Accuracy assessment of GOCE-based geopotential models and their use for modelling the gravimetric quasigeoid – A case study for Poland

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Abstract: The GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) has significantly upgraded the knowledge on the Earth gravity field. In this contribution the accuracy of height anomalies determined from Global Geopotential Models (GGMs) based on approximately 27 months GOCE satellite gravity gradiometry (SGG) data have been assessed over Poland using three sets of precise GNSS/levelling data. The fits of height anomalies obtained from 4th release GOCE-based GGMs to GNSS/levelling data were discussed and compared with the respective ones of 3rd release GOCE-based GGMs and the EGM08.

Furthermore, two highly accurate gravimetric quasigeoid models were developed over the area of Poland using high resolution Faye gravity anomalies. In the first, the GOCE-based GGM was used as a reference geopotential model, and in the second – the EGM08. They were evaluated with GNSS/levelling data and their accuracy performance was assessed. The use of GOCE-based GGMs for recovering the long-wavelength gravity signal in gravimetric quasigeoid modelling was discussed.

1. Introduction

During the last decade, the dedicated satellite gravity field missions, the CHAMP (CHAllenging Minisatellite Payload), GRACE (Gravity Recovery And Climate Experiment) and GOCE (Gravity field and steady-state Ocean Circulation Explorer) have significantly upgraded the modelling of the Earth gravity field and its temporal

variations by several orders of magnitude. A substantial improvement in modelling gravity in the long/medium wavelengths, e.g. up to degree and order (d/o) 200–220, and thereby the geoid has been achieved, for example, a considerable improvement of the gravity field has been observed from GOCE satellite mission data on a regional scale, e.g. Africa, Antarctica, South America, South-East Asia (Yi and Rummel, 2014), and on a local scale e.g. in Brazil (Guimaraes et. al., 2012), in southern Norway (Mysen, 2013) and in Japan (Odera and Fukuda, 2013). In March 2009, the GOCE being a core satellite mission of the European Space Agency (ESA) Living Planet Program (ESA, 1999) has been successfully launched. The objective of the mission was to provide a static global geopotential model enabling to determine geoid with 1–2 cm accuracy and gravity anomaly with 1 mGal accuracy for a spatial resolution of about 100 km (half wavelength) corresponding to d/o 200 (Drinkwater et al., 2003). Four generations of Global Geopotential Models (GGMs) based on GOCE data have been developed: GGMs based on ~2 months (1st release), ~8 months (2nd release), ~12 months (3rd release) and ~27 months (4th release) of effective GOCE data volume. Three different strategies were applied by ESA's GOCE High Level Processing Facility (HPF) for the determination of the Earth's gravity field models. They are denoted as the direct solution, time-wise solution and space-wise (only for 1st and 2nd release) solution (Rummel et al., 2004; Pail et al., 2011). In addition to ESA's solutions, models based on a combination of GOCE data and the complementary gravity field information from other satellites and terrestrial data were also developed by other institutions.

In order to assess the accuracy performance of GOCE-based GGMs, appropriate gravity field functionals should be evaluated. The accepted methods for estimating the accuracy of GOCE-based GGMs on a global scale are based on the comparison of gravity functionals determined from GOCE data with the corresponding ones obtained from the EGM08 (e.g. Hirt et al., 2011; Gruber et al., 2011; Pail et al., 2011; Yi and Rummel, 2014). On a local scale, the ground truth data (i.e. free-air gravity anomalies, GNSS/levelling data, deflections of the vertical) are mainly employed to estimate the accuracy of GOCE-based GGMs using numerous procedures, e.g. the Gauss' filter (Voigt et al., 2010), inverse distance weight filter (Godah and Krynski, 2013b) and spectral enhancement method (Gruber, 2009; Hirt et al., 2011). Over all, the spectral enhancement method based on the use of a high resolution GGM, e.g. the EGM08 (Pavlis et al., 2012), for compensating the medium/short wavelength gravity signal beyond the maximum d/o of GOCE-based GGMs is widely used as an acceptable method to estimate the accuracy of GOCE-based GGMs. However, since the performance of the EGM08 is varying from place to place and its accuracy depends on the quality of terrestrial data that have been included when developing this model (Pavlis et al., 2012), the assessed accuracy of GOCE-based GGMs with the use of the EGM08 on a global or local scale requires ensuring that the gravity functionals computed from the EGM08 are sufficiently accurate over the evaluation area. In the recent studies (e.g. Gruber et al., 2013; Rexer et al., 2014; Yi and Rummel, 2014) the accuracy of height anomalies obtained from the latest GOCE-based GGMs

is estimated by comparing them with the corresponding ones determined from the EGM08 truncated at the same d/o as well as with the corresponding ones obtained from GNSS/levelling data after removing the gravity signal beyond the applied maximum d/o of GOCE-based GGMs using the EGM08. They have indicated that the accuracy of height anomalies determined from 4th release GOCE-based GGMs at d/o 200 is at the level of 3–5 cm in the areas with the high performance of the EGM08 such as Europe, North America and Australia (see Pavlis et al., 2012).

The main aim of this contribution is to reliably assess the accuracy of height anomalies obtained from GOCE-based GGMs as well as to study the use of those GGMs for modelling the gravimetric quasigeoid. In particular, the accuracy assessment of GGMs based on approximately 27 months (4th release) GOCE satellite gravity gradiometry (SGG) data over an area where accurate terrestrial gravity data and GNSS/levelling data as well as the high performance of the EGM08 are available has been discussed. Two investigations have been conducted. In the first one, the height anomalies determined from GOCE-based GGMs as well as from GOCE-based GGMs extended with the EGM08 coefficients were compared with the corresponding ones obtained from GNSS/levelling data. In the second investigation, the highly accurate gravimetric quasigeoid model based on the GOCE-based GGM and terrestrial gravity data for the area investigated has been developed and compared with GNSS/levelling data.

The descriptions of the chosen study area and data used are given in the section 2. In sections 3 and 4, the evaluation methodologies are specified and their results are analysed. In the section 5, the conclusions concerning the possibility of using GOCE-based GGMs when computing gravimetric quasigeoid models using remove-compute-restore (RCR) procedure are drawn.

2. Study area and data used

The area of Poland has been selected as a case study area. It seems specifically suitable for the accuracy assessment of GOCE-based GGMs due to the availability of high-precision quasigeoid model (accuracy below 2 cm) (e.g. Krynski, 2007) and high-precision GNSS/levelling data distributed homogeneously as well as accurate and dense terrestrial gravity data. In addition, since a grid of 5'×5' terrestrial free-air gravity anomalies from Poland has been included when developing the EGM08, the quasigeoid model represented by the EGM08 performs almost as the existing quasigeoid model in Poland (Krynski and Kloch, 2009; Lyszkowicz, 2009). The data sets used throughout the computation and the evaluation are described in the sections 2.1–2.3.

