.

    To prove the utility of the above method in determining transient temperature, the temperature of the fluid in the open wind tunnel (Fig. 4) was measured by K-type sheathed thermocouples with outer diameters of
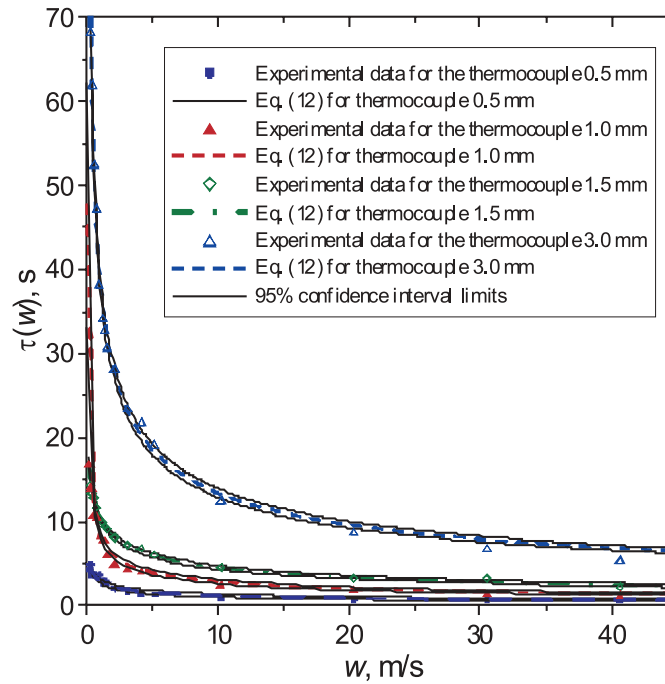
Figure 3. Time constants τ of sheathed thermocouples with outer diameters of 0.5 mm, 1.0 mm, 1.5 mm, and 3.0 mm as a function of air velocity w with 95% confidence interval limits.

0.5 mm, 1.0 mm, and 1.5 mm. Air in the tunnel was heated by a heat exchanger and its velocity was also altered. The thermocouples measured the temperature behind the heat exchanger and were placed very close together (i.e. they measured the same temperature, but did not influence each other). Variation in the air velocity was measured by the vane anemometer FV A915 S220. Both the velocity and temperature data were collected using the Ahlborn ALMEMO 5990-0 data acquisition system.

The comparison of the computed temperatures with the measured temperatures, when the time constants of the thermocouples are known, shows that the above method provides decent results (Fig. 5). The results are very accurate, especially for the thermocouples with outer diameters of 0.5 mm and 1.0 mm. In the case of the thermocouple with outer diameter 1.5 mm, there is a small difference between its computed temperature and the other computed temperatures. This difference is due to a large value of the time constant τ for the 1.5 mm thermocouple.
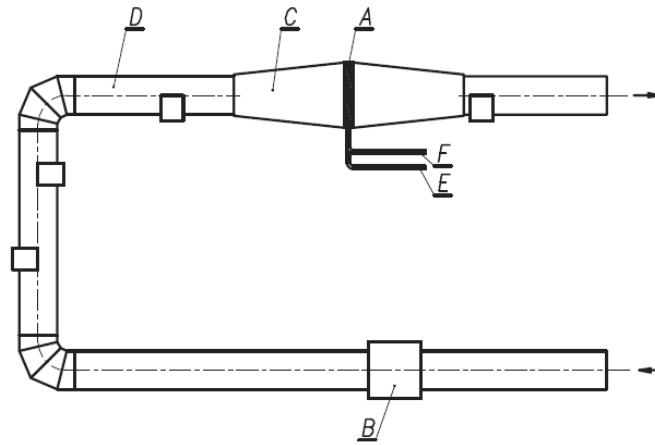
Figure 4. Diagram of open wind tunnel [19]: A – heat exchanger, B – fan, C – chamber, D – air channel, E – water outlet pipe, F – hot water feeding pipe.

Based on the computed time constants determined for various velocities for thermocouples with outer diameters of 0.5 mm, 1.0 mm, 1.5 mm, and 3.0 mm, one universal function can be obtained. Data were approximated using the method of least squares by the following function:

$$\tau = \frac{a - bd_t - c\ln w + d\left(\ln w\right)^2}{1 - ed_t + fd_t^2 - gd_t^3 + h\ln w} \; , \tag{13}$$

where τ is expressed in s, d_t in mm and w in m/s. The best estimates for the constants a, b, c, d, e, f, g and h, with uncertainty at the 95% confidence level are: $a = 0.997 \pm 0.729$, $b = $ -0.00797 $\pm$ 0.84119, $c = $ -0.316 $\pm$ 0.337, $d = 0.0269 \pm 0.1092$, $e = $ -1.76 $\pm$ 0.37, $f= 1.07 \pm 0.34$, $g = $ -0.196 $\pm$ 0.076, $h = 0.00192 \pm 0.02757$.

