Evaluation method of single blow experiment for the determination of heat transfer coefficient and dispersive Peclet number[*]

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Abstract   An evaluation method is developed for single blow experiments with liquids on heat exchangers. The method is based on the unity Mach number dispersion model. The evaluation of one experiment yields merely one equation for the two unknowns, the number of transfer units and the dispersive Peclet number. Calculations on an example confirm that one single blow test alone cannot provide reliable values of the unknowns. A second test with a liquid of differing heat capacity is required, or a tracer experiment for the measurement of the Peclet number. A modified method is developed for gases. One experiment yields the effective number of transfer units and approximate values of the two unknowns. The numerical evaluation of calculated experiments demonstrates the applicability of the evaluation methods.

Keywords: Heat exchanger; Single blow experiment; Evaluation method; Dispersion model

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Nomenclature

A –   area, m^2
a –   exponent
B –   capacity ratio, $B = V\rho c_p / V_w \rho_w c_w = C/C_w$
C –   capacity, J/K
C –   propagation velocity of thermal disturbances, m/s
c –   specific heat capacity, J/(kg K)
F –   transfer function
i –   counter
k –   counter
L –   length of flow path, m
M   –   dispersive thermal Mach number, M $= w/C$
N –   number of transfer units, $N = \alpha A \big/ \dot{W}$
N_d –   effective number of transfer units according to Eq. (1)
$N_{d,33}$ –   effective number of transfer units according to Eq. (33)
$N_{d,34}$ –   effective number of transfer units according to Eq. (34)
$N_{d,36}$ –   effective number of transfer units according to Eq. (36)
$N_{d,40}$ –   effective number of transfer units according to Eq.(40)
Nu   –   Nusselt number
m –   number of heat transferring surfaces
n –   number of completely mixed zones in the cascade model or
                   number of subsystems in Eqs. (25)–(28)
Pe   –   dispersive Peclet number, Pe $= \frac{wL\rho c_p}{\lambda_d} = \frac{\dot{W}L}{A_c \lambda_d}$
$\dot{q}_x$ –   axial dispersive heat flux, W/m^2
Re   –   Reynolds number
s –   Laplace variable
T –   dimensionless fluid temperature outside the heat exchanger,
                   $T = \frac{\theta - \vartheta_0}{\Delta \vartheta^*}$
t –   dimensionless fluid temperature inside the heat exchanger, $t = \frac{\vartheta - \vartheta_0}{\Delta \vartheta^*}$
V –   volume of fluid inside the flow channel, m^3
$\dot{V}$ –   volumetric flow rate, m^3/s
v_i –   volume ratio of subsystem i, $v_i = V_i \big/ \sum_{i=1}^{n} V_i$
$\dot{v}_i$ –   flow rate ratio of subsystem i, $\dot{v}_i = \dot{V}_i \big/ \sum_{i=1}^{n} \dot{V}_i$
$\dot{W}$ –   heat capacity rate, W/K
w –   mean flow velocity, m/s
x –   dimensionless flow length, $0 \leq x \leq 1$
z –   dimensionless time coordinate, $z = \tau/\tau_R$

Greek symbols

α –   heat transfer coefficient, W/(m^2 K)
α_d –   effective heat transfer coefficient according to Eq. (1), W/(m^2 K)

η	–	dimensionless dispersive heat flux, $\eta = \frac{\dot{q}_x L}{\lambda_d \Delta \vartheta^*}$
θ	–	fluid temperature outside the heat exchanger, K
ϑ	–	fluid temperature inside the heat exchanger, K
$\Delta\vartheta^*$	–	arbitrary temperature difference, K
λ	–	thermal conductivity, W/(m K)
ρ	–	density, m^3/kg
τ	–	time coordinate, s
τ_R	–	residence time, $\tau_R = V \big/ \dot{V}$, s

Subscripts

c	–	cross section
d	–	dispersive
p	–	isobaric
p	–	parabolic
w	–	wall
0	–	inlet
1	–	outlet

Superscripts

$^-$	–	Laplace transform

1 Introduction

The single blow experiment is widely used for the determination of heat transfer coefficients in thermal regenerators and recuperators [1–3]. Various evaluation methods are known and applied depending on the heat exchanger model used and the parameters considered. In the following an evaluation method is developed and discussed which considers the unity Mach number axial dispersion model which has recently [4] been suggested for the improvement of the usual simple thermal design calculation methods. In steady state processes in heat exchangers the model requires merely a simple correction of the mean temperature difference or preferably a correction of the mean heat transfer coefficient according to

$$\frac{1}{\alpha_d A} = \frac{1}{\alpha A} + \frac{1}{\dot{W} \mathrm{Pe}}; \qquad \frac{1}{N_d} = \frac{1}{N} + \frac{1}{\mathrm{Pe}}. \tag{1}$$

The degree of dispersion is expressed with the dispersive Peclet number. $\mathrm{Pe} = \infty$ means ideal plug flow. All known correlations and design charts [5] can further be applied if the corrected heat transfer coefficients are used. On the contrary, the transient single blow process can generally not be described with the above corrected heat transfer coefficient alone. This is shown in the following derivations.

2   Analysis of single blow experiment with axial dispersion

According to the hyperbolic dispersion model [4] the energy equation of the fluid can be expressed in dimensionless form as

$$\frac{\partial t}{\partial z} + \frac{\partial t}{\partial x} + \frac{1}{\text{Pe}}\frac{\partial \eta}{\partial x} + \sum_{i=1}^{m} N_i \left(t - t_{wi}\right) = 0 \; , \tag{2}$$

$$\eta + \frac{\text{M}^2}{\text{Pe}}\left(\frac{\partial \eta}{\partial z} + \frac{\partial \eta}{\partial x}\right) = -\frac{\partial t}{\partial x} \; . \tag{3}$$

An axial dispersion term is incorporated in Eq. (2), which takes the deviations from ideal plug flow into account. Equation (3) describes the dispersive heat flux, η, according to the heat conduction law of Chester [4, ref. 12] which considers finite propagation velocities C of thermal disturbances: $0 \le C^2 = (w/\text{M})^2 \le \infty$. For infinite velocity $C = \infty$ the dispersive Mach number M $= 0$, and Eq. (3) turns to the Fourier type conduction law, which is the basis of the former parabolic dispersion model [2,3].