2.1. Terrestrial gravity data

A grid of 1.5'×3' Faye gravity anomalies for the area bounded by the parallels of 48°N and 56°N and the meridians of 12°E and 26°E has been used in the computation of the gravimetric quasigeoid model. For the area within the boundary of Poland a new set of mean 1.5'×3' Faye gravity anomalies have been generated using almost 1 000 000 point gravity values from the Polish Geological Institute (Królikowski, 2006) unified and reprocessed in the Institute of Geodesy and Cartography within the grant PBZ-KBN-081/T12/2002 supported by the Polish State Committee for Scientific Research (Krynski, 2007). The terrain corrections were calculated using DTED2 and SRTM3 height data. The 1.5'×3' free-air gravity anomaly grid for neighbouring countries was obtained on the basis of different kinds of gravity data collected, developed and made accessible for geoid modelling by the Department of the Planetary Geodesy of the Space Research Center of the Polish Academy of Sciences (Lyszkowicz, 1994), i.e. the mean 5'×7.5' free-air anomalies for the areas of Ukraine, Czech, Slovakia, Hungary and Romania, the mean 5'×5' and 2'×3' free-air anomalies for Germany, the mean Bouguer and free-air gravity anomalies and heights in the 8 km × 8 km grid for Belarus, Ukraine and Lithuania and the 8 km × 8 km gravity data set for the whole area of interest obtained from Leeds. For preparing a new set of 1.5'×3' gravity anomalies for area surrounding Poland also the data from the geophysical marine missions and airborne gravimetry for the Baltic Sea (Krynski, 2007) were used. A map of Faye gravity anomalies over Poland and the neighbouring areas is given in Figure 1.
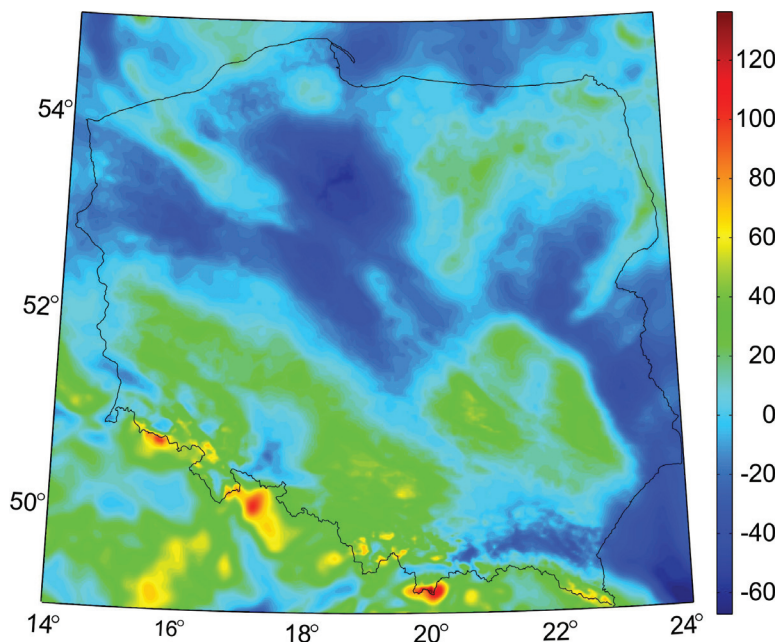

Fig. 1. Faye gravity anomalies for the area of Poland and adjacent areas [mGal]

2.2. GOCE-based Global Geopotential Models (GGMs)

Several GGMs based on GOCE satellite mission data have been released during the past few years. The main differences among those GGMs are the observation period, type of data used and the modelling procedure. In this study, recent two satellite-only GOCE-based GGMs, i.e. GO_CONS_GCF_2_TIM_R4 (TIM-R4 GGM) and GO_CONS_GCF_2_DIR_R4 (DIR-R4 GGM) have been investigated. The chosen models were developed and released for the public use by ESA. The TIM-R4 GGM is distinguished as a GOCE-only model in a rigorous sense, i.e. no external gravity field information is used, neither as a reference model, nor for constraining the solution (Pail et al., 2011), while the DIR-R4 GGM is a satellite-only model based on a combination of GOCE data together with GRACE and LAGEOS data (Bruinsma et al., 2013). Both models have shown high performance worldwide (Yi and Rummel, 2014). The height anomaly cumulated error of the DIR-R4 GGM at degree and order (d/o) 200 is about 1 cm, which indicates an improvement of about 60% with respect to its previous 3rd release, and it is about 3.2 cm for the TIM-R4 GGM. In local areas, both models have also shown significant improvement with respect to those of previous release, for example, the height anomaly determined using the TIM-R4 GGM at d/o 200 fits to the GNSS/levelling data within 4.5 cm in Germany and 10 cm in Japan (Gruber et al., 2013). Further information concerning these GGMs can also be found in the ESA's web page https://earth.esa.int/ and in the International Centre for Global Earth Models (ICGEMs) website http://icgem.gfz-potsdam.de/ICGEM/ ICGEM. The main characteristics of these models are summarized in Table 1.

Table 1. The main characteristics of the GOCE-based GGMs used

GGM	DIR-R4 GGM	TIM-R4 GGM
Name in ICGEM	GO_CONS_GCF_2_DIR_R4	GO_CONS_GCF_2_TIM_R4
Maximum degree/order (d/o)	260	250
Semi-major axis a [m]	6378136.46	6378136.30
GOCE GPS-SST data	d/o 100 ~28 months	d/o 130 ~26.5 months
GOCE SGG data	d/o 260 ~28 months	d/o 250 ~26.5 months
GRGS/CNES GRACE release 2	d/o 54 ~7 years	-
GFZ GRACE release 05	from d/o 55 to 180 9 years	-
LAGEOS-1/2 SLR data	d/o 3 ~25 years	-
Kaula regularization constraints beyond d/o	200	180
Time of releasing	March 2013	March 2013

