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Dynamic Approach to the Origin-Destination Matrix Estimation in Dense Street Networks

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Abstract

The article deals with some issues related with the travel demand modelling in dense street networks. Estimation of the trip distribution usually presented in a form of O-D matrix has been described as one of the most important stages of this process. The field of interest includes a short review as well as classification of the most popular and applicable methods in this area. The main emphasis has been placed on dynamic approach based on traffic counts. Advanced technologies used in traffic management systems should rely on real up-to-date and exact not only average travel demand information that provides efficient results. All above makes a background to formulate some assumptions for the original concept of the O-D matrix estimation.

1. Introduction

Trip distribution is basic input data for many problems related with designing the organizational changes, travel modelling and traffic prognoses within transportation systems. It is desired both for the long-term planning and short-term traffic management based on within-day dynamics. Such distribution is most likely expressed as a square origin-destination (O-D) matrix and makes a mathematical representation of travel demand.

Efficient traffic management requires information about the origins and destinations of trips taken by particular groups of users, especially when some serious events such as road incidents, car crashes, road works, and traffic emergences along

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with other untypical conditions occur. It is possible then to set the optimal diverse routes within the street network based on uch information. The O-D matrices, updated in the given reference periods, describe the trip distribution and their estimation may be based on the current traffic counts. A frequent input data update regarding real traffic flows within the street network plays a key role in the traffic management.

A model representation of the actual spatial characteristics of trips within the road network is a complex issue and the subject of many studies that require data on zoning and land use planning, information about traffic conditions within the network as well as all sorts of socioeconomic and demographic attributes. Such features can be determined by different levels of aggregation. It makes the estimated characteristics of trip distribution a close approximation of real traffic with various degrees of accuracy.

Traffic forecasting and travel behaviour modelling are very complex especially in dense street networks, which are characterized by high concentration of intersections that cause frequent disturbance of traffic. What is more, significant variations in the flow distribution due to its velocity and vehicle composition may also lead to congestion and to longer times of journey. The number of alternative paths for each O-D pair is much higher in dense street networks than in any other type of networks. Thus, the demand flow between origin and destination distributes to a larger number of paths causing a significant increase in computational complexity. In addition, a high concentration of elements that generate and absorb trips enlarges the size of the O-D matrix.

2. O-D Matrix as a Representation of Demand in Transportation Network

The problem with estimating the trip distribution is one of the essential stages in a four-step demand model that is among others used to prepare traffic prognoses. It is a sequential model that contains a mathematical representation of travel behaviour. This staged representation goes from generation of demand (step 1: trip generation), through the spatial distribution (step 2: trip distribution) and selecting a way of travelling and mode of transportation (step 3: mode choice), to loading the transportation network with trips (step 4: traffic assignment) [23, 28, 36]. All four steps of the travel demand model which also are the sub-models are shown symbolically in Fig.1. Even though the classical approach makes each step to obtain results from the previous one, the theory shows different variants that combine certain steps into whole or change the order of their realization [15, 36].

The study area in the travel modelling issues is usually divided into a number of discrete geographical units the traffic analysis zones and represented by the centres of gravity (centroids). They are defined as the points of concentration of the beginning or end of individual trips. The network model usually takes the form of a directed graph [24, 29, 36]. Particular cells of the two-dimensional O-D matrix represent the volume of demand flow, which is expressed in a number of trips between two zones. In practice the centres of gravity can be linked to the model of transportation network by so-called "connector links" or "centroid connectors" with specific characteristics. They are identified as to represent the average costs of access to the centres of gravity.

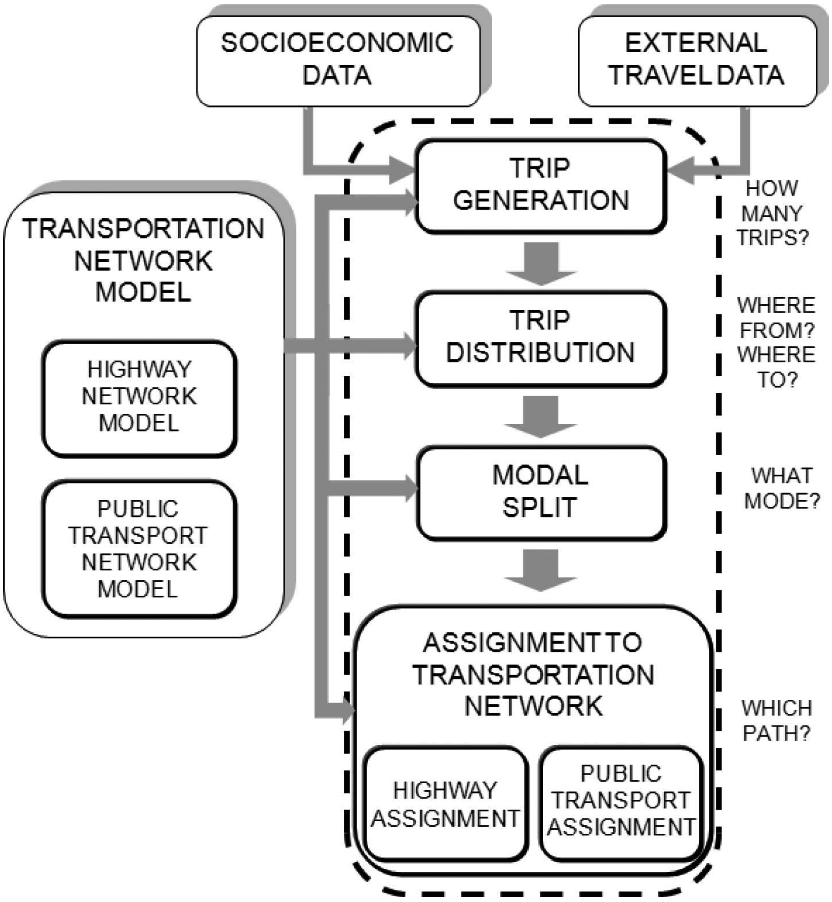

Fig. 1. Scheme of four-step demand model

The O-D matrix maps interactions between the transportation system and its surrounding for the part describing the problems to solve within the system. These tasks are in the form of demand for transport as expressed in O-D pairs [24, 25, 29]. A single element of the O-D matrix can be then presented as:

- a constant value for fixed demand,
- a random variable with known distribution for random demand,
- a deterministic function that describes demand,

- a probabilistic function that describes demand with the parameters defined by random variables.

Depending on the location of origins and destinations within the study area, one can distinguish the following types of trips [15]:

- internal trips (origins and destinations within the area),
- exchange trips (origins within the area and destinations outside the area, and vice versa),
- crossing trips (both origins and destinations outside the area, but travels take place partly within the area and therefore have influence on the transportation network),
- external trips, entirely performed outside the area.

Figure 2 contains identification of specific types of trips within the O-D matrix for the n zones within the study area boundary and $m-n$ external zones [15]. The main diagonal corresponds to intrazonal trips.