In this paper the new unity Mach number dispersion model [4] is applied, in which the propagation velocity equals the mean flow velocity and $\text{M}^2 = 1$. As discussed earlier [4], the mean value M $= 1$ is more appropriate for simultaneous backmixing and maldistribution than M $= 0$.

The energy equation of the wall i out of m adjacent walls is

$$\frac{\partial t_{wi}}{\partial z} = N_i B_i \left(t - t_{wi}\right) \; . \tag{4}$$

The number of transfer units, N_i, is formed with the individual heat transfer coefficient, α_i, and heat transfer surface, A_i. The heat transfer coefficient may vary with x, $\alpha_i = \alpha_i(x)$. The same applies to the heat capacity ratio, $B_i = B_i(x)$. In a heat exchanger the rear surface of the separating wall is considered to be adiabatic (evacuated flow channel).

The walls are thermally thin which mean zero heat conduction resistance perpendicular to the heat transfer surface and zero conductance parallel to the surface. For simplification axial wall heat conduction is neglected in this analysis, however, it is indirectly taken into account with the dispersive Peclet number of the fluid. Actual axial wall heat conduction during operation of the exchanger under steady state conditions as well as during the transient single blow experiment can be considered by an effective

slightly decreased dispersive Peclet number (higher apparent dispersion coefficient). The single blow experiment will automatically yield the effective Peclet number for design calculations.

The dimensionless fluid temperature inside the flow channel ($0 \leq x \leq 1$) is denoted with t, the fluid temperatures in the nondispersive fore and aft sections are denoted with T. The energy balances at the inlet and outlet cross-sections require sudden local changes in temperature [4] according to

$$x = 0 \quad \text{and} \quad x = 1 : \qquad T = t + \frac{\eta}{\text{Pe}} \; . \tag{5}$$

This equation is only valid for M = 1. Before the experiment all dimensionless temperatures have to be zero.

$$\tau \leq 0, \; z \leq 0 : \qquad T = t = t_w = 0 \; . \tag{6}$$

During the experiment an arbitrarily shaped temperature pulse is generated at the inlet

$$\tau > 0, \; z > 0 : \; T(x = 0) = T_0 = f(z) \; . \tag{7}$$

The solution of the equation system Eqs. (2)–(5) are obtained using the Laplace transformation

$$s\bar{t} + \frac{\mathrm{d}\bar{t}}{\mathrm{d}x} + \frac{1}{\text{Pe}} \frac{\mathrm{d}\bar{\eta}}{\mathrm{d}x} + \sum_{i=1}^{m} N_i \left(\bar{t} - \bar{t}_{wi} \right) = 0 \; , \tag{8}$$

$$\bar{\eta} \left(1 + \frac{s}{Pe} \right) + \frac{1}{\text{Pe}} \frac{\mathrm{d}\bar{\eta}}{\mathrm{d}x} = -\frac{\mathrm{d}\bar{t}}{\mathrm{d}x} \; , \tag{9}$$

$$s\bar{t}_{wi} = N_i B_i \left(\bar{t} - \bar{t}_{wi} \right) \; , \tag{10}$$

$$\bar{T} = \bar{t} + \frac{\bar{\eta}}{\text{Pe}} \; . \tag{11}$$

For the consideration of the sudden changes at inlet and outlet, the temperature T according to Eq. (5) is introduced as a variable. This hypothetic temperature within the dispersive region turns to the true fluid temperature in the fore and aft sections ($x \leq 0$, $x > 1$). Substitution in Eq. (8) $\bar{t}$ according to Eq. (11) and $\bar{t}_{wi}$ according to Eq. (10) yields

$$\left(\bar{T} - \frac{\bar{\eta}}{\text{Pe}} \right) \left(s + \sum_{i=1}^{m} \frac{1}{\frac{1}{N_i} + \frac{B_i}{s}} \right) = -\frac{\mathrm{d}\bar{T}}{\mathrm{d}x} \; . \tag{12}$$

Replacing $\bar{t}$ in Eq. (9) using Eq. (11) gives

$$\frac{\bar{\eta}}{\mathrm{Pe}} = -\frac{1}{\mathrm{Pe}+s}\frac{\mathrm{d}\bar{T}}{\mathrm{d}x} \ . \tag{13}$$

Eliminating $\bar{\eta}$ by substituting Eq. (13) into Eq. (12), rearranging, separating of variables and integration leads finally to

$$a\left(s\right) = \ln\frac{\bar{T}_0}{\bar{T}_1} = \int_{x=0}^{x=1} \frac{\mathrm{d}x}{\frac{1}{s+\sum_{i=1}^{m}\frac{1}{\frac{1}{N_i}+\frac{B_i}{s}}} + \frac{1}{\mathrm{Pe}+s}} \tag{14}$$

with the transfer function $F(s)$

$$e^{-a(s)} = F\left(s\right) = \frac{\bar{T}_1\left(s\right)}{\bar{T}_0\left(s\right)} = \frac{\int_0^\infty T_1 e^{-sz}\mathrm{d}z}{\int_0^\infty T_0 e^{-sz}\mathrm{d}z} \ . \tag{15}$$

In Eq. (14) the Peclet number has to be independent of x since Eq. (11) has been differentiated with $\mathrm{Pe} = const$. The number of transfer units, N_i, and the heat capacity ratio, B_i, may vary with x. The most general Eq. (14) could be used for the calculations of the outlet profiles if $B_i(x), N_i(x)$ and Pe are given or guessed. Comparing measured and calculated outlet profiles $T_1(z)$ could yield the unknown Peclet number and numbers of transfer units N_i. In this paper another approach is proposed in which the measured data are evaluated in the frequency domain.