2.3. GNSS/levelling data

The height anomalies obtained from GNSS/levelling data at 315 POLREF and 58 EUVN sites were used for the validation of quasigeoid models. Their accuracy is estimated to be 3–4 cm for POLREF sites and 2 cm for EUVN sites (e.g. Krynski, 2007, p. 46). In addition, the height anomalies at 184 sites of 868 km long GNSS/ levelling control traverse (Fig. 2) (ibid., pp. 150–163), established in 2003–2004 for verification and accuracy estimation of quasigeoid models in Poland as well as for evaluation of the interpolation algorithms used for application of GNSS/levelling quasigeoid models, were used. Based on the length of GNSS observation session and applied strategy of data processing, 44 stations of the GNSS/levelling control traverse have been classified as 1st order stations and 140 stations as 2nd order stations. The accuracy of height anomalies for either 1st or 2nd order stations is estimated to 1–2 cm (ibid., p. 47).
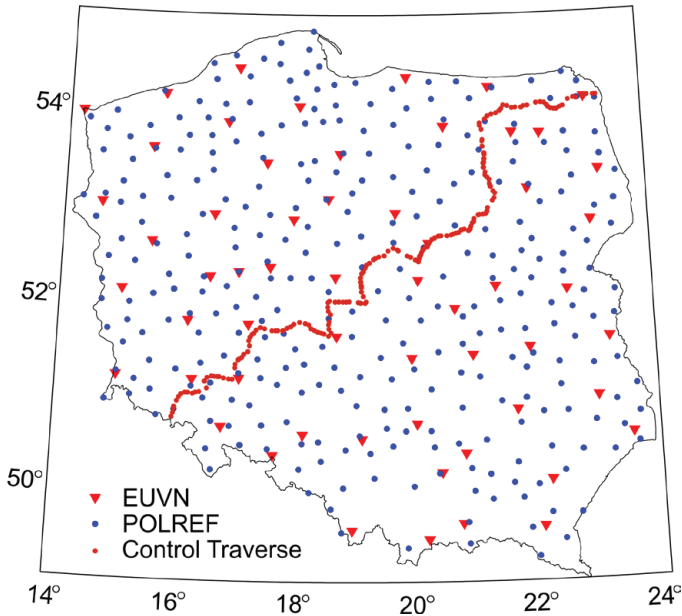

Fig. 2. POLREF, EUVN and the GNSS/levelling control traverse sites

3. Accuracy assessment of height anomalies determined from GOCE-based GGMs

The accuracy of height anomalies determined from GOCE-based GGMs is assessed with GNSS/levelling data. The height anomalies ζ_{GGM} can be calculated from GGMs as follows (Torge and Muller, 2012)

$$\zeta_{GGM} = \zeta_0 + \frac{GM}{a\gamma} \sum_{n=2}^{N_{max}} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} Y_{nm}(\varphi, \lambda) \tag{1}$$

with $Y_{nm}(\varphi, \lambda)$ – the spherical harmonic function given by

$$Y_{nm}(\varphi, \lambda) = (\Delta \overline{C}_{nm} \cos m\lambda + \Delta \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm}(\sin \varphi) \tag{2}$$

where GM is the product of the Newtonian gravitational constant G and mass of the Earth M, a is the semi-major axis of the reference ellipsoid, $\overline{P}_{nm}(\sin \varphi)$ are fully normalized associated Legendre functions of degree n and order m, $\Delta \overline{C}_{nm}$ are differences between fully normalised spherical harmonic coefficients of actual and the normal gravity field and $\Delta \overline{S}_{nm} = \overline{S}_{nm}$, r, φ, λ are the geocentric coordinates of the computation point P on the physical surface of the Earth, N_{max} is the applied maximum degree of geopotential model, γ is the normal gravity at the computation point. The additive constant ζ_0 is a bias determined as follows (Heiskanen and Moritz, 1967):

$$\zeta_0 = \frac{GM - GM_0}{R\gamma} - \frac{W_0 - U_0}{\gamma} \tag{3}$$

where M_0 is the mass of the reference ellipsoid, U_0 is the gravity potential of the ellipsoid and R is the mean radius of the reference ellipsoid. The values of these parameters are related to the Geodetic Reference System 1980 (Moritz, 2000). On the other hand, W_0 is the gravity potential of the Earth which together with M are numerical standards of the International Earth Rotation and Reference Systems Service Conventions (McCarthy and Petit, 2004).

Using Eq. (1) height anomalies ζ_{GGM} have been computed at each site of the EUVN and POLREF networks as well as the GNSS/levelling control traverse. The height anomalies differences

$$\Delta\zeta_1 = \zeta_{GNSS/levelling} - \zeta_{GGM} \tag{4}$$

between $\zeta_{GNSS/levelling}$ obtained from GNSS/levelling data and ζ_{GGM} determined from GOCE-based GGMs were computed. In order to make height anomalies in Eq. (4) spectrally compatible, the spectral enhancement method is used, of which the medium/short wavelength gravity signal beyond the applied maximum degree N_{max} is compensated from the EGM08 coefficients up to d/o 2190. It should be mentioned that the influence of the very short wavelength gravity signal (e.g. from d/o 2190 onward) in terms of the standard deviation of height anomalies differences does not exceed 3 mm for the POLREF, EUVN networks and the GNSS/levelling control traverse (see Section 4.3), and is neglected in this evaluation. The height anomalies differences $\Delta\zeta_2$ were thus obtained after compensating the medium/short wavelength gravity signal as follows:

$$\Delta\zeta_2 = \zeta_{GNSS/levelling} - \zeta_{GGM} - \zeta_h \tag{5}$$

With

$$\zeta_h = \frac{GM}{a\gamma} \sum_{n=N_{max}+1}^{2190} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} Y_{nm}(\varphi, \lambda) \tag{6}$$

Figure 3 illustrates the standard deviation values of the differences $\Delta\zeta_1$ and $\Delta\zeta_2$ using 4th release GOCE-based GGMs with d/o from 100 to 250/260 with 10 d/o step (*the applied maximum degree N_{max} = 100, 110, 120,..., 250, 260*).
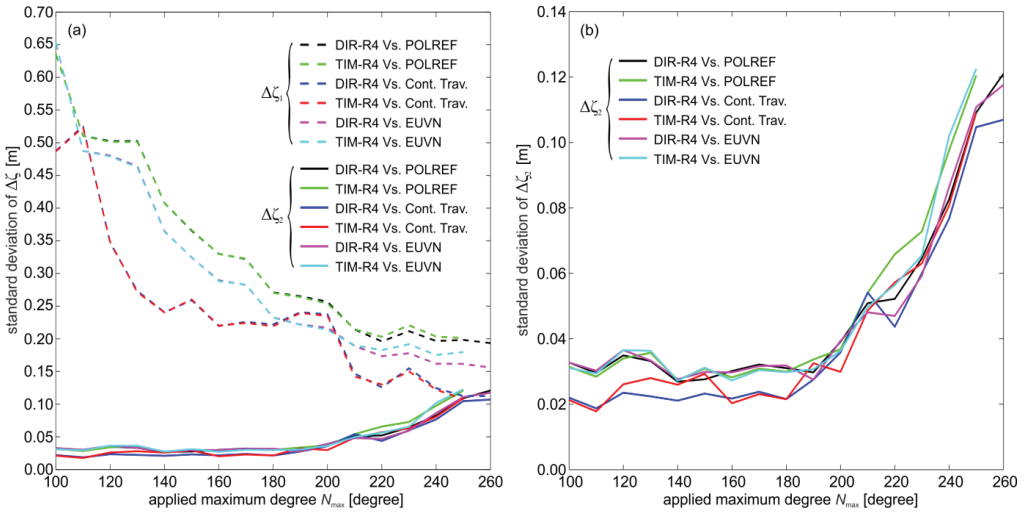