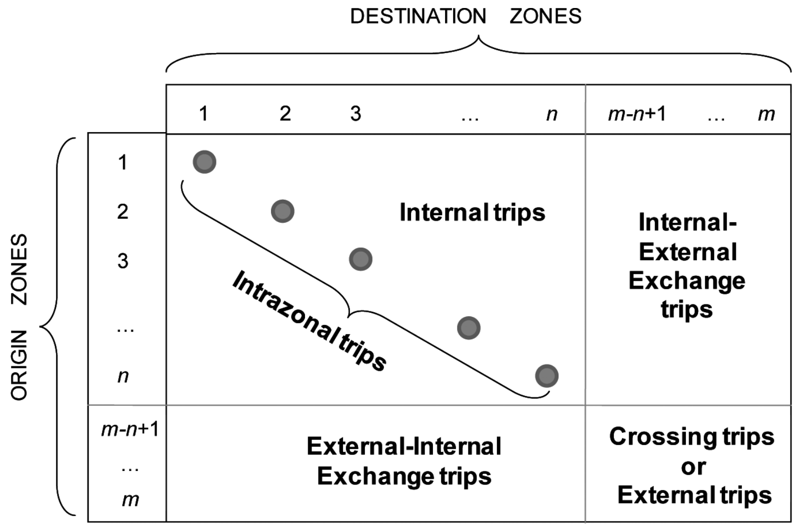

Fig. 2. Identification of specific types of trips within the O-D matrix (based on [15])

The matrix can be further disaggregated, for example, by modes of transport, specified time periods, different trip purposes, as well as groups of users (see Fig. 3). These matrices can be provided for the existing or future states (traffic predictions).

3. Methods of the O-D Matrix Estimation

Depending on the objective of the travel demand model and forecasts to prepare, as well as the precision of data available when estimating the O-D matrix one can use different methods that can be divided into two main groups: the conventional and unconventional ones [49]. Further division can be carried out assuming dif-

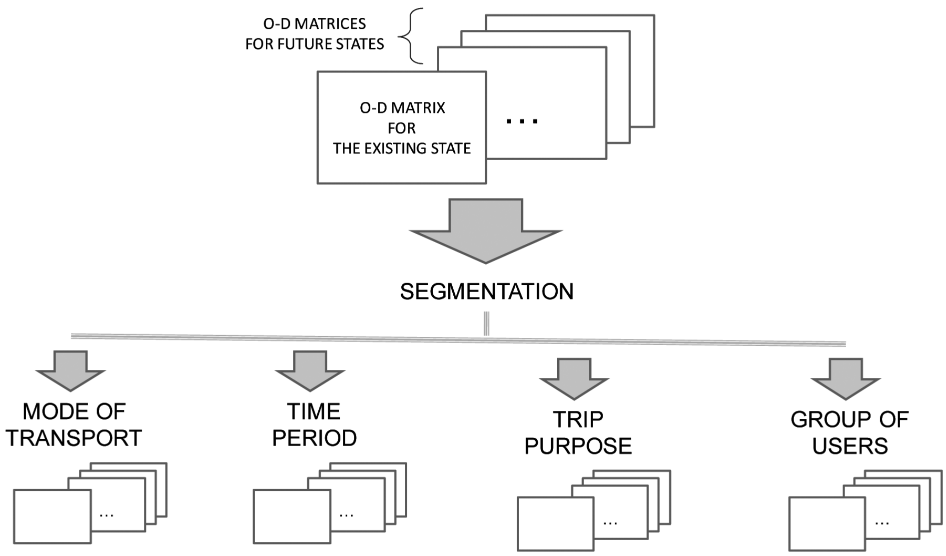

Fig. 3. Segmentation of O-D matrices

ferent criteria of classification. Taking a research technique that has been used as a basic criterion for grouping the conventional methods can be divided into direct and indirect ones. The first ones are used to obtain a real picture of trips within the sample (empirical methods) whilst the latter are used to build trip distribution models (theoretical methods) (see Fig. 4).

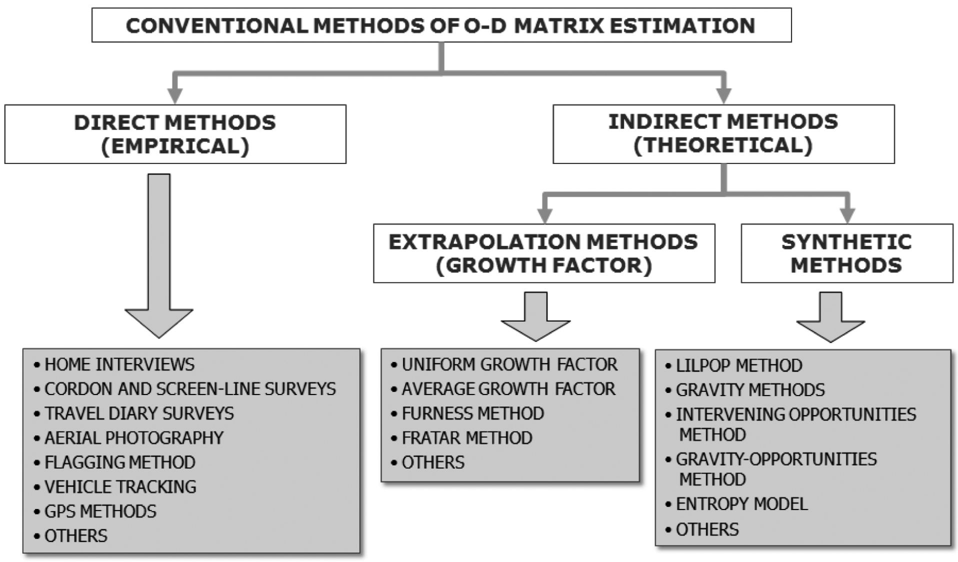

Fig. 4. Classification of conventional methods of O-D matrix estimation (based on [49])

Along with the techniques used in the empirical methods one should mention among others home interviews, cordon surveys, aerial photography, a flagging method (identification of vehicles at cordon and internal points by means of registration numbers, stickers, etc. [49, 54]), vehicle tracking, GPS or other modern technologies. The O-D matrices resulting from such methods are based on advanced statistical techniques, and their accuracy depends on the estimators that were used, the degree of the network aggregation, the sample size, etc.

Theoretical methods can be divided into two groups: growth factor methods and synthetic ones. There is a variety of the first group of methods, also referred to as extrapolation ones. These often use some previously known O-D matrices (prior matrices), as well as either the new numbers of trips from origins to destinations, or specific growth factors. This group contains a variety of methods of different complexity, from the simple ones based on uniform growth factors equal for the entire study area to the sophisticated modifications of iterative Furness procedures or the Fratar method [36].

In the case of growth factor methods one needs to remember that they can be applied in state of relative stability only. It limits their usage to the short-term planning horizons, and each error in the matrix for the base year can cause significant deviations in the forecasting models. In addition, the estimation of reliable values of growth factors requires an extensive database containing, among others, the results of comprehensive traffic and transportation studies, information on the condition of vehicles, hypotheses on development of motorization region- and nationwide, annual average daily traffic and the length of public roads, as well as the characteristics of the regions in terms of urbanization and the type of land use [35]. There is also the need of forecasting the development of the transportation sector, GDP and demographic prognoses, as well as information about the split of tasks among the means of transportation.