3   Evaluation method

The principle of the method is explained and tested for the simplest case of one wall ($m = 1$) and constant values of B, N and Pe. Constant B and N presumes thermally thin walls in which heat conduction resistances perpendicular to the surface can be neglected. Then Eq. (14) simplifies to

$$\frac{1}{a\left(s\right)} = \frac{1}{s + \frac{1}{\frac{1}{N}+\frac{B}{s}}} + \frac{1}{\mathrm{Pe}+s} \ . \tag{16}$$

As also previously proposed for the evaluation of tracer experiments [6], the characteristics of the function $a(s) = -\ln F(s)$ at the mean point $s = 0$ are used for the evaluation of measured temperature profiles. For $B > 0$ and $s = 0$ the transfer function $F(s = 0) = 1$ and $a(s = 0) = a_0 = 0$.

This result represents the heat balance: Heat input (area under inlet pulse) equals heat output (area under outlet pulse).

The main information for the determination of N and Pe are obtained from the derivatives of $a(s)$. Differentiating $a(s)$ of Eq. (16) yields

$$a_0' = \left. \frac{\partial a}{\partial s} \right|_{s=0} = 1 + \frac{1}{B} \, , \tag{17}$$

$$a_0'' = \left. \frac{\partial^2 a}{\partial s^2} \right|_{s=0} = -\frac{2}{N} \frac{1}{B^2} - \frac{2}{\text{Pe}} \left(1 + \frac{1}{B} \right)^2 \, . \tag{18}$$

The derivatives of $a(s)$ from Eq. (14) are given in the appendix, Eqs. (A1) and (A2). The limiting case $N = 0$ (no heat transfer) and/or $B = \infty$ (no wall heat capacity) describes the tracer experiment [6] if T is the tracer concentration. The Eq. (16) turns to $1/a = 1/s + 1/(\text{Pe} + s)$, Eq. (17) to $a_0' = 1$ and Eq. (18) $a_0'' = -2/\text{Pe}$. The Eq. (17) and (18) are also valid for the parabolic dispersion model ($M = 0$, Pe_p) and the cascade model (n mixed zones) if Pe is replaced according to

$$\text{Pe} = 2n = \frac{\text{Pe}_p^2}{\text{Pe}_p - 1 + \exp\left(-\text{Pe}_p\right)} \, . \tag{19}$$

The transfer functions of the two models are given in the appendix, Eqs. (A3) and (A4). Under steady state conditions in heat exchangers the relationship Eq. (19) is valid as well, however, exactly only for uniform heat flux along the heat transfer surface.

3.1   Pulse signals

Solving Eq. (15) for $a(s)$ and differentiating twice with respect to s, yields for $B > 0$ finally the experimental derivatives:

$$a_0' = \frac{R_1}{Q_1} - \frac{R_0}{Q_0} \, , \tag{20}$$

$$a_0'' = \frac{S_0}{Q_0} - \frac{S_1}{Q_1} + \left(\frac{R_1}{Q_1} \right)^2 - \left(\frac{R_0}{Q_0} \right)^2 \, , \tag{21}$$

where

$$Q = \int_0^\infty T \mathrm{d}z = \frac{\dot{V}}{V} \int_0^\infty T \mathrm{d}\tau \, , \tag{22a}$$

$$R = \int_0^\infty Tz\mathrm{d}z = \left(\frac{\dot{V}}{V}\right)^2 \int_0^\infty T\tau\,\mathrm{d}\tau\,, \qquad (22\mathrm{b})$$

$$S = \int_0^\infty Tz^2\mathrm{d}z = \left(\frac{\dot{V}}{V}\right)^3 \int_0^\infty T\tau^2\mathrm{d}\tau\,. \qquad (22\mathrm{c})$$

With Eqs. (22), (20) and (17) either the mean residence time, τ_R, or the capacity ratio, B, can be determined, if B or τ_R, respectively, are known from the experimental data. Substituting Eq. (21) into Eq. (18) yields the linear relationship between the unknowns $1/N$ and $1/\mathrm{Pe}$. In most cases B can easily be determined from the design data of the exchanger and the fluid properties. Then the following equation is recommended for evaluation

$$-\frac{a_0''}{2\,(a_0')^2} = \frac{1}{\mathrm{Pe}} + \frac{1}{N}\frac{1}{(1+B)^2} = \psi\,. \qquad (23)$$

The advantage of Eq. (23) is that in the left-hand ratio the dimensionless time z in Q, R, S can be replaced by the real time τ or any other time scale. Equation (23) reveals that one single blow experiment yields obviously only one equation for the two unknowns N and Pe. Consequently two such experiments are required with different values of B and the same N and Pe (same Re and Pr). In the limiting case $B \to \infty$ any value of N would provide the correct Pe. This is the analog to the tracer experiment, if T is the tracer concentration. The same values of Pe are obtained as with the method proposed in [6]. For liquids the capacity ratio may assume values of $1 < B < 10$. With gases the ratio is much smaller, e.g., $0.001 < B < 0.01$. Then the derivatives $a_0' \to \infty$ and $a_0'' \to \infty$ and the Eqs. (20)–(22) may no longer be applicable. This limiting case ($B \to 0$) is considered separately later in this paper.

3.2   Step-like signals

Till now the input signal and consequently also the outlet signal have been regarded as a temperature pulse starting and ending with the initial uniform temperature. Cases may arise in which step-like signals are preferred. In such cases Eq. (22) cannot be applied and the following approach is recommended.

The governing energy equations reveal that the solutions for the temperatures are also valid for their time derivatives. For that reason the measured

step functions starting with zero and ending with a fixed value can be differentiated numerically with respect to time and the obtained pulse functions can be evaluated in the described way.

It is more convenient to calculate the moments Q', R', S' directly from the original step functions

$$Q' = \int_{T_0}^{T_\infty} \mathrm{d}T = T_\infty - T_0 \;, \tag{24a}$$

$$R' = \int_{T_0}^{T_\infty} z\mathrm{d}T \;, \tag{24b}$$

$$\frac{\dot{V}}{V} \int_{T_0}^{T_\infty} \tau\,\mathrm{d}T, S' = \int_{T_0}^{T_\infty} z^2 \mathrm{d}T = \left(\frac{\dot{V}}{V}\right)^2 \int_{T_0}^{T_\infty} \tau^2\,\mathrm{d}T \;, \tag{24c}$$

and use them in Eqs. (20) and (21).