Equation (13) was determined using the TableCurve 3D software [20], with a time constant coefficient of determination r^2 of 0.997. The function $\tau = f(d_t, w)$ and its related experimental data are presented in Fig. 6.

6 Conclusions

The method presented in this paper for measuring the transient temperature of a fluid can be used for the on-line monitoring of fluid temperature change with time. This method, where the thermometer is modelled
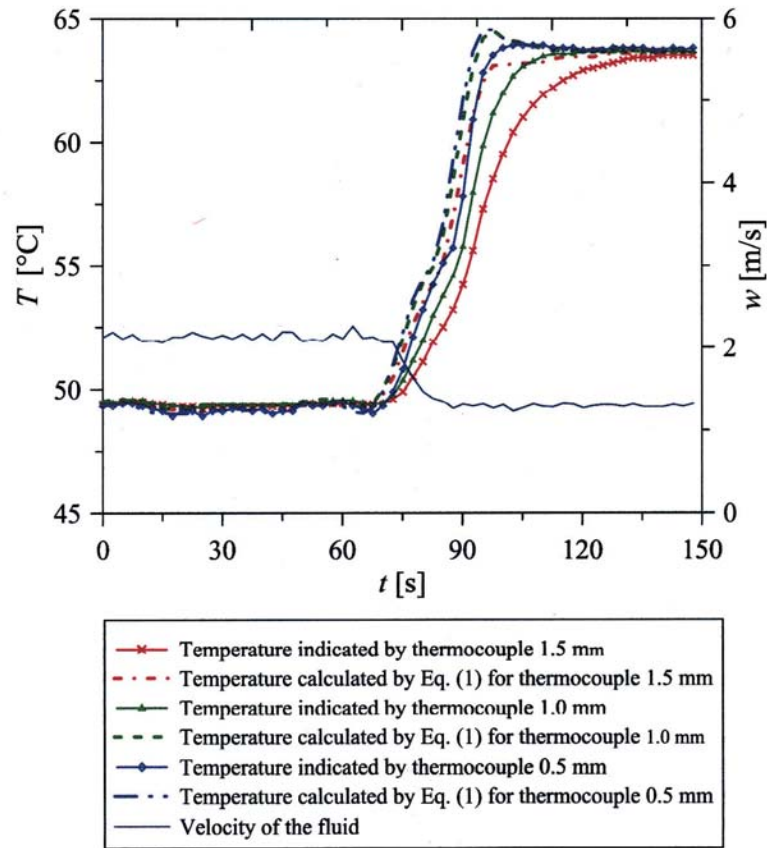
Figure 5. Temperature of the air measured by the thermocouples with outer diameters of 0.5 mm, 1.0 mm, and 1.5 mm and temperature calculated by Eq. (1) when the velocity of the air was changed.

using the ordinary first-order differential equation, is appropriate for thermometers that have small time constants. In such cases, the delay of the thermometer is small in comparison to the changes of the temperature of the fluid. When the delay of the thermometer is big, considering the thermometer as a second-order inertia device is more appropriate. Substantial stability and accuracy of the computed actual fluid temperature from the measured thermometer temperature can be achieved by using a 9-point digital filter. The technique proposed in this paper can also be used when the
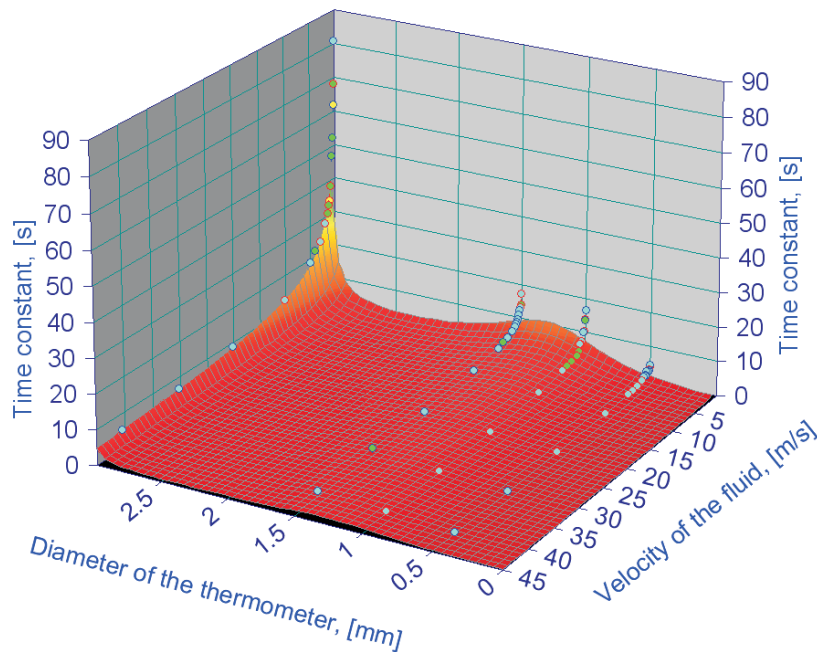
Figure 6. Time constant τ as a function of the fluid velocity w and the outer diameter
of the thermocouple d_t.