The dynamically developing regions are characterized by frequent changes in the structure of the transportation system. They may be associated for example with the land use development. When analysing such regions one needs to apply models that consider the significant determinants affecting trip generation. In these models one analyses travel behaviours of the system users in the context of distance, time or travel costs, as well as attractions of the zones in terms of the purpose and the accessibility of the transportation system. The users are divided into homogenous groups based on purpose of travel. They can be further split according to for example income level, car ownership or household size and structure.

The solutions resulting from the use of synthetic methods should oscillate between two extreme models, referred to as the minimum and maximum (proportional) models [37]. The assumption for the minimum model is that at first for each purpose of travel demand is satisfied within the same zone – the maximal intrazonal trips occur. Then the outnumbered demand flows are shifted to the nearest zones with the great attractiveness determined by number of trip attractors within given purpose. This is a model with the smallest dispersion, which provides the lowest travel costs.

In turn the demand flows in the maximum model (the model with the largest dispersion) spread out proportionally to the number of trip attractors in particular zones. Assuming that the attractiveness of all zones is equal, the model produces the most probable trip distribution in the conditions of a random selection of destination. The probability of its occurrence increases with the growth of number of zones. The proportional model often makes a starting point for many other synthetic methods.

The model proposed by Lilpop was one of the first synthetic models applied in Poland producing real trip distributions. It uses a dispersion factor, determined experimentally, that is equal for all O-D pairs [37]. This, however, does not take into account the influence of distance or travel time on the volume of trips. It results in a fact that the demand flows between remote zones are considerably overestimated and the ones between the nearby zones are underestimated.

The most currently used methods the gravity methods are based on the assumption that the number of trips between the two zones is proportional to their productions and attractions. It also takes into account spatial interaction in form of so called a deterrence function. It associates the number of trips for the O-D pair with the cost of travelling between origin and destination zones [23, 36, 37]. Over the years the method used for the first time by Casey [17] has undergone many modifications. Depending on the form of used deterrence function, as well as the limitations and conditions to model one can formulate different variants of gravity methods.

Other assumptions come with the intervening opportunities model, where the number of trips between two zones depends on attractiveness of the zones and the probability of destination choice at the earliest accessible opportunity. Such probability decreases with the distance increasing between a given O-D pair. Stouffer [48] has introduced the concept for the first time and applied it to describe the migration and location of the residential and public areas in cities. However, it is Schneider [38] who has developed this theory to the form currently used in modelling of the travel behaviour of users. For many years this method has been developed in Poland by Zipser [3, 30].

There are also models that combine the elements of the gravity and intervening opportunities methods in a form of gravity-opportunity models. Wills [53] has formulated first assumptions for such problem. He proposed a flexible model in which classical forms of gravity models and intervening opportunities were treated as special cases. The choice between these models is determined empirically by certain parameters affecting the manner of the O-D matrix estimation. Tamin and Willumsen [50] have made its generalization.

Some computational methods, including the gravity methods and Furness method, can be generalized by the entropy model. The theory of entropy assumes that in the transportation system depending on the level of aggregation there are microstates (e.g. characteristics of individual users, their origin and destinations, mean of transportation, travel time, etc.), mesostates (e.g. the number of trips for particular O-D pair), and macrostates (e.g. the total number of trips on various sections of the network, the total number of trips generated or attracted in each zone, etc.). To determine a reliable measure of activity of travellers it is often easier conduct observations at a higher level of aggregation. The O-D matrix is estimated at the level of mesostates.

By using the theory of entropy one can generalize both, the gravity models and the models of intervening opportunities [36]. The analysis of transportation systems may use a maximum entropy criterion that allows one to expect the trip distribution with the greatest possible value of the measure of uncertainty (entropy), associated with the probability distribution. Such procedures can be applied to estimate the demand flows; they can also be used at the lower levels of aggregation (e.g. assigning flows to the transportation network) [20].

According to the classic methods, estimating the O-D matrices typically requires surveys on a large scale. Additional information, such as detailed sociological and demographic data describing attractiveness of the zones within the area, and characteristics of individual groups of users with the homogeneous travel behaviours usually are necessary to model travel demands. Hence, these methods are expensive and time-consuming. Results obtained in such manner can be easily out of date due to the dynamic nature of traffic within urban areas. The O-D matrices obtained from surveys do not include temporal variations of travel demand resulting for example from the time of a day, seasons, organizing the mass events, or weather conditions. For the long-term planning such forms are sufficient enough, while classic methods do not work for the necessity of continues updating of input data according to the current situation on the road (traffic control and management, emergency situations).

For that reason, for several decades now, the O-D matrices estimation has been supported by additional techniques and sources of information. Monitoring of urban areas spreading increasingly used to obtain data on the volume of traffic flows on particular links is becoming an attractive alternative to collect additional data for the trip distribution estimation. There have also been attempts to build the matrix based on new technologies such as genetic algorithms [39], neural networks [57], or fuzzy logic techniques [26].

Due to the high complexity of the travel demand modelling process and the necessity to repeat certain stages of the process during model calibration and verification, special IT tools supporting the computational process started to be developed. Classic methods and their modifications were implemented in a number of computer applications for travel demand forecasting and complex traffic analyses. Among the most popular software packages used in Poland, one should mention VISUM, EMME/2 and SATURN [61]. Specialised packages are usually sufficient for everyday applications. However, in case of non-standard travel demand modelling or when new computational procedures are being developed or modified, it may well turn out that the traditional package is not enough and a new software tool needs to be developed.

4. The Dynamic Approach to the Trip Distribution

Estimation of the O-D matrix can also be considered in terms of demand variability over time. Then models can be divided into static and dynamic ones. It is assumed that demand for travel and supply of the transportation system within the first group does not change in time. The latter methods assume variability of demand and supply over time, and therefore they are more complex and require much more input data. These models are perceived to be superior to static approaches due to their potential ability to capture the spatial and temporal evolution of traffic on the network. When estimating the forecasting models, frequently used to plan the transportation systems, both models can be applied. On the other hand, dynamic models are mainly used in traffic control and traffic management (as in Advanced Traffic Management Systems, Advanced Traveller Information Systems, etc.). In research investigations, these models have been used for diverse applications such as High Occupancy Vehicle (HOV) and High Occupancy Toll (HOT) lane analyses, congestion pricing, work zone management, evacuation planning and vehicle routing. They can base on the link traffic counts [46].

There are many works and publications on the dynamic estimation of the O-D matrix [2, 4, 7, 11, 12, 19, 41]. They are both, theoretical and practical, and most of them were verified on the rural road networks, where the number of intersections with traffic lights was negligible, and the number of paths for a given O-D pair was limited [9]. Dynamic estimation of the O-D matrix allows providing for changes in demand over time.

In addition, the problem of changes in time itself can be modelled in different ways. It is common for the dynamic approach that the referred period is divided into a sequence of smaller time periods (time intervals) and, due to the traffic volume variability over time, the estimation process is carried out separately for each time interval. Current traffic volumes are recorded in these intervals [58, 60]. The values of a priori O-D matrix for particular time interval are derived from estimated trip tables for the previous interval. There are also some other approaches to these issues, as for example in [11]. The graph describing the structure of the transportation network in the space-time continuum has been extended by introducing a supplementary matrix. For each time interval the matrix identifies the number of intervals needed for passing through a given link, occupied by a vehicle. Using such interpretation the dynamic problem can be reduced to the static problem within a particular space-time continuum.