3.3 Combined systems

If the exchanger consists of a number of n subsystems in series and/or parallel flow arrangements (e.g., tube bundle plus two headers), combining rules are required for the N_i, Pe_i and B_i of the subsystems and the overall values or the measured derivatives a'_0 and a''_0 of the combined system. The rules are used to extract parameters of the subsystem of interest (e.g., tube bundle) from the measured parameters of the combined system (e.g., tube bundle plus headers).

Suitable combining rules have been derived from mass and energy balances. For arbitrary flow arrangements the first derivative

$$a'_0 = 1 + \frac{1}{B} = 1 + \sum_{i=1}^{n} \frac{v_i}{B_i} \;. \tag{25}$$

The combining rule for n subsystems in series flow arrangement is

$$-\frac{1}{2}a''_0 = \frac{1}{N}\frac{1}{B^2} + \frac{1}{\mathrm{Pe}}\left(1 + \frac{1}{B}\right)^2 = \sum_{i=1}^{n} v_i^2 \left[\frac{1}{N_i}\frac{1}{B_i^2} + \frac{1}{\mathrm{Pe}_i}\left(1 + \frac{1}{B_i}\right)^2\right] \;. \tag{26}$$

For n subsystems in parallel flow arrangement

$$\left(a_0'\right)^2 - a_0'' = \frac{2}{N}\frac{1}{B^2} + \left(1 + \frac{2}{\mathrm{Pe}}\right)\left(1 + \frac{1}{B}\right)^2 =$$
$$= \sum_{i=1}^{n}\frac{v_i^2}{\dot{v}_i}\left[\frac{2}{N_i}\frac{1}{B_i^2} + \left(1 + \frac{2}{\mathrm{Pe}_i}\right)\left(1 + \frac{1}{B_i}\right)^2\right]. \qquad (27)$$

For parallel arrangement of one forward flow system f and one backflow system b, where the total volumetric flow rate $\dot{V} = \dot{V}_f - \dot{V}_b > 0$ and the total volume $V = V_f + V_b$, the derivatives and overall parameters of the complete system can be expressed as

$$-\left(a_0'\right)^2 - a_0'' = \frac{2}{N}\frac{1}{B^2} + \left(\frac{2}{\mathrm{Pe}} - 1\right)\left(1 + \frac{1}{B}\right)^2 =$$
$$= \frac{v_f^2}{\dot{v}_f}\left[\frac{2}{N_f}\frac{1}{B_f^2} + \left(\frac{2}{\mathrm{Pe}_f} - 1\right)\left(1 + \frac{1}{B_f}\right)^2\right]$$
$$+ \frac{v_b^2}{\dot{v}_b}\left[\frac{2}{N_b}\frac{1}{B_b^2} + \left(\frac{2}{\mathrm{Pe}_b} + 1\right)\left(1 + \frac{1}{B_b}\right)^2\right]. \qquad (28)$$

The systems f and b can consist of subsystems (parallel or series) for which first Eqs. (25)–(27) have to be applied. The derivatives on the left hand side of Eqs.(25)–(28) are determined from the measured profiles using Eqs. (20)–(??). The overall values N and Pe are valid for the single blow process and do not necessarily compare with the steady state values. For the tracer experiment where $B_i = \infty$ and $B = \infty$, the Eqs. (26)–(28) yield the correct overall mean value of Pe for $s = 0$, defined in [6]. For the example of maldistribution with backflow the correct value Pe = 245/73 [6, tab. 1] can directly be calculated using the above Eqs. (27) and (28).

4   Calculated example

To demonstrate the application of the derived equations, an example with calculated experimental data is presented. As in the previous example [7] the evaluation of tracer experiments, the shell side of a shell and tube exchanger with two baffles is considered. Fluid flow and heat transfer are assumed to follow the cascade model with $n = 3$ identical mixed zones. The fluid is water which gives the estimated capacity ratio $B = 4$. The

number of transfer units $N = 2.4$. An inlet temperature pulse is given as

$$
\begin{aligned}
0 \leq z \leq z_1 : \quad & T_0\left(z\right) = \frac{\pi}{2z_1}\sin\left(\frac{\pi}{z_1}z\right) \\
z > z_1 : \quad & T_0\left(z\right) = 0
\end{aligned}
\tag{29}
$$

For the calculation of the 'experimental' outlet pulse Eq. (29) is Laplace transformed yielding

$$
\bar{T}_0\left(s\right) = \frac{1}{2}\frac{1 + e^{-sz_1}}{1 + \left(\frac{sz_1}{\pi}\right)^2} .
\tag{30}
$$

Applying Eqs. (15), (A4) and (30) gives the transformed outlet signal $\bar{T}_1\left(s\right)$, from which the outlet temperature profile $T_1(z)$ is calculated with help of numerical inversion [8]. The calculated experimental inlet and outlet signals are shown in Fig. 1 for $z_1 = 5$ ($T_0(z)$ and $T_1(z)$, $B = 4$).
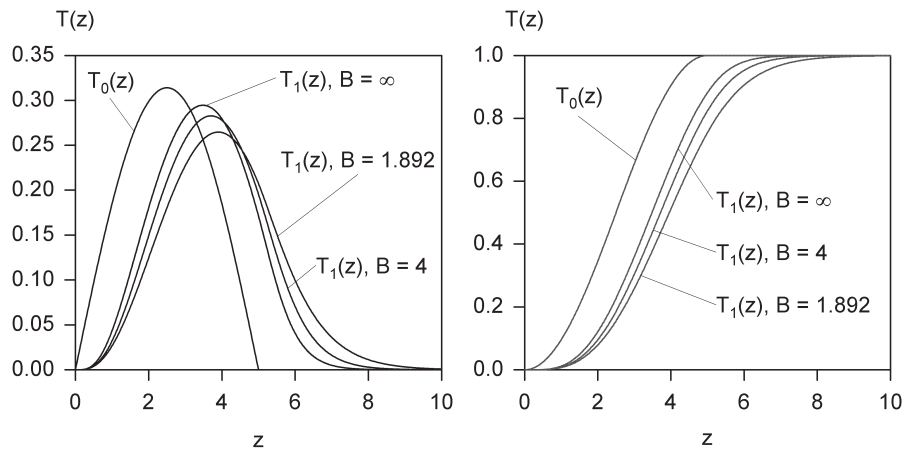

Figure 1: Calculated inlet and outlet profiles of three single blow experiments: Tracer ($B = \infty$), single blow water ($B = 4$) and single blow methanol ($B = 1.892$). The given data of the cascade model are $N = 2.4$, $n = 3$, $z_1 = 5$. For the dispersion model $\mathrm{Pe} = 6$; right) Equivalent step-like profiles found by integration of profiles on the left. They are valid for the same data N, n, Pe, B as on the left.