Fig. 3. (a) Standard deviation of differences between height anomalies obtained from GNSS/levelling data and from the GOCE-based GGMs (*the applied maximum degree N_{max} = 100, 110, 120,..., 250, 260*); dashed lines refer to $\Delta\zeta_1$ – only GOCE-based GGMs, solid lines refer to $\Delta\zeta_2$ – GOCE-based GGMs extended with the EGM08 coefficients and (b) Zoom into standard deviation of $\Delta\zeta_2$

The results presented in Figure 3 show consistency for both 4th release GOCE-based GGMs investigated. The discrepancy between the fits of GOCE-based GGMs to GNSS/levelling data (dashed lines in Fig. 3) results from the differences in the distribution of sites (Fig. 2), their number (Table 2), and accuracy of height anomalies in POLREF, EUVN, and control traverse data sets. A considerable reduction of standard deviations of height anomalies differences after compensating the medium/short wavelength gravity signal using the EGM08 is observed. The height anomalies determined from GOCE-based GGMs in terms of standard deviation of differences fit to GNSS/levelling data at the level of 11–20 cm at the maximum d/o considered (260) and 22–26 cm at d/o 200. It shows that GOCE observables provide also valuable information for modelling the gravity field beyond d/o 200 corresponding to spatial resolution specified in the objectives of GOCE mission. When considering the medium/short wavelength gravity signal the standard deviations of height anomalies

differences remain almost constant (about 2–4 cm) up to d/o 200, which reflects the suitability of using GOCE data only (as in TIM-R4 GGM) in modelling the gravity field in the spectral range from d/o 100 to 200. Beyond d/o 200 they start explicitly growing up and reach 12 cm at the maximum d/o considered (260). This may indicate reasonably large commission error of those coefficients since they were estimated with the use of Kaula rule (see Table 1) as well as the signal noise is expected to become higher at spectral bands beyond d/o 200 (Rummel, 2010).

Additionally, in a similar way as in Eq. (5), the previous 3rd release of direct and time-wise GGMs (DIR-R3 and TIM-R3) and the EGM08 have also been evaluated at d/o 200 with the corresponding GNSS/levelling data. Table 2 shows the statistics of the obtained differences.

Table. 2. Statistics of height anomalies differences between obtained from GNSS/levelling data and the corresponding ones determined from the EGM08 (up to d/o 2190) and recent developed GOCE-based GGMs (truncated at 200 d/o) extended with the EGM08 (from d/o 201 to 2190) [m]

GNSS/levelling sites	Statistics	EGM08	3rd release GOCE-based		4th release GOCE-based	
			TIM	DIR	TIM	DIR
POLREF 315 sites	Min	0.007	−0.160	−0.177	−0.036	−0.035
	Max	0.170	0.352	0.329	0.207	0.209
	Mean	0.100	0.089	0.090	0.093	0.095
	Std dev.	0.027	0.084	0.091	0.037	0.039
EUVN 58 sites	Min	0.019	−0.071	−0.126	−0.007	0.025
	Max	0.173	0.256	0.278	0.168	0.191
	Mean	0.092	0.089	0.095	0.088	0.090
	Std dev.	0.028	0.079	0.085	0.036	0.039
Traverse (1st+2nd order) 184 sites	Min	0.027	−0.172	−0.212	−0.025	−0.023
	Max	0.126	0.284	0.282	0.122	0.136
	Mean	0.075	0.090	0.088	0.055	0.057
	Std dev.	0.022	0.101	0.105	0.030	0.036
Traverse (1st order) 44 sites	Min	0.043	−0.126	−0.103	0.006	0.020
	Max	0.126	0.269	0.248	0.122	0.136
	Mean	0.087	0.060	0.066	0.064	0.067
	Std dev.	0.019	0.112	0.107	0.027	0.032

The results presented in Table 2 indicate that 4th release GOCE-based GGMs are distinctly superior with respect to their previous 3rd release. This confirms the subsequent accuracy improvement of GOCE-based GGMs with increasing amount of GOCE observations data used in developing GOCE-based GGMs over the study area

(Godah and Krynski 2012, 2013a). As an example, Figure 4 illustrates the improvement in the standard deviation of differences between height anomalies obtained from GNSS/levelling control traverse data (184 stations) and the corresponding ones obtained from the 1st–4th release TIM GGM.
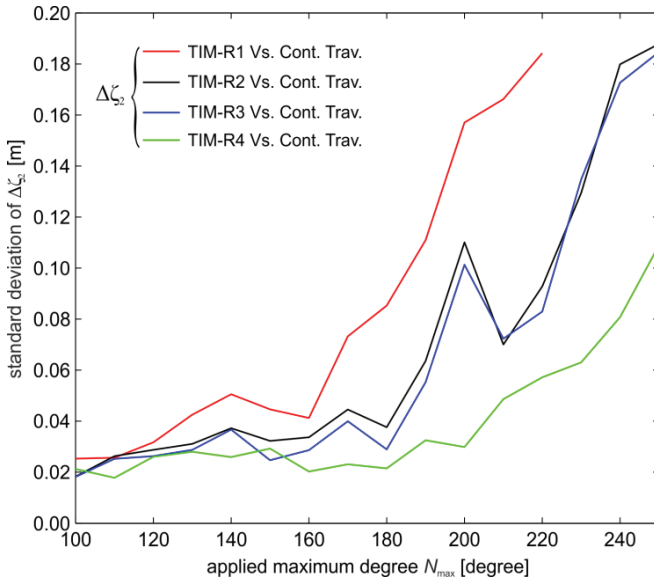

Fig. 4. Standard deviation of differences between height anomalies obtained from the GNSS/levelling control traverse data (184 stations) and from TIM-R*y* GGMs (where *y* denotes the 1st, 2nd, 3rd and 4th release) extended with the EGM08 coefficients (*the applied maximum degree* N_{max} = 100, 110, 120, ..., 250, 260)

The standard deviations of differences between height anomalies obtained from GNSS/levelling data and the corresponding ones determined from 4th release GOCE-based GGMs truncated at d/o 200 extended with the coefficients of the EGM08 from d/o 201 to 2190 range from 2.7 to 3.9 cm. The comparison of these standard deviations of differences with their cumulative error specified in Section 2.2 indicates that the estimated cumulated error of the TIM-R4 GGM seems realistic, while the one of the DIR-R4 GGM seems too optimistic. The EGM08 fits best to the GNSS/levelling data. This is due to the fact this model was fed with high quality terrestrial gravity data from the investigated area and their contribution to the model covers also the frequency band below d/o 200. When comparing the results for the EGM08 with those from GOCE-based GGMs shown in Table 2, it should be noted that the statistics for height anomalies differences $\Delta\zeta_2$ are also a subject of errors contained in the compensated medium/short wavelength gravity signal when using GOCE-based GGMs. However, regarding the obtained results, the TIM-R4 GGM has shown slightly better fit to GNSS/levelling data. Thus, the TIM-R4 spherical harmonic coefficients

truncated to d/o 200 have been chosen to recover the long-wavelength component of the gravimetric quasigeoid model over the study area.