4.1. Estimation of O-D demand flows using traffic counts

From a certain point of view estimation of the O-D matrix, based on traffic counts is an inverse problem to the traffic assignment, as it is shown schematically in fig.5. The assignment problem focuses on calculating the link flows with the O-D flows, network and path choice models as input data. Conversely, the problem under study is that of estimating the O-D flows starting with measured link flows, as well as using the network and path choice models. Estimation is therefore reduced to finding a reliable O-D matrix, which, when assigned to the network, reproduces the observed traffic counts.

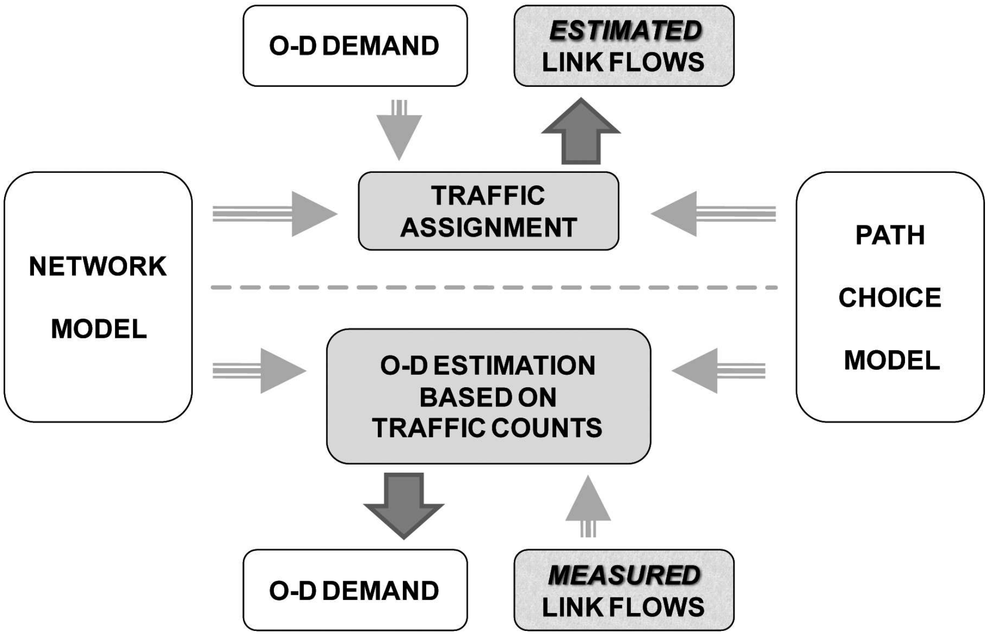

Fig. 5. Comparison of two processes: estimation of O-D matrix based on traffic counts with traffic assignment

The general relation between the link volumes and the OD-flows can be written as [1]:

$$x_{ij}(t) = \frac{1}{R} \sum_{(a,b) \in E} \delta_{ij}^{ab}(t) \cdot x^{ab}(t) \tag{1}$$

where:

$x_{ij}(t)$ – traffic volumes on link $(i, j) \in \boldsymbol{L}$ over time t,

$\boldsymbol{L}$ – set of links within the transportation network,

R – the car occupancy factor,

$\boldsymbol{E}$ – set of O-D pairs,

$x^{ab}(t)$ – O-D flows for $(a, b) \in \boldsymbol{E}$ over time t,

$\delta_{ij}^{ab}(t)$ – fraction of trips for O-D pair $(a, b) \in \boldsymbol{E}$ that uses link $(i, j) \in \boldsymbol{L}$ over time t.

It is assumed for such methods that for a certain subset of links $\hat{\boldsymbol{L}} \in \boldsymbol{L}$ and for a specific time interval t (e.g. peak hour, average day) the volume of traffic is known. Using formula (1) for each link $(i, j) \in \hat{\boldsymbol{L}}$, where $\hat{\boldsymbol{L}}$ is a set of links with the obtainable traffic counts, one can build a system of n equations with m unknown variables (n – number of observed links, m – number of elements of the O-D matrix). Typically the number of unknown variables is greater than the number of equations – the system is underspecified (or degenerate) and may have many possible solutions. In such case the total number of O-D pairs can normally exceed the number of links for which traffic counts have been collected [1]. It means that there are many O-D matrices, which, when assigned to the network, reproduce the observed traffic counts – they give the same result. Therefore, searching for O-D matrix is equivalent to searching for a matrix satisfying the given boundary conditions and properly formulated optimization criteria. Often, to find a unique solution, one needs additional data such as in the form of so-called a priori or target matrix, obtained by other synthetic methods or a historical matrix (for an earlier time period).

Selecting the proper objective function is not simple. Usually, the function represents a measure of distance between the model solution ($x_{ij}(t)$ or $x^{ab}(t)$) and the real or assumed values ($\hat{x}_{ij}(t)$ or $\hat{x}^{ab}(t)$). During the estimation process the function is being minimized. Some methods are focused on differences between estimated demand flows $x^{ab}(t)$ and elements of a priori matrix $\hat{x}^{ab}(t)$. The distance between them may be expressed – in a general form – as a function $F_1\left(x^{ab}(t), \hat{x}^{ab}(t)\right)$. The observed set of traffic counts $\hat{x}_{ij}(t)$ may also be assumed to be very exact reproduction of traffic counts obtained as an assignment of the estimated O-D matrix. Hence, such values of demand flows $x^{ab}(t)$ are sought which give the smallest differences between the estimated link flows $x_{ij}(t)$ and the real ones $\hat{x}_{ij}(t)$. This assumption may be expressed as a criterion $F_2\left(x_{ij}(t), \hat{x}_{ij}(t)\right)$. Although the assumptions and concepts of various methods used to estimate the O-D matrix differ among each other, their generalized form may be as follows [1]:

$$F\left(x^{ab}(t), x_{ij}(t)\right) = \gamma_1 F_1\left(x^{ab}(t), \hat{x}^{ab}(t)\right) + \gamma_2 F_2\left(x_{ij}(t), \hat{x}_{ij}(t)\right) \tag{2}$$

where:

$\hat{x}^{ab}(t)$ – real or assumed values of O-D flows for $(a, b) \in \boldsymbol{E}$ over time t, element of a priori O-D matrix,

$\hat{x}_{ij}(t)$ – observed traffic counts on link $(i, j) \in \boldsymbol{L}$ over time t,

F_1, F_2 – distance measures in functional form,

γ_1, γ_2 – weights determining a degree of reliability of data used in the model.

If the a priori O-D matrix is more reliable, as it has been for example built based on reliable surveys and still contains largely valid data, then γ_1 should take much higher value compared to γ_2. It will make the modelled O-D matrix more convergent to the a priori matrix. In such situation, greater deviations between the traffic volumes obtained from the traffic assignment and observed ones are accepted. However, if the observed volumes are more reliable than the information contained in the a priori matrix, the value of γ_2 should be much greater than the parameter γ_1. Thus, the values of weights in question are closely related to the modelling approaches and assumptions.