4.1   Test of Equation (23)

In order to test the validity of Eq. (23) the outlet profiles are also calculated using the Eq. (16) for the dispersion model. Not only the given values $N = 2.4$ and $\mathrm{Pe} = 2n = 6$ but also various pairs of N and Pe are used.

The calculations reveal that pairs which fulfill Eq. (23) nearly yield exactly the same outlet profile shown in Fig. 1. Minor deviations lie within the drawing and undoubtedly within the measurement accuracy. The correct pair 2.4/6 does not provide a better fit than other pairs from Eq. (23). Pairs which do not fulfill Eq. (23) yield more or less remarkably deviating outlet profiles. The results confirm the Eq. (23) and demonstrate that even a sophisticated curve matching method which compares measured and calculated outlet profiles could not provide a reliable pair of N and Pe. Thus another experiment for a second equation for N and Pe is definitely required.

Two additional experiments are considered, the tracer experiment and a single blow test with another liquid.

4.2   Tracer experiment

This experiment is performed with the same liquid water under the same flow conditions (Re, Pr) as the first single blow test ($B = 4$). Due to the analogy of heat and mass transfer the tracer experiment can be regarded and evaluated as an adiabatic single blow experiment with $B = \infty$ (and/or $N = 0$). The temperatures are the tracer concentrations. The dispersive Peclet number is the same as for the single blow test. The same equations are applied as in the previous single blow experiment. Using the same inlet profile, the outlet profile is calculated as described before. The outlet concentration $T_1(z)$, $B = \infty$, is drawn in Fig. 1.

4.3   Single blow with methanol

The first single blow test has been performed with water (20 °C), $\rho\,c_p = 4177.5$ kJ/(m^3 K), Pr = 7.004, $\nu = 1.003 \times 10^{-6}$ m^2/s, $B = 4$. The second single blow is carried out with methanol (20 °C), $\rho\,c_p = 1976.0$ kJ/(m^3 K), Pr = 7.200, $\nu = 0.740 \times 10^{-6}$ m^2/s, $B = 1.892$. The Reynolds and Prandtl numbers have to be the same. The Prandtl numbers in both tests can easily be adapted with a slight temperature change. For Re the flow velocity has to be changed according to the ratio of viscosities. With the same inlet profile as function of z ($z_1 = 5$), the same Re and Pr (and consequently N and Pe), and with $B = 1.892$ the outlet profile is calculated as described before and given in Fig. 1 as $T_1(z)$, $B = 1.892$.

4.4   Numerical evaluation

The calculated inlet and outlet profiles in Fig. 1 are regarded as experimental curves and are numerically evaluated using Eqs. (20)–(23). The numerical results for Q, R, S and ψ are presented in Tab. 1 together with the exact values of ψ, calculated directly from the given values of B, N, Pe $= 2n$, using Eq. (23). Each experiment yields one Eq. (23) with the related values of B and ψ. Combining the equations yields one experimental pair of N and Pe. The final results are given in Tab. 2.

Table 1: Results of the numerical evaluation of the calculated profiles shown in Fig. 1 for the tracer experiment ($B = \infty$), the single blow with water ($B = 4$) and the single blow with methanol ($B = 1.892$). Q_0, R_0 and S_0 are the same for all three experiments. In brackets are the exact values from analytical integrations using Eq. (29) for $T_0(z)$.

Item	$B = \infty$	$B = 4$	$B = 1.892$
Q_0, Eq. (22)	$1.000099(1.0\bar{0}\dots)$		
R_0, Eq. (22)	$2.500061(2.5\bar{0}\dots)$		
S_0, Eq. (22)	$7.433874\ (7.433941)$		
Q_1, Eq. (22)	1.000112	1.000112	1.000112
R_1, Eq. (22)	3.500129	3.750138	4.028688
S_1, Eq. (22)	13.767425	15.819532	18.424865
ψ, Eqs. (20), (21) and (23)	0.166853	0.183484	0.216610
ψ_{exact}, Eq. (23)	$0.16\bar{6}\dots$	$0.183\bar{3}\dots$	0.216485

Table 2: Final results N and Pe of the evaluation of three pairs of calculated experiments. 1. Tracer ($B = \infty$) and single blow with water ($B = 4$). 2. Two single blow with water ($B = 4$) and methanol ($B = 1.892$). 3. Tracer ($B = \infty$) and methanol ($B = 1.892$).

No.	B	B	N	Pe	n	Pe_p
1	∞	4	2.4051 (+0.2%)	5.9933 (-0.1%)	2.9967 (-0.1%)	4.7399 (-0.2%)
2	4	1.892	2.4019 (+0.08%)	5.9941 (-0.1%)	2.9971 (-0.1%)	4.7408 (-0.1%)
3	1.892	∞	2.4090 (+0.4%)	5.9933 (-0.1%)	2.9967 (-0.1%)	4.7399 (-0.2%)

The number n of the cascade model and Pe_p of the parabolic model are

calculated from the 'measured' Pe with help of Eq. (19). The deviations from the correct values $N = 2.4$, $n = 3$, $Pe = 6$, $Pe_p = 4.7470$ are inaccuracies during numerical inversion and numerical integration. The same values and accuracy of ψ, N and Pe would be obtained when different dimensionless pulse durations z_1 or even differently shaped inlet pulses would be applied for the three calculated experiments.

The above results clearly demonstrate that in the case of liquids reliable values of N and Pe (and n, Pe_p) can be obtained if two experiments are carried out at the same Re and Pr, namely either one single blow plus one tracer experiment ($B = \infty$), or two single blow experiments with different liquids (different B).