thermometer time constant is a function of fluid velocity. A comparison
of the computed temperatures of air, whose velocity was varied, based on
measurements by three thermocouples with different outer diameters, gave
confidence to the accuracy of the presented method.

Received 10 October 2011

References

[1] NICHOLAS J.V., WHITE D.R.: *Traceable Temperatures. An Introduction to Tem-
   perature Measurement and Calibration*, 2nd edn. Wiley, New York 2001, 140–145.

[2] MICHALSKI L., ECKERSDORF K., McGHEE J.: *Temperature Measurement*, Wiley,
   Chichester 1991.

[3] WIŚNIEWSKI S.: *Temperature Measurement in Engines and Thermal Facilities*,
   WNT, Warszawa 1983 (in Polish).

[4] TALER J.: *Theory and Practice of Identification of Heat Transfer Processes*. Zakład
   Narodowy imienia Ossolińskich, Wrocław 1995 (in Polish).

[5] KABZA Z., KOSTYRKO K., ZATOR S., ŁOBZOWSKI A., SZKOLNIKOWSKI W.: *Room Climate Control*, Agenda Wydawnicza, Pomiary Automatyka Kontrola, Warszawa 2005 (in Polish).

[6] LITTLER D. J., *et al.*: *Instrumentation, Controls & Testing. Modern Power Station Practice*. Pergamon Press, Oxford 1991.

[7] CHILDS P.R.N.: *Practical Temperature Measurement*, Buterworth-Heinemann, Oxford 2001.

[8] GERASHCHENKO O.A., GORDOV A.N., LAKH V.I., STADNYK B.I., YARYSHEV N.A.: *Temperature Measurements*. Naukova Dumka, Kiev 1984 (in Russian).

[9] JE-CHIN HAN, SANDIP DUTTA, SRINATH V.EKKAD: *Gas Turbine Heat Transfer and Cooling Technology*, Chapter 6, Experimental Methods, Taylor&Francis, New York, London 2000, 531–584.

[10] SZÉKELY V., RESS S., POPPE A., TÖRÖK S., MAGYARI D., BENEDEK ZS., TORKI K., COURTOIS B., RENCZ M.: *New approaches in the transient thermal measurements*. Microelectronics J. **31**(2000), 727–733.

[11] CROCKER D.S., PARANG M.: *Unsteady temperature measurement in an enclosed thermoconvectively heated air*. Int. Comm. Heat Mass Transfer **28**(2001), 1015–1024.

[12] PAO C. CHAU: *Process control, A First Course with MATLAB*. Cambridge University Press, Cambridge 2002.

[13] JAREMKIEWICZ M., TALER D., SOBOTA T.: *Measurement of transient fluid temperature in installations of power plants*. Pomiary Automatyka Kontrola **55**(2009), 288–291.

[14] ASME: *Policy on reporting uncertainties in experimental measurements and results*. J. Heat Transfer **122**(2000), 411–413.

[15] MOFFAT R.J.: *Describing the uncertainties in experimental results*. Experimental Thermal and Fluid Science **1**(1988), 3–17.

[16] *TableCurve 2D v.5.0, Automated Curve Fitting & Equation Discovery*. AISN Software Inc., 2000.

[17] *WT4401-S & WT4401-D Benchtop Wind Tunnels*. Omega, Stamford, CT, USA, www.omega.com

[18] SANITJAI S., GOLDSTEIN R.J.: *Forced convection heat transfer from a circular cylinder in crossflow to air and liquids*. Int. J. of Heat and Mass Transfer **47**(2004), 4795–4805.

[19] TALER D.: *The Dynamics of Tubular Heat Exchangers*. Uczelniane Wydawnictwa Naukowo-Dydaktyczne AGH, Cracow 2009 (in Polish).

[20] *TableCurve 3D v.4.0*. SYSTAT Software Inc., 1993–2002.