4.2. Methods of O-D matrix estimation based on traffic counts

Taking the applied research technique into account, one can divide the methods of O-D matrix estimation based on traffic counts into three main categories as follows (Table 1) [1]:
- traffic modelling based approaches,
- statistical inference approaches,
- gradient based solution techniques.

One of the first models based on a concept of traffic modelling have been developed by Van Zuylen and Willumsen [51, 54]. They were based on the assumption of the proportional assignment, which does not take the degree of traffic congestion into account. It is Fisk [22] who has introduced the idea of extending the model of entropy by Van Zuylen and Willumsen. He took the traffic congestion into consideration by introducing the boundary conditions in the form of equilibrium assignment that satisfies the first principle of Wardrop [52]. Further development of such methods went mainly towards the use of both, the gravity methods and intervening opportunities to estimate the O-D matrix [50].

Table 1

General classification of methods of O-D matrix estimation based on traffic counts

RESEARCH TECHNIQUE		RESEARCHER
traffic modelling based approaches		• Van Zuylen, Willumsen (1980) • Fisk (1988, 1989) • Tamin, Willumsen (1989) • Sherali, Sivanandan, Hobeika (1994) • Sherali, Narayanan, Sivanandan (2003)
statistical inference approaches	**maximum likelihood method**	• Spiess (1987)
	generalized least squares method	• Cascetta (1984) • Bell (1991) • Bierlaire, Toint (1995)
	Bayesian inference	• Maher (1983)
gradient based solution techniques		• Spiess (1990) • Chen (1994)

Along with the development of technology these methods have undergone some modifications. Sherali [42] was one of those who worked on using a linear programming approach in estimating the O-D matrix based on the link traffic counts. Optimization of the objective function introduced in his model was to minimize the sum of travel costs and deviations between the assigned and observed values (traffic counts and the prior O-D matrix). In the following publications Sherali has adapted the model to a situation in which traffic volumes are not known on all links [40]. The solution can be determined only for certain, fixed points that were estimated heuristically by fitting iteratively a nonlinear model with a sequence of linear programming techniques. Sherali has also developed other models used in estimating the O-D matrix in the dynamic approach [41].

It is assumed for the methods using statistical inference that the link traffic counts are treated as realizations of independent Poisson random variables. Most commonly these models use techniques of maximum likelihood [45], a method of generalized least squares [7, 14], and Bayesian inference [31].

Gradient techniques are usually used in street networks of considerable size and are based on iterative correction of the a priori matrix according to the gradient of appropriately formulated objective function [44]. These models are not restricted only to the proportional assignment ones, but also they are equally well applicable with the equilibrium assignment.

The other important feature distinguishing the various models used to estimate the O-D matrix is a way of treatment of congestion in the estimation process. Typically, the congestion is included in a link cost function (often expressed in time units) that depends on traffic volumes. In the modelling process, the congestion can be treated exogenously (the proportional assignment) or endogenously (the equilibrium assignment). The first one is used in networks with no congestion or when the level of congestion is predictable. It is characterized by relative stability [1]. The latter should be considered in the networks with congestion, and depends on traffic conditions.

Most models use only the relation between the cost and traffic volumes that affect only the analysed link. In more general cases one also needs to take into account the influence of traffic volumes on the neighbouring links, as in the form of variance inequalities. Given these issues one can classify the methods of estimating the O-D matrix based on traffic counts by a structure and complexity of network and possibilities of path choice [18]. Thus, one can build various models for the following groups:

1. a simple road network with no choice of alternative routes – in these methods one can use information about the turning probabilities usually in form of relative size of turning volumes at intersections [5]. They heavily affect the accuracy of turning flow estimates and have been considered as one of the critical input;
2. a dense road network with a route choice, in which there is no traffic congestion (the network without congestion) – when estimating the matrix one assumes that trip distribution (proportional) and traffic assignment can be independent processes. The assignment may be estimated for instance based on the observed travel times or velocities and assumptions concerning the route choice. To determine the measures of convergence one can apply some of the aforementioned statistical methods (Table 1) [7, 12, 14, 31, 45, 51];
3. a dense road network with a route choice, in which there is severe traffic congestion (the network with congestion) – in these methods one cannot accept the assumption of proportional assignment, since the route choice and the O-D matrix estimation are interdependent. The process is iterative, which means that the results from the traffic assignment are used to estimate the O-D matrices. These

matrices are then assigned again on the network, revised, etc. [8]. Therefore, the other assumptions of the route choice model should be taken into account. One of the possible solutions to ensure this interaction is bi-level programming approach that is to determine the proportion of route choice while estimating the O-D matrix [22, 32, 55]. In this method, upper-level problem uses one of the previously mentioned statistical techniques (such as generalized method of least squares) to find the most appropriate O-D matrix, while the lower-level problem assigns traffic to the network endogenously (according to for example Deterministic User Equilibrium or Stochastic User Equilibrium) based on the results of earlier estimation. However, the bi-level programming issues can encounter difficulties with the computational complexity while estimating the O-D matrix for the networks of large sizes.

Nearly all models to estimate the O-D matrices based on the traffic counts use some prior information, which can be presented in the form of the a priori O-D matrix or total number of trips generating and absorbing in particular zones. The a priori matrix can be estimated by classical methods of modelling based on empirical research; this may also be a historical trip table (often no longer valid) [1]. Hence, there is the need to take the a priori matrix and its typology into account. This is yet another characteristic that can be used to divide the models into these where the a priori matrix is obligatory (as in [7, 31, 44, 45]) and these where it is facultative (as in [14, 22]).

In addition to those mentioned above, there are many other criteria of dividing the methods of estimating the O-D matrix. An example of such might be the degree of network elements aggregation (methods for aggregated and non-aggregated networks), the use of networks by various means of transportation (the method for unimodal and multimodal networks), size of the network (methods for large and small networks), or the location of the analysed zone (methods for urban and rural networks).

4.3. Generalized estimation of the O-D matrices based on traffic counts

The process of estimating the O-D matrix based on information about the traffic counts can be generalized and represented schematically as in Fig. 6. In the dynamic approach the analysed time period is divided into intervals during which the traffic counts are recorded. Hence, the presented estimating process should be carried out separately for each time interval due to the variability of demand over time.

The input data include a description of the network structure using graph theory and the O-D matrix structure as a set of O-D pairs. Due to the dynamic approach to the problem, values of the traffic counts in the following time intervals also become the input data. Additional data can be related to for example technical and traffic characteristics of nodes and links. In a real situation, a set of links $\hat{\boldsymbol{L}} \subset \boldsymbol{L}$ for which one can obtain the traffic counts is often much smaller than the set of all links $\boldsymbol{L}$.

Therefore, there are different methods to estimate the missing link flows [43]. The essential problem concerns also the proper traffic count locations [10, 21, 56].