Another way for liquids in heat exchangers is the combination of single blow and traditional steady state experiments. Numerous measurements with variations of Re and Pr have to be carried out, preferably under constant conditions in the other flow channel (rear side of the wall). The single blow yields ψ(Re, Pr) for $B > 0$, e.g., $B = 4$. The steady state experiment provides N_d(Re, Pr) $= \psi$(Re, Pr) for $B = 0$. Applying the well known Wilson plot technique [9] with a common least square estimation one can develop correlations for Nu(Re, Pr) and Pe(Re, Pr), where Pe may usually depend on geometry only, $Pe = const$.

If gases are involved steady state and single blow experiments yield both nearly the same information as B approaches zero. Special considerations are required for the evaluation of single blow experiments since for $B \to 0$ $a'_0 \to \infty$ and $a''_0 \to \infty$, and the above developed evaluation method cannot be applied.

5   Single blow with gases

5.1   Limiting case B = 0

First the limiting case $B = 0$ is considered in which the dimensionless wall temperature remains constant at $t_w = 0$. The solution, Eq. (16), simplifies to

$$\frac{1}{a(s)} = \frac{1}{N+s} + \frac{1}{Pe+s} \, . \tag{31}$$

For $s = 0$

$$\frac{1}{a_0} = \frac{1}{N} + \frac{1}{Pe} = \frac{1}{N_d} \, . \tag{32}$$

The exponent a_0 equals the effective dispersive number of transfer units N_d for steady state operation, given with Eq. (1).

Integrating the measured inlet and outlet pulses yields Q_0 and Q_1 (Eq. (22)). The Eq. (15) gives

$$a_0 = \ln\left(\frac{Q_0}{Q_1}\right) = N_d \,. \tag{33}$$

This equation describes also steady state cooling in a heat exchanger with constant wall temperature, if Q_0 and Q_1 are the mean inlet and outlet temperatures.

5.2 Cross-flow corrections for changing wall temperature

The value of N_d from Eq. (33) is too low as the true mean temperature difference between gas and wall is smaller than in case of constant wall temperature. To account for small changes of wall temperature, the transient single blow process is regarded as a steady state cross-flow process in a heat exchanger. Q_0 is the inlet and Q_1 the outlet temperature. The hot fluid is the gas with flow path x, the cold fluid is the wall with time τ or z as flow direction coordinate. The heat capacity flow rate of the wall is C_w/τ_1 with τ_1 as the duration of the measured inlet pulse. Pure cross-flow (unmixed-unmixed) is the appropriate flow arrangement, as no wall heat conduction in the x-direction is assumed. Introducing the single blow parameters, the known equation for pure cross-flow [5] can be written as

$$Bz_1\left(1 - \frac{Q_1^*}{Q_0^*}\right) =$$
$$\frac{1}{N_d}\sum_{m=0}^{\infty}\left[\left(1 - e^{-N_d}\sum_{j=0}^{m}\frac{1}{j!}N_d^j\right)\left(1 - e^{-N_d Bz_1}\sum_{j=0}^{m}\frac{1}{j!}(N_d Bz_1)^j\right)\right] \tag{34}$$

with

$$Q^* = \int_{z=0}^{z_1} T dz \,. \tag{35}$$

For $Bz_1 \to 0$ N_d from Eq. (34) approaches that of Eq. (33), calculated with Q_0^* and Q_1^*.

If one assumes infinite thermal conductivity of the wall, $\lambda_w = \infty$, the Eq. (34) has to be replaced by the known equation [5] for mixed-unmixed

cross-flow (wall = mixed fluid):

$$N_d = -\ln\left\{1 + \frac{1}{Bz_1}\ln\left[1 - Bz_1\left(1 - \frac{Q_1^*}{Q_0^*}\right)\right]\right\} , \qquad (36)$$

which equation yields the upper limit of N_d: $N_{d,36} \geq N_{d,34} \geq N_{d,33}$ (with Q^* in Eq. (33)).

5.3   Calculated example

The evaluation method for gases is tested with a calculated single blow experiment. It is first assumed that the real process follows the dispersion model with Pe $= 12$, $N = 3$ and $B = 0.002$. These data are similar to those of earlier experiments with air in a wind tunnel [3].

In example 1 the previous inlet profile according to Eqs. (29) and (30) is used with a longer dimensionless pulse duration $z_1 = 25$. Applying Eq. (16) yields $\bar{T}_1(s)$ and numerical inversion gives the calculated 'experimental' outlet profile $T_1(z)$ shown in Fig. 2 together with $T_0(z)$. Integrations according to Eq. (35) yield Q_0^* and Q_1^*. Then $N_{d,34}$ and $N_{d,36}$ are calculated using Eqs. (34) and (36), respectively. Then the same example is calculated with the 'experimental' inlet profile from the cascade model with $n = 6$. Both results are presented in Tab. 3.

In example 2 it is tested whether or not the method can be applied to cases in which both profiles are incomplete (not back to zero) as shown in Fig. 3. The equations for the inlet pulse are given as

$$T_0(z) = \frac{1}{36}z\exp\left(-\frac{z}{6}\right) , \qquad (37)$$

$$\bar{T}_0(s) = \frac{1}{(1 + 6s)^2} , \qquad (38)$$

$$Q_0^* = 1 - \left(1 + \frac{1}{6}z_1\right)\exp\left(-\frac{z_1}{6}\right) . \qquad (39)$$

The results of the calculation are also given in Tab. 3.