4. The gravimetric quasigeoid model and its accuracy assessment

In this work, the gravimetric quasigeoid model QGM$_{Tim-R4+Terr}$ based on the TIM-R4 GGM truncated at d/o 200 and Faye gravity anomalies specified in section 2.1 has been developed over the study area using the least squares collocation (LSC) method (Moritz, 1980) and RCR procedure (see e.g. Torge and Muller, 2012). The GRAVSOFT package (Tscherning et al., 1992) has been used for this purpose. In particular, GEOCOL, TC, COVFIT and EMPCOV programs were applied. The main computation steps and their results are given in the sections 4.1 and 4.2. The accuracy of the developed gravimetric quasigeoid model is assessed in the section 4.3.

4.1. Calculation of residual gravity anomalies

The residual gravity anomalies Δg_{res} have been obtained from Faye gravity anomalies Δg_{Faye} (see section 2.1) after removing the long-wavelength component Δg_{GGM} of the Earth gravity field calculated from the TIM-R4 GGM truncated at d/o 200 (Torge and Mueller, 2012)

$$\Delta g_{res} = \Delta g_{Faye} - \Delta g_{GGM} \tag{7}$$

where

$$\Delta g_{GGM} = \frac{GM}{a^2} \sum_{n=2}^{200} (n-1) \left(\frac{a}{r}\right)^{n+2} \sum_{m=0}^{n} Y_{nm}(\varphi, \lambda) \tag{8}$$

Table 3 shows the statistics of gravity anomalies and residual gravity anomalies. The removal of the long-wavelength gravity field from Faye gravity anomalies using the TIM-R4 GGM truncated at d/o 200 resulted in the substantial smoothing reflected in the reduction in both dispersion (by 23%) and standard deviation (by 43%).

Table 3. Statistics of gravity anomalies and the residual gravity data [mGal]

Statistics	Min	Max	Mean	Std dev.
Δg_{Faye}	−67.23	139.30	4.27	22.139
$\Delta g_{Faye} - \Delta g_{GGM}$	−49.29	101.20	−0.69	12.701

4.2. Computation of the gravimetric quasigeoid model

The gravimetric geoid heights N computed using the RCR procedure are expressed as

$$N = N_{GGM} + N_{res} + N_{ind} \tag{9}$$

where N_{GGM} is the reference geoid height determined from the TIM-R4 GGM truncated to d/o 200

$$N_{GGM} = \frac{GM}{a\gamma_0} \sum_{n=2}^{200} \left(\frac{a}{R}\right)^{n+1} \sum_{m=0}^{n} Y_{nm}(\varphi, \lambda) \tag{10}$$

with R being the mean radius of the Earth and γ_0 is the normal gravity on the ellipsoid, and N_{ind} is the indirect effect (Grushinsky, 1976)

$$N_{ind} \approx -\frac{\pi G \rho}{\overline{\gamma}} H_P^2 \tag{11}$$

where H_P is the height of the computation point, G is the Newtonian gravitational constant and ρ is a constant mass density ($\rho \approx 2670$ kgm^{-3}).

The residual geoid heights N_{res} are determined from the residual gravity anomalies given by Eq. (7) with the use of the LSC method (Moritz, 1980)

$$N_{res} = C_{N_{res}\Delta g_{res}} C_{\Delta g_{res}\Delta g_{res}}^{-1} \Delta g_{res} \tag{12}$$

where $C_{\Delta g_{res}\Delta g_{res}}$ is the auto-covariance matrix of observations Δg_{res}, $C_{N_{res}\Delta g_{res}}$ is the matrix of the cross-covariance between residual geoid heights N_{res} and observations Δg_{res}.

In order to evaluate N_{res} in Eq. (12), the auto-covariance function for residual gravity anomalies Δg_{res} is required to be estimated first. The estimation of this covariance function is based on the empirical covariance function (Tscherning, 2013)

$$\text{cov}_{\Delta g_{res}\Delta g_{res}}(\overline{\psi}_{jk}) = \frac{1}{N_{jk}} \sum_{j,k}^{N_{jk}} \Delta g_{res}(\varphi_j, \lambda_j) \Delta g_{res}(\varphi_k, \lambda_k) \tag{13}$$

where N_{jk} is the number of pairs for each interval

$$\overline{\psi}_{jk} - \Delta < \overline{\psi}_{jk} \leq \overline{\psi}_{jk} + \Delta \tag{14}$$

and $\overline{\psi}_{jk}$ is the spherical distance (Heiskanen and Moritz, 1967)

$$\cos \overline{\psi}_{jk} = \sin\varphi_k \sin\varphi_j + \cos\varphi_k \cos\varphi_j \cos(\lambda_j - \lambda_k) \qquad (15)$$

while Δ is chosen suitably to the resolution of gravity data available (Sanso, 2013).

In practice, modelling the covariance function means the fitting of the empirical covariance function to the analytical covariance function model and determining its parameters. The well-known analytical Tscherning/Rapp model of the covariance function for the anomalous potential (Tscherning and Rapp, 1974) was used in this study. The complete model is presented as follows (Tscherning, 2013)

$$\mathrm{cov}(T(P),T(Q)) = \alpha \sum_{n=2}^{N_{\max}} \sigma_n^2 \left(\frac{R_E^2}{r_P r_Q} \right)^{n+1} P_n(t) + \sum_{n=N_{\max}+1}^{\infty} \frac{A}{(n-1)(n-2)(n+4)} \left(\frac{R_B^2}{r_P r_Q} \right)^{n+1} P_n(t) \quad (16)$$

where R_E is the mean Earth's radius, R_B is the radius of the Bjerhammar sphere, $P_n(t) = P_n(\cos\psi_{P,Q})$ is the Legendre polynomial of the degree n with the spherical distance $\psi_{P,Q}$ between the points $P(r_P, \varphi_P, \lambda_P)$ and $Q(r_Q, \varphi_Q, \lambda_Q)$, r_P and r_Q are the radial distances of the points P and Q from the Earth's centre, σ_n is the error degree variance for the anomalous potential, A is a constant in units of $(\mathrm{m/s})^4$ and α is the scale factor of the error degree variance.