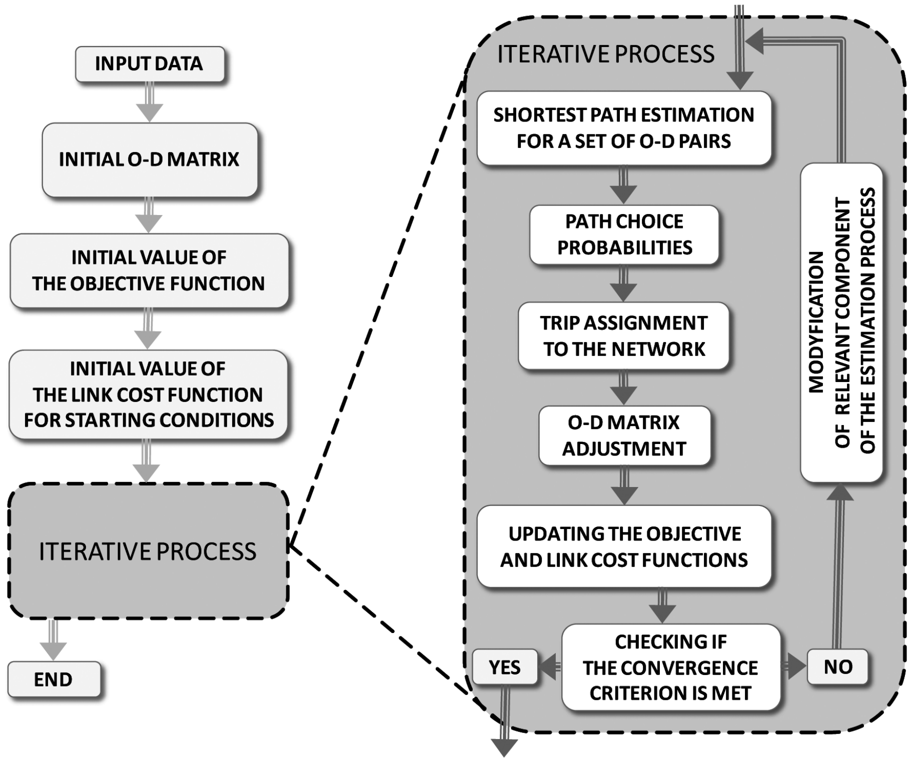

Fig. 6. Scheme of O-D matrix estimation based on traffic counts

Estimation of the initial O-D matrix depends mainly on the chosen estimation algorithm. It often includes values determined based on previous demand matrices (obtained empirically or analytically) or estimated from the observed traffic counts.

As it has already been mentioned, selecting the right objective function is not a simple task. Typically, it represents the measure of convergence of the obtained results and the real or assumed values of the O-D flows (formula 2). Mostly to ensure the highest degree of convergence one can use the following methods: least squares [2, 13, 14], maximum entropy [6, 51], or maximum likelihood [45]. The objective function is usually defined for all links, but it can also be determined only for a critical link (for instance with the largest deviations from the empirical values). The initial value of this function is often estimated based on the previous O-D matrix.

For each element of the network (such as links or nodes) one can assume a value of the cost function, which typically depends on variability of traffic counts over time. Such function represents the cost of travelling on this particular element of the network. The ways of its estimation can differ due to level of complexity –

from assuming some constant parameters (such as, so called, "the time penalties" for turning flows) to applying sophisticated methods (dynamic estimation of the delays for particular elements of the network).

The iterative process includes finding the shortest path and analysis of their choice probability for the particular O-D pairs. It also covers traffic assignment, correction of the O-D matrix and update of the objective function. One also needs to consider estimating the value of the link cost function based on new theoretical traffic volumes as well as to check the conditions of convergence achievement.

Algorithms of finding the shortest paths in the network are among the matters with the highest computational complexity, which increases with the size of the network. The time of estimation in the dynamic methods is very important. Despite the increasing technical possibilities of the IT tools one seeks the algorithms with the lowest computational complexity. For each of the O-D pairs the paths with a minimum temporal value of the link cost function are estimated. One needs then to determine the probability of path choice by users for the particular O-D pairs. In the dense networks for some O-D pairs one can find a few, a dozen or even more paths. A fraction of the demand flows are then assigned to the selected paths. Typically this process also is iterative. The paths for each O-D pair can be determined by one of the shortest (cheapest) path methods [27, 34, 47], taking travel times or costs in form of properly expressed link cost function as a temporal objective function to minimize.

The choice of a proper method of traffic assignment depends on the purpose of research, problem specific character as well as traffic conditions and organization. At this stage one can go for more or less complex models applied both in the statistical and dynamical methods. The problem of the dynamic traffic assignment (DTA) to the network by determining the time-varying traffic volumes can be solved both analytically [16] and by using simulation tools [33, 57].

The optimal instantaneous traffic assignment expressed in the form of a matrix $\left[x_{ij}(t)\right]^*$ is the one that follows the formula:

$$\left[x_{ij}(t)\right]^* = \arg\min_{x_{ij}(t)} F\left(x^{ab}(t), x_{ij}(t)\right) \tag{3}$$

After establishing a model traffic assignment to the network, the correction of the O-D matrix takes place. Then one calls for the update of the objective function and estimating the value of the link cost function based on a new volume of traffic. These changes are necessary to evaluate the temporal objective function and to achieve the assumed convergence. If the degree of convergence satisfies the established conditions then the estimation can be completed. Otherwise, one needs to correct a method of determining the trip distribution model. This may involve a change of:

- values of parameters or of the functional form of the link cost function,
- method to find the shortest paths in the network,
- the traffic assignment method,

- the network structure,
- other factors affecting the obtained solution.

The estimating process is repeated for the next time period for which the values of the traffic counts have been observed.

5. Concept of the Method to Estimate the O-D Matrix Using Traffic Counts

The proposed method is consistent with the scheme presented in Fig. 7. Dynamic character of this approach lies in the fact that elements of O-D matrix are estimated mainly on the basis of trip tables determined for the previous time intervals.