If the single blow process follows the dispersion model, the cross-flow correction method yields for both examples very good values of the effective number of transfer units $N_d = N_{d,34}$. The relative error falls below $\pm 0.5\%$. This demonstrates that the simple cross-flow correction is quite effective. The higher values of $N_{d,36}$ from Eq. (36) confirm that with $\lambda_w = \infty$ higher heat transfer coefficients would be required than with $\lambda_w = 0$.
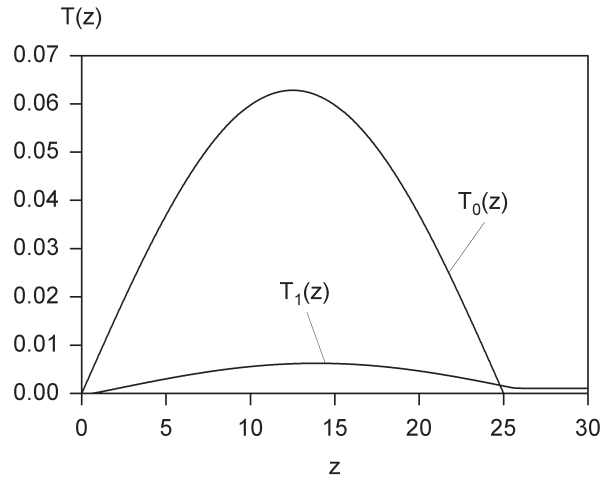
Figure 2: Calculated profiles of example 1 for single blow with a gas ($B = 0.002$). $T_0(z)$ from Eq. (29). $T_1(z)$ for dispersion model with $N = 3$ and Pe $= 12$. For cascade model with $n = 6$ $T_1(z)$ is qualitatively the same.
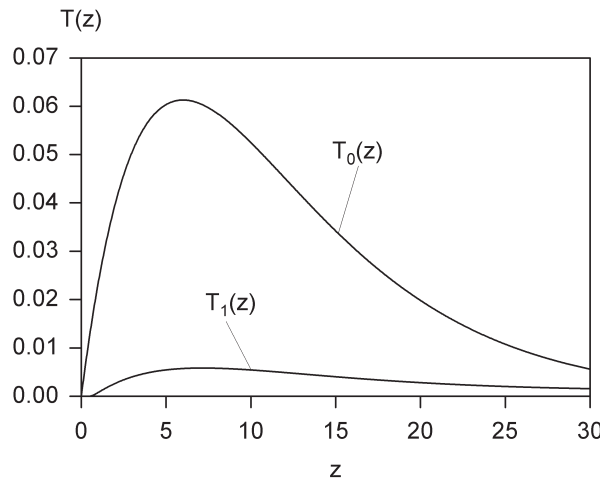

Figure 3: Calculated profiles of example 2 for single blow with a gas ($B = 0.002$). $T_0(z)$ from Eq. (37). $T_1(z)$ for dispersion model with $N = 3$, Pe $= 12$ and cascade model (qualitatively) with $n = 6$.

Looking at the results of the experiments which follow the cascade model with $n = $ Pe$/2 = 6$ shows that the obtained N_d are higher, $N_{d,34} > 2.4$. The reason is that the wall temperature remains nearly constant during

the single blow process and the exponent $a_0(B = 0, s = 0)$ is used for the evaluation instead of $a_0''(B > 0, s = 0)$. The relationship Eq. (19) is not valid here but the following equation for $t_w = const$:

$$N_d = \left(\frac{1}{N} + \frac{1}{\mathrm{Pe}} \right)^{-1} = n \ln \left(1 + \frac{N}{n} \right) = \ln \left[\frac{1}{F_p \left(\sigma = N/\mathrm{Pe}_p \right)} \right] \qquad (40)$$

with $1/F_p$ from Eq. (A3, $\sigma = N/\mathrm{Pe}_p$) is a good approximation. This relationship between Pe, n and Pe_p depends also on N. In the limiting case $N \to 0$ (tracer) Eq. (40) turns to Eq. (19). With $N = 3$ and $n = 6$ Eq. (40) yields $N_d = 2.433$ which compares well with the 'measured' values of Tab. 3. The corresponding Peclet number Pe $= 12.867$.

The Eq. (40) in connection with the cross-flow correction is also con-

Table 3: Results of numerical evaluation of calculated examples. Example 1: Pulse Eq. (29), Fig. 2. Example 2: Pulse Eq. (37), Fig. 3. Single blow process according to dispersion model with Pe $= 12$, according to cascade model with $n = 6$. $a_0 = N_{d,31}$ for $t_w = const$, $N_{d,32}$ is the actual result after cross-flow correction, $N_{d,34}$ is the upper limiting value for $\lambda_w = \infty$.

Example	Model	$N_{d,33}$, (33), (35)	$N_{d,34}$, (34), (35)	$N_{d,36}$, (36), (35)
1	Pe $= 12$	2.2742 (-5.2%)	2.4109 (+0.45%)	2.4994 (+4.1%)
pulse (29)	$n = 6$	3.3020 (-4.1%)	2.4421 (+1.8%)	2.5360 (+5.7%)
2	Pe $= 12$	2.2569 (-6.0%)	2.3915 (-0.36%)	2.4768 (+3.2%)
pulse (37)	$n = 6$	2.2837 (-4.9%)	2.4215 (+0.90%)	2.5118 (+4.7%)
1 + 2	Pe $= 12$	2.2656 (-5.6%)	2.4012 (+0.05%)	2.4881 (+3.7%)
	$n = 6$	2.2928 (-4.5%)	2.4318 (+1.3%)	2.5239 (+5.2%)

firmed with the data of one single blow experiment with air of ref [3]. Evaluating the profiles of Fig. 5 [3], according to the cross-flow correction method yields $N_{d,34} = 2.5084$ ($\lambda_w = 0$) and $N_{d,36} = 2.6052$ ($\lambda_w = \infty$). The curve matching method [3] developed for the parabolic dispersion model with consideration of axial wall heat conduction in the walls (copper plates) yielded $N = 3.172$ and $1/\mathrm{Pe}_p = 0.1103$. Introducing these values in Eqs. (A4) and (40) gives $N_{d,40} = 2.5363$ and the reasonable relationship $N_{d,32} = 2.5084 < N_{d,40} = 2.5363 < N_{d,36} = 2.6052$. Further evaluations of the experiments [3] are planned and beyond the scope of this paper.