Figure 4 shows the empirical and analytical fitted covariance functions for the residual gravity anomalies Δg_{res} which exhibit a very good agreement. It reflects the homogeneity of the distribution of the used residual gravity anomalies.
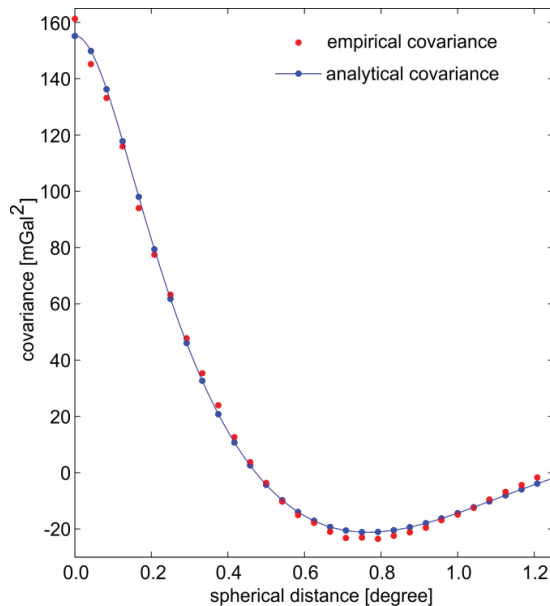

Fig. 5. Empirical and analytical fitted covariance functions for residual gravity anomalies Δg_{res}
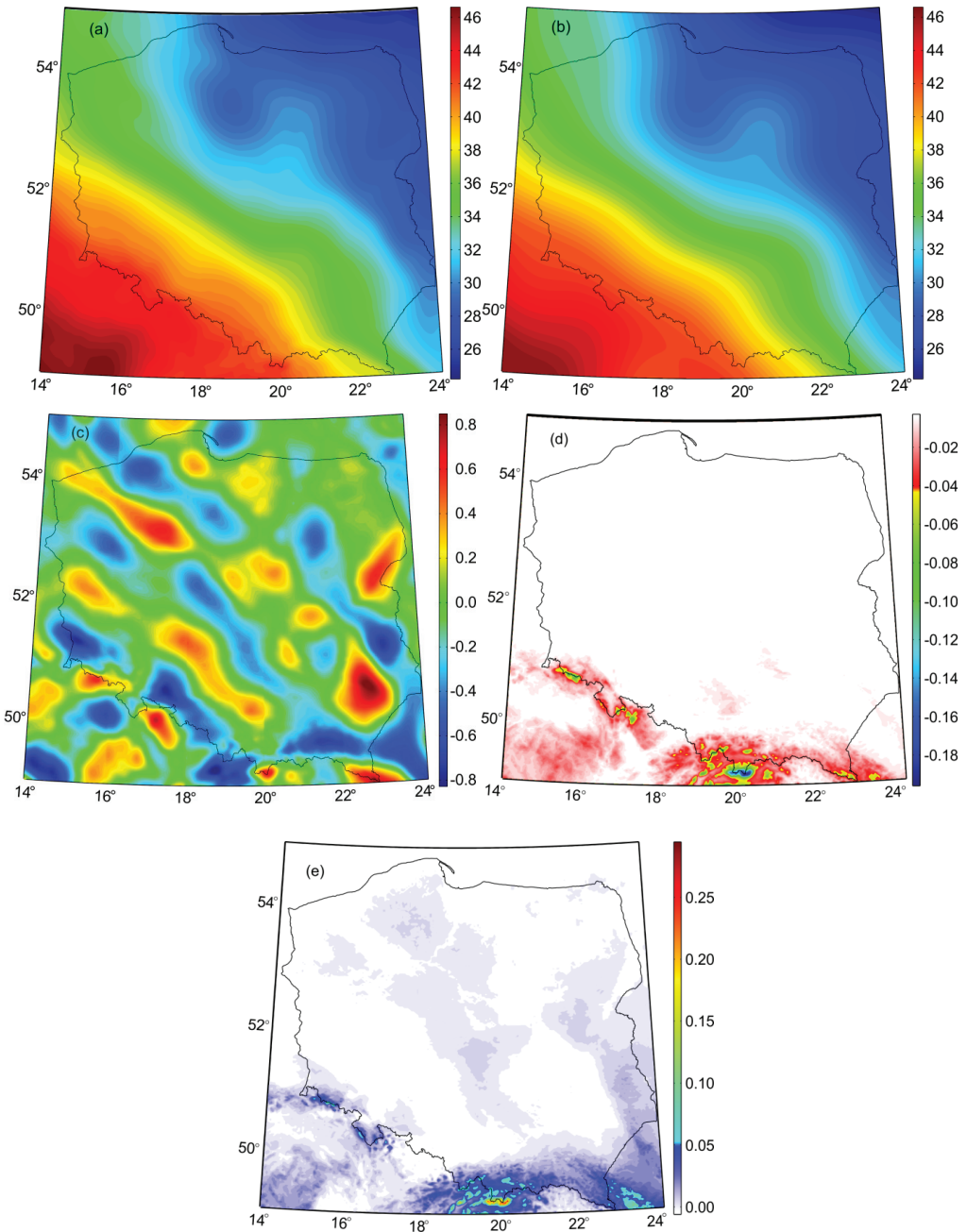
Fig. 6. (a) The gravimetric quasigeoid model QGM$_{Tim-R4+Terr}$, (b) the reference geoid model,
(c) the residual gravimetric geoid model, (d) the indirect effect, and (e) the geoid-to-quasigeoid
separation [m]

Covariance function with parameters estimated through the fitting procedure, i.e. $R_B = -6.49046$ km, the variance of gravity anomalies at zero altitude of 160.75 mGal^2, the error degree variance scale factor of 7.8994 and $N_{max} = 200$, has been used to calculate the residual geoid heights N_{res} on a 1.5'×3' grid from the residual gravity anomalies Δg_{res}.

The gravimetric geoid model has been computed by combining the reference geoid heights, the residual geoid heights and the indirect effect using Eq. (9). Finally, the gravimetric geoid model has been converted to a gravimetric quasigeoid model as follows (Heiskanen and Moritz, 1967)

$$\zeta - N = -\frac{\Delta g_B}{\bar{\gamma}} H \qquad (17)$$

where H and Δg_B are the height and Bouguer anomaly at the computation point, respectively, and $\bar{\gamma}$ is the mean normal gravity.

Figure 6 depicts the gravimetric quasigeoid model QGM$_{Tim-R4+Terr}$, the reference geoid model obtained from the TIM-R4 GGM, the residual gravimetric geoid model, the indirect effects on the geoid and the geoid-to-quasigeoid separation. The major contribution to the gravimetric quasigeoid (Fig. 6a) comes from the reference geoid model (Fig. 6b). The residual geoid heights illustrated in Figure 6c representing the medium/short wavelength gravity signal from the terrestrial gravity data starting from d/o 201 range from –80 cm to +80 cm. The indirect effect (Fig. 6d) ranges from zero in flat areas to 20 cm in the mountains (the southern area of Poland) with an average of 0.4 cm over the study area, while the geoid-to-quasigeoid separation (Fig. 6e) is at the level of ±1 cm for the majority of the territory of Poland and reaches up to 29 cm in the mountains.