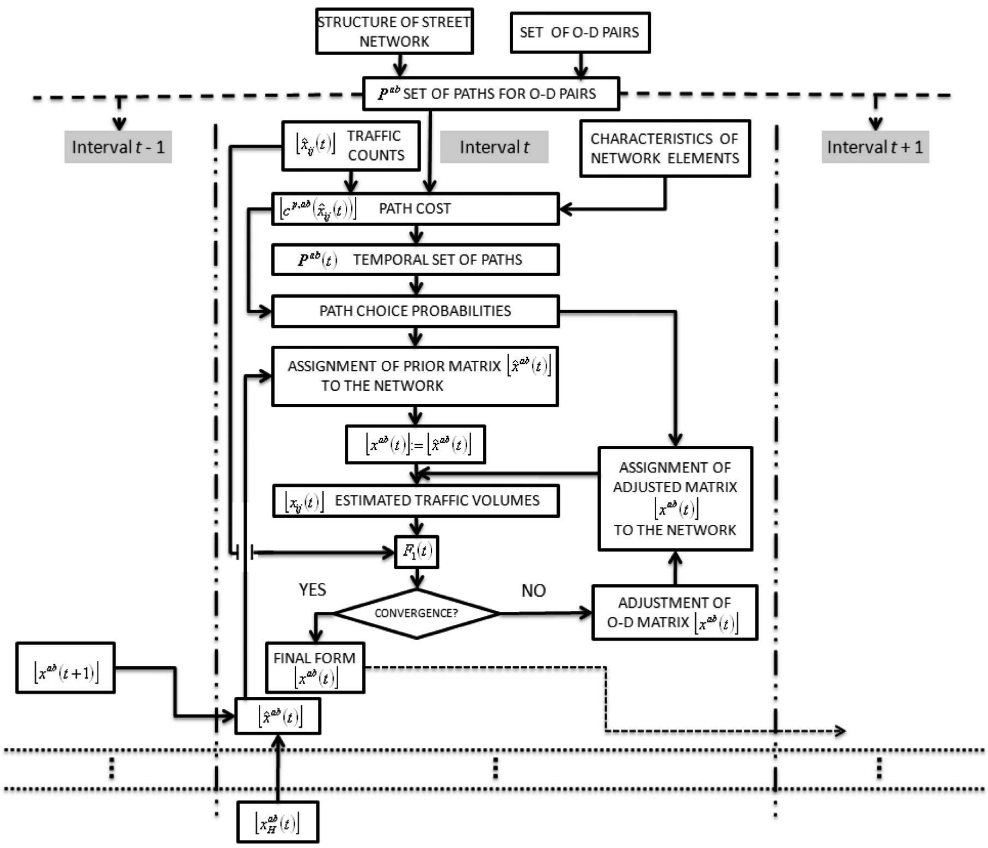

Fig. 7. Scheme of proposed method to estimate O-D matrix based on traffic counts

The method requires, among others, the knowledge of:

- the structure of the road network in the form of a directed graph,
- characteristics of the network elements (such as capacity, velocity, etc.),

- traffic counts for all links of the analysed network in the following time intervals,
- the structure of the O-D matrix as a set of O-D pairs,
- elements of the prior O-D matrices obtained on the basis of trip tables estimated over previous intervals.

Following are the notations – in alphabetical order – used to express the short outline of the method described in this section.

(a, b)	–	O-D pair, $(a, b) \in \boldsymbol{E}$,
$c_{ij}\left(\hat{x}_{ij}(t)\right)$	–	generalized cost of travelling on the link $(i, j) \in \boldsymbol{L}^{\mathbf{p,ab}}$ over time t,
$c^{p,ab}\left(\hat{x}_{ij}(t)\right)$	–	generalized cost of travelling between a and b on the p-th path, $p \in \boldsymbol{P}^{\mathbf{ab}}$,
$\boldsymbol{E}$	–	set of O-D pairs,
$E_{ij}(t)$	–	travel demand components of traffic volumes on link $(i, j) \in \boldsymbol{L}$ over time t,
(i, j)	–	link within the street network, $(i, j) \in \boldsymbol{L}$,
$\boldsymbol{L}$	–	set of links within the street network,
$\boldsymbol{L}^{\mathbf{p,ab}}$	–	set of links between $(a, b) \in \boldsymbol{E}$, belonging to the p-th path, $\mathbf{p} \in P^{ab}$, $L^{p,ab} \in \boldsymbol{L}$,
l_{cr}	–	critical link, $l_{cr} \in \boldsymbol{L}$,
P^{ab}	–	set of paths for O-D pair $(a, b) \in \boldsymbol{E}$,
$P^{ab}(t)$	–	set of paths for O-D pair $(a, b) \in \boldsymbol{E}$ over time t,
p^{*}	–	optimal path for the pair $(a, b) \in \boldsymbol{E}$,
R	–	the car occupancy factor,
T	–	reference period,
t	–	time interval, $t \in T$,
$x^{ab}(t)$	–	estimated O-D flows for $(a, b) \in \boldsymbol{E}$ over time t,
$x_H^{ab}(t)$	–	historical O-D flows for $(a, b) \in \boldsymbol{E}$ over time t,
$x_{ij}^{ab}(t)$	–	estimated O-D flows for $(a, b) \in \boldsymbol{E}$ on link $(i, j) \in \boldsymbol{L}$ over time t,
$\hat{x}^{ab}(t)$	–	real or assumed values of O-D flows for $(a, b) \in \boldsymbol{E}$ over time t, element of a priori O-D matrix,
$x_{ij}(t)$	–	estimated traffic volumes on link $(i, j) \in \boldsymbol{L}$ over time t,
$\hat{x}_{ij}(t)$	–	observed traffic counts on link $(i, j) \in \boldsymbol{L}$ over time t,
β_{gr}^{ab}	–	assumed critical value of coefficient $\beta^{p,ab}(t)$,
$\beta^{p,ab}(t)$	–	coefficient of increase in the generalized cost of travelling between a and b on the p-th path, $p \in \boldsymbol{P}^{\mathbf{ab}}$,
$\delta_{ij}^{ab}(t)$	–	fraction of trips for O-D pair $(a, b) \in \boldsymbol{E}$ that uses link $(i, j) \in \boldsymbol{L}$ over time t,
ε_1	–	accuracy factor.

The analysed reference period T has been divided into intervals t. For the current version for each O-D pair $(a, b) \in \boldsymbol{E}$, one can establish any number of paths.

The set of paths determined by the set of O-D pairs and the graph representing the network structure is initially the same for all time intervals. Only in the later stage it will be limited to the most rational alternatives in particular intervals.

The main assumption is that the users choose the optimal route according to a particular criterion or set of criteria, taking into account the current traffic volumes $\hat{x}_{ij}(t)$. Such selection is based on travel costs on particular paths. The generalized cost of travelling between a and b on the p-th path can be written as:

$$c^{p,ab}\left(\hat{x}_{ij}(t)\right) = \sum_{(i,j)\in L^{p,ab}} c_{ij}\left(\hat{x}_{ij}(t)\right) \tag{4}$$

However, this relation is satisfied providing that the generalized travel cost covers the characteristics that are additive.

In dense street network there is a significant number of paths for many pairs $(a, b) \in \boldsymbol{E}$. Thus, the temporal coefficient $\beta^{p,ab}(t)$ is being analysed for each path $p \in \boldsymbol{P}^{\mathbf{ab}}$ that $(a, b) \in \boldsymbol{E}$. Such coefficient puts a limit on the number of analysed paths. It describes the degree of increase in the generalized cost of travelling on the path to the generalized cost of travelling on the optimal path p^*, as it is shown in the following formula:

$$\beta^{p,ab}(t) = \frac{c^{p,ab}\left(\hat{x}_{ij}(t)\right)}{c^{p^*,ab}\left(\hat{x}_{ij}(t)\right)} \tag{5}$$

where the optimal path p^* is specified for each pair $(a, b) \in \boldsymbol{E}$ as:

$$p^* : c^{p^*,ab}\left(\hat{x}_{ij}(t)\right) = \min_{p\in P^{ab}}\left\{ c^{p,ab}\left(\hat{x}_{ij}(t)\right)\right\} \tag{6}$$

The probability of choosing the particular path by the users decreases with the increase of values of the coefficient $\beta^{p,ab}(t)$. Some paths $p \in \boldsymbol{P}^{\mathbf{ab}}$ can not be selected within a specified time interval at all. Thus, number of paths for each pair $(a, b) \in \boldsymbol{E}$ may vary in the following intervals t. This can be written in the form of a set of temporal paths $\boldsymbol{P}^{ab}(t)$. Assuming a critical value β^{ab}_{gr} for each pair $(a, b) \in \boldsymbol{E}$ one can formulate a condition of joining the p-path into the set $\boldsymbol{P}^{\mathbf{ab}}(t)$ as:

$$\boldsymbol{P}^{ab}(t) = \left\{p : \beta^{p,ab}(t) < \beta^{ab}_{gr},\ p \in \boldsymbol{P}^{ab}, (a,b) \in \boldsymbol{E}\right\} \tag{7}$$

Paths, for which this condition is not met are rejected in a given interval t and are not consider in the traffic assignment.