5.4   Determination of N and Pe from N_d

For design calculations (steady state) the measured values of $N_d(1/N_d = 1/N + 1/\text{Pe}$, Eq. (1)) and the corresponding heat transfer coefficient α_d can directly be used without knowing N and Pe. For the calculation or evaluation of transient processes also Pe and N have to be known. For their determination a second test with another gas would not provide new information since $(1+B)$ remains nearly the same. Hence an additional tracer experiment could be recommended with evaluation according to the described method for liquids with $B = \infty$. Incomplete inlet concentration profiles as in example 2 for gases are not allowed and usually do not occur. The outlet profiles look similar to those in Fig. 1. The evaluation would yield the correct value $\text{Pe} = 12 = 2n$ for both the dispersion model ($\text{Pe} = 12$) and the cascade model experiment ($n = 6$). If the process (gas) follows the dispersion model ($\text{Pe} = 12$) one would obtain the correct $N \approx 3$ from the measured $N_d \approx 2.4$ (single blow) and $\text{Pe} \approx 12$ from the tracer experiment. However, if the single blow process (gas) follows the cascade model ($n = 6$) the measured mean value $N_d = 2.4318$ (Tab. 3) would yield only the approximate value $N = 3.05$. This fundamental inaccuracy cannot be avoided. With lower Pe the error increases.

     If only a rough estimation of N and Pe from the measured effective value N_d is needed, the following simple approach appears useful. Two equations for N and Pe are required. The first is Eq. (1) and the second is Eq. (16) with one given value of $s = s_1$.

     If the process follows the dispersion model any value of s will yield the correct result, provided the quantity $a(s_1)$ can properly be determined from the measured profiles. As one or both profiles may not reach zero within the range of measurements (see Figs. 2 and 3) one has to select a sufficiently large positive value $s_1 > 0$, to ensure that the rest of the profiles can be cut off at z_1 without affecting the numerically Laplace transformed temperatures $\bar{T}(s_1)$ and consequently $a_1 = a(s_1)$. Substituting in Eq. (16) N according to Eq. (1) and solving for $1/\text{Pe}$ yields

$$
\begin{aligned}
\frac{1}{\text{Pe}_{1,2}} &= D \mp \sqrt{D^2 + \frac{2D(a_1 - s_1) - 1}{s_1(2a_1 - s_1)}}, \\
D &= \frac{1}{2}\left(\frac{1}{N_d} + \frac{B}{s_1}\right).
\end{aligned}
\tag{41}
$$

The experimental values $a_1(s_1)$ have to be determined numerically according to Eq. (15) from the measured profiles. For this test the values of a_1 are calculated from Eqs (16) and (A4). The value $s_1 = 0.5$ is used for

both examples. The mean values of N_d and the exact value $N_d = 2.4$ are considered. The results are presented in Tab. 4.

Table 4: Calculated N and Pe using Eq. (16) with $s = 1/2$ from given $N_d = 2.4$ and 'measured' mean values $N_{d,34}$ of Tab. 3. All values of correct solutions with relative deviations (%) from $N = 3$ or Pe $= 12$, respectively.

Model of experiment	$N_{d,34}$	$Pe_{1,2}$ (41)	$N_{1,2}$ (1)
$N = 3$	2.4000	12.0000 (0%)	3.0000 (0%)
Pe $= 12$		2.9644	12.6050
$N = 3$	2.4012	11.8886 (-0.93%)	3.0089 (+0.3%)
Pe $= 12$		2.9731	12.4822
$N = 3$	2.4318	11.2417 (-6.3%)	3.1030 (+3.4%)
$n = 6$		3.0650	11.7710

The results of table 4 show very good results for N and Pe if the single blow process follows exactly the unity Mach number dispersion model. Two pairs of N and Pe are found for one experiment. Usually the higher number will be the Peclet number. If the process follows the cascade model the obtained N and Pe are only approximations. The resulting N is 3.4% too high. A higher accuracy cannot be expected from such simple evaluation method. As the values of N and Pe are not needed for design calculations the errors can be tolerated. More precise results for Pe and N can be obtained from an additional tracer experiment or from a sophisticated curve matching method [3] which would have to be adapted to the unity Mach number dispersion model. But also such method cannot yet guarantee 100% reliability.

6   Conclusions

1. Single blow tests can be evaluated with consideration of the unity Mach number axial dispersion model.

2. One single blow test alone cannot provide reliable values for the dispersive Peclet number and the related true heat transfer coefficient.

3. With liquids reliable results for both unknowns can be obtained from two experiments: Either two single blow tests with different liquids, or

one single blow and one tracer experiment using the same or another liquid.

4. With one liquid a combination of single blow and steady state experiments can provide reliable correlations for heat transfer coefficient and dispersive Peclet number.

5. With gases one single blow experiment yields a suitable value for the effective heat transfer coefficient which can be used for design calculations. Additionally approximate values of heat transfer coefficient and Peclet number can be obtained.

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Appendix

Derivatives of $a(s)$ from Eq. (14)

$$a_0' = \left.\frac{\partial a}{\partial s}\right|_{s=0} = 1 + \int_{x=0}^{1} \sum_{i=1}^{m} \frac{\mathrm{d}x}{B_i} \, , \qquad (A1)$$

$$a_0'' = \left.\frac{\partial^2 a}{\partial s^2}\right|_{s=0} = -2 \int_{x=0}^{1} \sum_{i=1}^{m} \frac{\mathrm{d}x}{N_i B_i^2} - \frac{2}{Pe} \int_{x=0}^{1} \left(1 + \sum_{i=1}^{m} \frac{1}{B_i}\right)^2 \mathrm{d}x \, . \quad (A2)$$

Parabolic dispersion model ($M = 0$, Pe_p)

$$\begin{aligned} \frac{1}{F(s)} &= \frac{1}{2}\left(1 + \frac{1+2\sigma}{\sqrt{1+4\sigma}}\right)\exp\left[-\frac{\mathrm{Pe}_p}{2}\left(1 - \sqrt{1+4\sigma}\right)\right] + \\ &\quad \frac{1}{2}\left(1 - \frac{1+2\sigma}{\sqrt{1+4\sigma}}\right)\exp\left[-\frac{\mathrm{Pe}_p}{2}\left(1 + \sqrt{1+4\sigma}\right)\right] \\ \sigma &= \frac{1}{\mathrm{Pe}_p}\left(s + \frac{1}{\frac{1}{N} + \frac{B}{s}}\right) \end{aligned} \qquad . \qquad (A3)$$

Cascade model (n mixed zones)

$$\frac{1}{F(s)} = \left[1 + \frac{1}{n}\left(s + \frac{1}{\frac{1}{N} + \frac{B}{s}}\right)\right]^{n} \, . \qquad (A4)$$