4.3. Accuracy assessment of the gravimetric quasigeoid model

Height anomalies $\zeta_{TIM-R4+Terr}$ at the sites of the POLREF and EUVN networks as well as the GNSS/levelling control traverse (Fig. 2) have been obtained from the QGM$_{Tim-R4+Terr}$. The differences $\Delta\zeta_3$ between those height anomalies and the corresponding ones obtained from GNSS/levelling data

$$\Delta\zeta_3 = \zeta_{GNSS/levelling} - \zeta_{TIM-R4+Terr} \qquad (18)$$

were computed to evaluate the quality of the gravimetric quasigeoid model. In order to verify the obtained differences $\Delta\zeta_3$ as well as to estimate the influence of the omitted gravity signal beyond d/o 2190 for GNSS/levelling dataset used, the gravimetric quasigeoid model QGM$_{EGM08+Terr}$ has been developed for the area investigated. It was determined on the basis of the same Faye gravity anomalies using LSC method and following the same computation steps as in the case of developing the QGM$_{Tim-R4+Terr}$ and the EGM08 up to d/o 2190. The differences $\Delta\zeta_4$ between height

anomalies obtained from GNSS/levelling data and the corresponding ones $\zeta_{EGM08+Terr}$ obtained from the gravimetric quasigeoid model $QGM_{EGM08+Terr}$

$$\Delta\zeta_4 = \zeta_{GNSS/levelling} - \zeta_{EGM08+Terr} \qquad (18)$$

were calculated. Graphical representations of the obtained differences $\Delta\zeta_3$ and $\Delta\zeta_4$ are depicted in Figures 6 and 7 and their statistics are given in Table 4.

Table. 4. Statistics of differences $\Delta\zeta_3$ and $\Delta\zeta_4$ between height anomalies obtained from GNSS/levelling data and the corresponding ones obtained from combined gravimetric quasigeoid models [m]

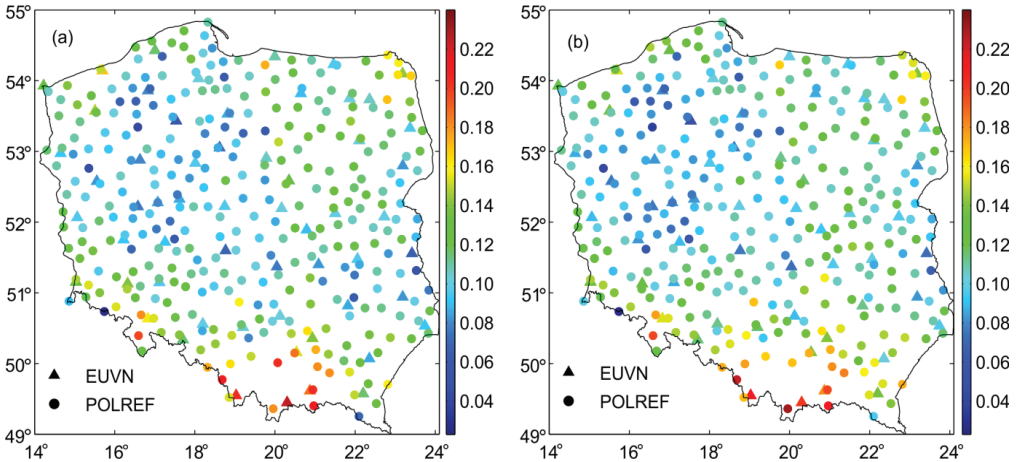| Statistics	$\Delta\zeta_3 = \zeta_{GNSS/levelling} - \zeta_{TIM-R4+Terr}$				$\Delta\zeta_4 = \zeta_{GNSS/levelling} - \zeta_{EGM08+Terr}$			
	POLREF	EUVN	TRAVERSE		POLREF	EUVN	TRAVERSE	
	315 sites	58 sites	44 sites (1st order)	184 sites (1st+2nd order)	315 sites	58 sites	44 sites (1st order)	184 sites (1st+2nd order)
Min	0.023	0.057	0.065	0.040	0.020	0.047	0.069	0.037
Max	0.215	0.224	0.152	0.153	0.231	0.215	0.146	0.153
Mean	0.113	0.107	0.109	0.098	0.114	0.106	0.105	0.095
Std dev.	0.028	0.033	0.021	0.024	0.029	0.031	0.019	0.021

Fig. 7. Differences between height anomalies obtained from GNSS/levelling data at the POLREF and EUVN sites and the corresponding ones obtained from gravimetric quasigeoid models (a) $\Delta\zeta_3$, and (b) $\Delta\zeta_4$ [m]
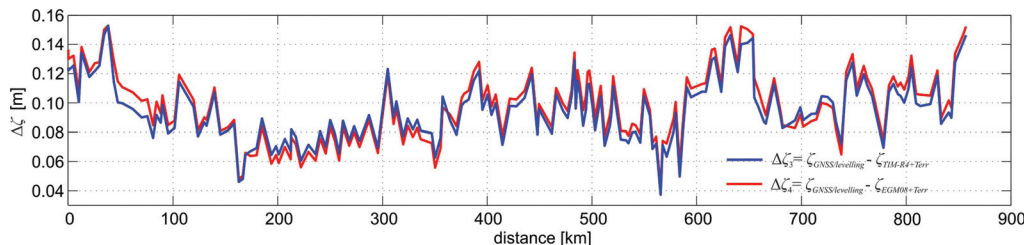
Fig. 8. Differences $\Delta\zeta_3$ and $\Delta\zeta_4$ between height anomalies obtained from GNSS/levelling data and the corresponding ones obtained from gravimetric quasigeoid models at sites of the control traverse

In spite of the spatial resolution inconsistency of GGMs used when developing the gravimetric quasigeoid models $QGM_{Tim-R4+Terr}$ and $QGM_{EGM08+Terr}$, the statistics in Table 4 exhibit quite similar performance for both gravimetric quasigeoid models developed. The fit of those gravimetric quasigeoid models to GNSS/levelling data in terms of standard deviation of differences ranges from 2.1 to 3.3 cm and from 1.9 to 3.1 cm for the $QGM_{Tim-R4+Terr}$ and the $QGM_{EGM08+Terr}$, respectively. The distribution of their differences with respect to GNSS/levelling data is very similar (see Figs. 6 and 7). This indicates that the TIM-R4 GGM is adequate for modelling the long-wavelength component (e.g. up to d/o 200) of the geoid over the area with high performance of the EGM08 such as the area of Poland.

The comparison of statistics presented in Tables 2 and 4 shows that the fit of $QGM_{Tim-R4+Terr}$ to GNSS/levelling data in terms of standard deviations of height anomalies differences have clearly improved (about 3–9 mm) with regard to the corresponding fit obtained from the TIM-R4 GGM extended with the EGM08 coefficients. It may reveal that extending GOCE-based GGMs with the EGM08 coefficients is not correct in a theoretical sense, because correlations of the coefficients of those GGMs have not been taken into account. These correlations can be obtained from the degree variances and error degree variances of those models. It has been shown that this effect can be substantially reduced by using terrestrial gravity data instead of extending a GOCE-based GGM with the EGM08 coefficients. On the other hand, the fits of the $QGM_{EGM08+Terr}$ and the EGM08 to GNSS/levelling data in terms of the standard deviation of height anomalies differences are almost the same. The dispersion of their differences is below 3 mm. It may imply that the contribution of terrestrial gravity data in a spectral band exceeding d/o 2190, to the determination of quasigeoid model, estimated with respect to GNSS/levelling dataset used, can be regarded merely as a noise.