The simplified form of probability of estimated temporal paths' choice for the pair $(a, b) \in \boldsymbol{E}$ based on a selected criterion, such as the generalized travel cost, may be estimated under assumption that, before making the trip, the user chooses a sequence of road segments to follow toward the destination. In practice, various random utility models can be used to obtain such probability [59]. The most popular of them belong to the logit or probit models.

In this version of method the a priori O-D matrix $\left[\hat{x}^{ab}(t)\right]$ is determined on the basis of both historical O-D matrix $\left[x_H^{ab}(t)\right]$ and trip table $\left[x^{ab}(t-1)\right]$ estimated over the previous time interval. Values of historical O-D matrix elements depend on the estimated matrix for the same period of day on other days (especially on the same day of the week – this figure may have greater weight). Determined a priori O-D matrix $\left[\hat{x}^{ab}(t)\right]$ is then assigned to the network in accordance with the path choice probability distribution specified earlier. The values of traffic volumes $x_{ij}(t)$ on link $(i, j) \in \boldsymbol{L}$ over time t are results of this process. Thus, the prior O-D matrix $\left[\hat{x}^{ab}(t)\right]$ becomes the first estimate of O-D matrix $\left[x^{ab}(t)\right]$.

The sum of the absolute values of deviations between the observed $\hat{x}_{ij}(t)$ and assigned flows $x_{ij}(t)$ is the objective function $F_1(t)$. The function is being calculated for each time interval t. Therefore, it can be written as:

$$F_1(t) = \sum_{(i,j)\in L} \left|\hat{x}_{ij}(t) - x_{ij}(t)\right| \tag{8}$$

The value of the objective function $F_1(t)$ is compared with the accuracy factor ε_1, which being the measure of convergence determines the acceptable level of deviation. This level may be determined depending on the aim of modelling and the degree of data aggregation. If the value of the objective function $F_1(t)$ is bigger than the assumed accuracy factor ε_1, one must modify the elements of a priori O-D matrix $\hat{x}^{ab}(t)$. It is iterative process that involves the following stages:

1) identification of a critical link in street network.

A critical link appears when an absolute deviation of the assigned flow value from the measured one reaches the highest value, as it is shown in the following formula:

$$l_{cr} : \left|\hat{x}_{l_{cr}}(t) - x_{l_{cr}}(t)\right| = \max_{(i,j)\in L} \left|\hat{x}_{ij}(t) - x_{ij}(t)\right| \tag{9}$$

2) specification of travel demand components of traffic volumes $x_{l_{cr}}(t)$ on a critical link.

   The travel demand components create a set $\boldsymbol{E}_{l_{cr}}(t) \subset \boldsymbol{E}$, determined as:

$$\boldsymbol{E}_{l_{cr}}(t) = \left\{(a,b) : \sum_{(a,b)} x_{l_{cr}}^{ab}(t) = x_{l_{cr}}(t),\ \ l_{cr} \in \boldsymbol{L},\ (a,b) \in \boldsymbol{E}\right\} \tag{10}$$

   where estimated O-D flows $x_{l_{cr}}^{ab}(t)$ for $(a, b) \in \boldsymbol{E}$ on a critical link $l_{cr} \in \boldsymbol{L}$ over time t may be calculated, according to the formula (1), as:

$$x_{l_{cr}}^{ab}(t) = \frac{1}{R}\delta_{l_{cr}}^{ab}(t) \cdot x^{ab}(t) \tag{11}$$

3) adjustment of estimated O-D matrix $\left[x^{ab}(t)\right]$.

The current version of the method assumes that the process of adjustment is applied only to the elements of a set $\boldsymbol{E}_{l_{cr}}(t) \subset \boldsymbol{E}$. Thus, some values of estimated O-D matrix $\left[x^{ab}(t)\right]$ are changed in such a way as to equalize the values of estimated traffic volumes $x_{l_{cr}}(t)$ on a critical link and values of traffic counts $\hat{x}_{l_{cr}}(t)$ for this link. The distance between values of estimated O-D matrix $\left[x^{ab}(t)\right]$ and the prior matrix $\left[\hat{x}^{ab}(t)\right]$ for O-D pairs belonging to the set $\boldsymbol{E}_{l_{cr}}(t) \subset \boldsymbol{E}$ may be taken into consideration as well.

4) assignment of adjusted O-D matrix $\left[x^{ab}(t)\right]$ to the network.
This process is carried out in accordance with the previously estimated path choice probability distribution. It leads to estimation of new values of traffic volumes $x_{ij}(t)$ and re-calculation of the objective function $F_1(t)$.

Iterative process is repeated until a satisfactory convergence criterion is met. This is equivalent to achieving such a value of the objective function $F_1(t)$ that is equal or less than accuracy factor ε_1. The estimated O-D matrix $\left[x^{ab}(t)\right]$, which resulted in obtaining such value, is regarded as a final form and may be used to estimate trip table $\left[x^{ab}(t+1)\right]$ over the next time interval. The optimization process is to minimize the objective function for each time interval t. This is to obtain such an O-D matrix (for each interval) that after assigning to the network gives the volumes with the highest degree of convergence with the observed results.

6. Conclusions

The article deals with the overview of methods of estimating the O-D matrix in dense street networks, including their specific characteristics and application. A particular attention has been paid to estimating the O-D matrix based on the traffic counts. Due to the possibility of applying to the dynamic problems, such methods can be used in effective road and traffic management.

A review of the presented methods of estimating the trip distribution indicates the variety of theories used for this purpose. One needs to keep in mind that choosing the right one depends on several factors, dependent or independent from each other. The most important among others would include:

- the structure and type of input data as well as their degree of aggregation,
- limitations and requirements of the particular methods,
- the structure and degree of congestion in the transportation network,
- the purpose of modelling and its practical application,
- dynamics of the transportation system development in the analysed area,
- accessibility of IT tools.

A schematic approach to the components of methods used to estimate the O-D matrix will help to isolate issues directly related to the computational process that affect the accuracy of obtained matrices, and their convergence with a real situation. Depending on the sub-models used, better or worse results may be expected.

A described here concept of a method used to estimate the O-D matrix in the dynamic approach has a modular structure. At first the simple sub-models have been used. Applying more sophisticated procedures requires additional research. The aim of the method is its high flexibility, which helps to verify the results obtained by different (more or less complex) sub-models and their combinations.

Further studies should focus on a development of various components. They should also analyse the influence of particular sub-model application on the convergence of the obtained results with the actual data. The model can also be a part of more complex management system that uses information about the current network volumes to control traffic. After performing a suitable modification some procedures could be also useful to the forecasting models and real-time navigation systems.

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