5. Summary and Conclusions

In the paper, the 4th release GOCE-based GGMs based on time-wise (TIM-R4 GGM) and direct (DIR-R4 GGM) solutions were evaluated with the use of three sets of precise GNSS/levelling data from the area of Poland. Consistent results have been observed for both models. The fits of height anomalies determined from these models to GNSS/levelling data in terms of standard deviations of differences are at the level of 11–20 cm at the maximum d/o considered (260), and 22–26 cm at d/o 200. It reveals that GOCE-based GGMs contain a valuable information on the gravity field also in the spectral band from d/o 200 to 260. When compensating the medium/short wavelength (from d/o $N_{max}+1$ to 2190) gravity signal in GOCE-based GGMs truncated to d/o N_{max} using the EGM08, the standard deviations of height anomalies differences for N_{max} from d/o 100 to 200 are practically constant below 4 cm, which indicates the superiority of GOCE-based GGMs over the existing satellite-only GGMs in the spectral band d/o 100-200. From d/o 200 onward those standard deviations of height anomalies differences start increasing. They can reach 11–12 cm at the maximum d/o considered (260) which is because the noise contained in GOCE data becomes higher, growing in the spectral band from d/o 200 to 260. The fit of 4th release GOCE-based GGMs truncated to d/o 200 and extended with the EGM08 coefficients to GNSS/levelling data ranges from 2.7 to 3.9 cm in terms of standard deviation of height anomalies differences. This result is consistent with those published by Yi and Rummel (2014) and Gruber et al. (2013). It exhibits clear improvement (60–70%) with respect to the corresponding results obtained from the previous 3rd release of GOCE-based GGMs. At d/o 200 the TIM-R4 GGM shows slightly better performance as compared to the DIR-R4 GGM.

The assessed accuracy of height anomalies obtained from the gravimetric quasigeoid model based on the combination of TIM-R4 GGM truncated to d/o 200 and Faye gravity anomalies is at the level of 2.1–3.3 cm in terms of standard deviations of height anomalies differences. It indicates an improvement by 3–9 mm compared to the one obtained from GOCE-based GGMs 4th release extended with the EGM08 coefficients. The obtained result from a highly accurate gravimetric quasigeoid model based on the same Faye gravity anomalies and the EGM08 up to d/o 2190 has indicated that the contribution of terrestrial gravity data to the very short-wavelength component (from d/o 2190 to about d/o 3600 which corresponds to the resolution of Faye gravity anomalies used) of the gravity field is negligible for the GNSS/levelling dataset used.

The analysis of the accuracy of height anomalies obtained from the resulting gravimetric quasigeoid models indicates that the gravimetric quasigeoid model based on the TIM-R4 GGM is slightly worse than the one based on the EGM08 for the area of Poland. It also reveals that the GOCE data cannot improve the modelling of the gravimetric quasigeoid for the areas with high performance of the EGM08, e.g. Poland, but such areas could be suitable to evaluate GOCE-based GGMs, in particular to estimate the accuracy of height anomalies obtained from those models.

On the other hand, when 1–2 cm accuracy of geoid at d/o 200 obtained from GOCE mission is achieved, the GOCE-based GGMs might be considered in such areas as an independent tool to assess the accuracy of regional/local geoid/quasigeoid models as well as to detect outliers among GNSS/levelling data.

The results obtained also indicate that extending GOCE-based GGMs with the EGM08 coefficients by simple merging of the spectra seems not to be recommended for exact accuracy assessment of height anomalies determined from 4th release GOCE-based GGMs since the correlations of the coefficients of both the EGM08 and GOCE-based GGMs have not been considered when combining those models. However, recovering the medium/short wavelength gravity signal using accurate terrestrial gravity data provides more reliable accuracy assessment for height anomalies obtained from 4th release GOCE-based GGMs.

Overall, an accuracy of 2.1–3.3 cm of height anomalies obtained from 4th release GOCE-based GGMs at maximum d/o 200 could be expected at any place on the Earth, except the poles and their adjacent areas that were not flown over by the GOCE satellite. This is because the GOCE-based GGMs are completely independent of the local terrestrial data and expected to provide homogeneous and uniform information of the Earth's gravity field. The assessed accuracy may also show that the cumulative error of the TIM-R4 GGM (3.2 cm) seems more realistic than the one of the DIR-R4 GGM (1.0 cm). It should be noted that, the assessed accuracy of those GGMs is also a subject of the accuracy of gravity signal beyond d/o 200 and the error of GNSS/levelling data used in the analysis. The 4th release GOCE-based GGMs will considerably improve the determination of height anomalies in the areas where the EGM08 performs poorly such as Africa, South America and South-East Asia. However, the results obtained can still be further verified using similar research in different areas of the world. It also suggests future investigations concerning the influence of the reference gravity field in modelling the gravimetric quasigeoid using GOCE-based GGMs and the EGM08 at the same spectral band.

Acknowledgments

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Oszacowanie dokładności modeli geopotencjału wyznaczonych na podstawie danych z misji GOCE oraz ich wykorzystanie do modelowania grawimetrycznej quasigeoidy – opracowanie dla obszaru Polski

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Streszczenie

Misja GOCE (Gravity Field and Steady-State Ocean Circulation Explorer) przyczyniła się do znacznego poprawienia znajomości pola siły ciężkości Ziemi. W artykule przedstawiono wyniki oszacowania dokładności anomalii wysokości, wyznaczonych z globalnych modeli geopotencjału opracowanych na podstawie blisko 27 miesięcy pomiarów z satelitarnej misji gradiometrycznej GOCE. Do oszacowania wykorzystano trzy zbiory dokładnych danych satelitarno-niwelacyjnych z obszaru Polski. Omówiono wyniki wpasowania wartości anomalii wysokości otrzymanych z czwartej wersji globalnych modeli geopotencjału wyznaczonych na podstawie danych misji GOCE do danych satelitarno-niwelacyjnych oraz porównano je z wynikami odpowiedniego wpasowania trzeciej wersji globalnych modeli geopotencjału otrzymanych z GOCE oraz z modelu EGM08.

Ponadto, wykorzystując wysokorozdzielczy zbiór grawimetrycznych anomalii Faye'a, wyznaczono dla obszaru Polski dwa grawimetryczne modele quasigeoidy o wysokiej dokładności. W pierwszym przypadku jako model referencyjny użyto model utworzony na podstawie danych z misji GOCE, w drugim – model EGM08. Wygenerowane modele quasigeoidy porównano z danymi satelitarno-niwelacyjnymi oraz oszacowano ich dokładność. Omówiono przydatność otrzymanych na podstawie danych z misji GOCE globalnych modeli geopotencjału do odtworzenia długofalowego sygnału grawimetrycznego przy modelowaniu grawimetrycznej quasigeoidy.